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TREATISE

ON Florian Cajori

ARITHMETIC,

IN

THEORY AND PRACTICE.

WITH

AN APPENDIX,

CONTAINING AN INTRODUCTION TO MENSURATION.

BY JAMES THOMSON, LL.D.

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INTRODUCTION TO MODERN GEOGRAPHY; ETC.

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ADVERTISEMENT TO THE SECOND EDITION.

THE object of the following Treatise, is to present a full and regular course of whatever is useful in Arithmetic. With this view, rules are given for performing all the requisite operations, and are illustrated by many examples; and numerous exercises are prescribed, to afford the pupil that practice which alone can produce expertness and accuracy in the management of numbers.

The Definitions and Rules, which it may perhaps be proper for the learner to commit to memory, are in the largest type employed in the work; the Examples and Exercises, and the principal illustrations, are in a character somewhat smaller; the less important illustrations and remarks are in a type still smaller, and may perhaps be omitted by the younger pupil; and the notes contain miscellaneous information, which may often be interesting to the reader.

The reasons of the rules and operations are explained, not in strict, formal demonstrations, but generally by simple and easy illustrations of particular cases and examples; and it is hoped, that the subject will thus be rendered as intelligible and attractive as possible. This part of Arithmetic is too generally neglected, both in treatises on the subject, and in teaching: and thus one of the principal divisions of Mathematical Science is converted into a mere practical art; and what is calculated to call forth and improve the reasoning powers of the pupil, is degraded into a dry exercise of memory.

Of the Examples and Exercises, some are proposed in purely abstract terms, being intended merely to afford practice to the learner in the rules and modes of calculation. To these are subjoined, in those parts of the work in which it could be conveniently done, other questions, which will not only afford the pupil farther exercise on the rules which precede them, but will also furnish him with many important facts in Commerce, Geography, Astronomy, Chronology, Chemistry, and other departments of knowledge. As the information contained in these questions has been all derived from authentic sources, its correctness may be depended on; and it is hoped, that what is thus presented may excite in the young reader a desire to enrich his mind, by the acquisition of farther information of a similar nature.

Some subjects are omitted, which in several works on Arithmetic are treated at considerable length. Such is *Barter*,—an application of the Rule of Proportion, which is of scarcely any use as a part of Mercantile Arithmetic. Neither has the method of calculating *Annuities at Simple Interest* been introduced; as it is unjust in principle, and, in real transactions, all calculations respecting annuities are made according to the principles of *Compound Interest*. Many applications of the rules to the circumstances of bodies in motion, and to other subjects in Natural Philosophy, which have been given in works on this science, have also been omitted, as they require a degree of knowledge to render them intelligible and useful, which is to be acquired only by the study of a considerable course of mathematical and physical science.

Instead of these, however, other subjects have been introduced, which are little known, but which cannot fail to be interesting to the more advanced pupil. Of this kind are Continued Fractions, and the Theory of Arithmetical Scales. The principal and most important parts of the work, also, have been explained and exemplified more largely than the rest; and in these parts a greater number of exercises have been left unwrought, for the improvement of the pupil. Thus, the Simple and Compound Rules, Proportion, Practice, Interest, and Exchange, have

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been treated at considerable length; and much care has been taken to render them as simple and easy as possible. Specimens of the most common and useful kinds of merchants' accounts, such as Invoices, Sales, Accounts Current, &c. according to modern and approved forms, are likewise given, and will be found to form useful exercises for pupils who may be intended for the counting-house.

In preparing the present edition for the press, considerable changes have been made in the manner of expressing and illustrating the rules, and in various other respects. These have partly resulted from the Author's own experience, and have partly been suggested by several teachers of the first respectability in this country, who have kindly favoured him with hints for the improvement of the work. Some additional tables have likewise been introduced, and a tract given on Mensuration, which cannot fail to be useful to the majority of learners. At the desire also of many teachers, the answers are, in this edition, annexed to all the questions, except a few of the simplest, which are of such a kind, that the idle pupil might copy the answers, without working the exercises: the answers to these are given at the end of the book. With respect to the entire work, no exertion has been spared to attain neatness of appearance; and, above all, to secure that accuracy which is so desirable both to the teacher and the pupil, in every work which contains calculations.

As so much care and labour have been employed in improving this edition, it has been STEREOTYPED; that is, plates have been cast for the entire work, which will in future be employed, instead of moveable types, in printing it. This mode of printing prevents the possibility of errors creeping into future impressions; and as any mistakes which may escape observation in a first impression, can be corrected in the plates, it affords the means of ultimately rendering the typography of a book completely perfect.

College, Belfast, Jan. 1, 1825.

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A

TREATISE ON ARITHMETIC,

IN

THEORY AND PRACTICE.

NOTATION AND NUMERATION.

ARITHMETIC is a science which explains the properties, and shows the uses of numbers.

Unity, or a *unit*, is the number one. Every other number is an assemblage of units.*

For facilitating the management of numbers in Arithmetic, they are generally expressed by signs or characters.

The method of expressing numbers by characters, is termed *Notation*.

The method of discovering the values of numbers already expressed by characters, is called *Numeration*.

In modern arithmetic, all numbers are expressed by means of ten characters: the cipher, or zero, 0, which has no value; and the nine significant figures, or digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, which denote respectively the numbers one, two, three, four, five, six, seven, eight, nine.

When any of these figures stands by itself, or when it is followed by no other figure, it expresses merely its simple value; but when it is followed by one figure, it signifies ten times its simple value; when by two, one hundred times; when by three, one thousand times; and so on, by a tenfold increase for each figure that follows it.—The increased value thus denoted by a figure in consequence of its position, is called its *local* value.

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* This definition, though it might appear otherwise, is applicable to fractions as well as to whole numbers. Thus, the fraction $\frac{7}{10}$ is an assemblage of seven units, each of which is one-tenth of the integer. The use of the term *unity* to denote the number *one*, appears to be improper; if it be, however, it has the sanction of long custom. Some late writers, probably to avoid this use of the word, employ in the same sense the term *unit*, but without the indefinite article; perhaps because it is not used before *two*, *three*, *ten*, &c. But, besides the sanction of custom, may not the article be used before it with the same propriety as before the words *hundred*, *thousand*, *million*, &c.? It seems to be as awkward, and as incorrect, to talk of increasing a number by *unit*, as of increasing it by *hundred* or *thousand*.

Thus the figure *5*, when it stands by itself, or is followed by no other figure, denotes simply five; but when placed to the left of one figure, it expresses ten times five, or fifty; when to the left of two figures, it expresses ten times fifty, or five hundred; when to the left of three, ten times five hundred, or five thousand, &c.; as in the number *5555*. In like manner, in the number 7854, the 4 signifies simply four units, or four; the *5*, five tens, or fifty; the 8, eight hundred; and the 7, seven thousand.—The names of the local values of figures will be known from the following table:—

NUMERATION TABLE.

I.	Units	1 Units 2 Tens 3 Hundreds
II.	Thousands	4 Units 5 Tens 6 Hundreds
III.	Millions	7 Units 8 Tens 9 Hundreds
IV.	Billions	10 Units 11 Tens 12 Hundreds
V.	Trillions	13 Units 14 Tens 15 Hundreds
VI.	Quadrillions	16 Units 17 Tens 18 Hundreds
&c.		&c.

From this table it appears, that if a line of figures be divided into periods of three figures each, commencing at the right hand, the first period will contain units, the second thousands, the third millions, &c.; and it is usual and convenient thus to divide the figures by which large numbers are expressed, for the purpose of facilitating their numeration. The periods succeeding those contained in the table, are *quintillions*, *sextillions*, *septillions*, *octillions*, and *nonillions*; and analogical names might be formed for the still higher periods. Those already given, however, are more than sufficient to express any number which it is ever necessary to designate in language. The local value of any figure used in expressing a number, is at once discovered from this table. Thus, 6 in the eighth place from the right hand, expresses six tens of millions, or sixty millions; and, conversely, sixty millions will be expressed by the figure 6 in the eighth place.*

* This method of dividing lines of figures into periods, and of naming those periods, is that which is used by the French and Italians. It is strongly recommended by its simplicity and elegance; and has been adopted in the treatise on Arithmetic by Mr. Anderson, in the Edinburgh Encyclopedia; and in Professor Leslie's "Philosophy of Arithmetic." In other English works, the periods are made to consist of six figures each, and have the same names as those in the table given above, except thousands, for which there is not a distinct period. The two methods agree as far as hundreds of millions, and it is rarely necessary to name larger numbers. For the use of those who pre-

The cipher, or zero, having no value, is used in combinations of figures, to fill places where no value is to be expressed, and thus to make the other figures occupy those places in which they will express the intended values. Thus the figures 365, combined in this order, denote three hundred and sixty-five; but the expression 306050, which contains the same significant figures, means three hundred thousand, no tens or thousands, six thousand, no hundreds, five tens, and no units; or three hundred and six thousand, and fifty.—From these principles we have the following rule:—

To express in Words the Numbers denoted by Lines of Figures.

RULE (1.) Commencing at the right hand side, divide the given figures into periods of three figures each, till not more than three remain. (2.) Then the first period towards the right hand contains units or ones; the second, thousands; the third millions, &c. as in the Numeration Table: and therefore commencing at the left side, annex to the value expressed by the figures of each period, except that of the units, the name of the period.

Thus, the expression 37053907, becomes by division into periods, 37,053,907, and is read *thirty seven millions, fifty three thousand, nine*

---

for the English method, the following table is subjoined; and the answers of the Exercises are given according to both methods at the end of the work. It is scarcely necessary to say, that the rules and directions given in the text will be applicable in this method, if the periods be made to consist of six figures each instead of three, and if the second period be called millions, the third billions, &c. as in the following table:—

COMMON NUMERATION TABLE.

I. Units	1	Units
	2	Tens
	3	Hundreds
	4	Thousands
	5	Tens of Thousands
	6	Hundreds of Thousands
II. Millions	7	Units
	8	Tens
	9	Hundreds
	10	Thousands
	11	Tens of Thousands
	12	Hundreds of Thousands
III. Billions	13	Units
	14	Tens
	15	Hundreds
	16	Thousands
	17	Tens of Thousands
	18	Hundreds of Thousands
IV. Trillions	19	Units
	20	Tens
	21	Hundreds
	22	Thousands
	23	Tens of Thousands
	24	Hundreds of Thousands
V. Quadrillions	25	Units
	26	Tens
	27	Hundreds
	28	Thousands
	29	Tens of Thousands
	30	Hundreds of Thousands
VI. Quintillions	31	Units
	32	Tens
	33	Hundreds
	34	Thousands
	35	Tens of Thousands
	36	Hundreds of Thousands
&c.		&c.

hundred and *seven*, the term *units* or *ones* at the last being omitted. By practice the pupil will soon find it unnecessary to divide into periods any lines of figures except those of considerable magnitude.

Exercises in Numeration.

Write down in words, or name, the numbers signified by the following expressions:—

Ex. 1. 24	Ex. 4. 1000	Ex. 7. 9790	Ex. 10. 4055070
2. 144	5. 1728	8. 37048	11. 300405
3. 365	6. 2240	9. 30009	12. 79503046

Ex. 13. 8005600480	Ex. 22. 40657200
14. 557290000	23. 100000001000
15. 680000042	24. 60606060700707
16. 93090093	25. 102030405060708
17. 113355	26. 901001101201301
18. 785398	27. 200030040538
19. 7030462	28. 738207600051926 45
20. 24902490	29. 10001000010000100
21. 9003008005	30. 40050060700809000

Ex. 31. 79462890064003000 47264791

To express Numbers by Figures.

Rule (1.) Make a sufficient number of ciphers or dots, and divide them into periods; (2.) Then, commencing at the left, place in their proper positions beneath the dots or ciphers, the significant figures necessary for expressing the proposed number. (3.) If any places remain unoccupied, let them be filled with ciphers.

Thus, the method of expressing the number, two hundred and five millons, twenty thousand, seven hundred and nine, will be found in the following manner:

000,000,000 . . ., . . ., . . ,

2 5 2 7 9, or 2 5 2 7 9, and thence by filling the unoccupied places, 205,020,709. By practice the learner will soon be able, in most cases, to dispense with the dots or ciphers.

Exercises in Notation.

Express the following numbers in figures:

Ex. 1. Fifty-two.
2. Three hundred and thirteen.
3. Five hundred and four.
4. One thousand and twenty-four.
5. Two thousand and forty-eight.
6. One thousand, eight hundred and fifteen.
7. Seven thousand, eight hundred, and fifty-four.
8. Three thousand and eight.

Ex. 9. Five thousand and seventy.
10. Four thousand, five hundred, and four.
11. Twenty thousand and eighty-four.
12. Six hundred and fifty thousand, and ninety.
13. Seven millions, seven thousand, and ten.
14. Sixty-four millions, three hundred.
15. Eleven millions, two thousand.
16. One hundred and ten millions, and twenty thousand.
17. One billion, ten millions, two hundred thousand.
18. One trillion, one hundred billions, two hundred millions.
19. One million, and fifty thousand.
20. One billion, two hundred thousand.
21. One trillion, seven hundred thousand.
22. One quadrillion, and nineteen millions.
23. Seventy billions, ten thousand, and eighty-eight.
24. Nine hundred billions, sixty-eight millions, and twenty.
25. The following numbers express the distances of the principal primary planets from the sun, in miles;—express them in figures:—Mercury, thirty-seven millions; Venus, sixty-nine millions; the Earth, ninety-five millions; Mars, one hundred and forty-five millions; Jupiter, four nundred and ninety-four millions; Saturn, nine hundred and seven millions; and the Georgium Sidus, or Herschel, one thousand eight hundred and ten millions.*

Such is the facility with which large numbers are expressed, both by figures and in language, that we have generally a very limited and inadequate conception of their real magnitudes. The following considerations may, perhaps, assist in enlarging the ideas of the pupil, on this subject:—To count a million, at one per second, would require between twenty-three and twenty-four days of twelve hours each. The seconds in six thousand years, are less than one-fifth of a trillion.† A quadrillion‡ of leaves of paper, each the two-hundredth part of an inch in thickness, would form a pile, the height of which would be three hundred and thirty times the moon's distance from the earth. Let it also be remembered, that a million is equal to a thousand repeated a thousand times; and a billion§ equal to a million repeated a thousand times.

In the ancient Roman notation, I signified one, V five, X ten, L fifty, and C one hundred. To these characters were added, at a later period, D, signifying five hundred, and M, one thousand. When any charac-

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* In several of the subsequent rules, examples are introduced which are calculated to exercise the pupil in notation. To familiarize him with numeration, the teacher should frequently require him to read off in words, the answers of the exercises which he performs.

† A billion, in the common notation.

‡ A thousand billions, in the common notation.

§ In the other notation a billion is equal to a million repeated a million of times.

ter was followed by another of equal, or of less value, the compound value was equal to the simple values of *both taken together*; but when a character preceded one of greater value, both together expressed a value equal to the *difference* of their simple values. Thus, II expressed two; XI, eleven, and IX, nine; CX, one hundred and ten, and XC, ninety. We find also IↃ put for 500; and by every such Ↄ annexed, the value is made ten times as great. Thus, IↃↃ signifies 5000; IↃↃↃ, 50,000, &c. CIↃ was also used to express 1000, and the prefixing of C and the annexing of Ↄ, increased its value ten times. Thus, CCIↃↃ signified 10,000; CCCIↃↃↃ, 100,000, &c. A line drawn over the top of a letter, made it signify as many thousands as the letter itself expressed units. Thus, $\overline{V}$ expressed 5000; $\overline{C}$ 100,000, &c. The following table, together with the preceding observations, will give an adequate idea of the Roman notation:

I	1	IX	9	LXXX . .	80
II	2	X	10	XC . . .	90
III	3	XX	20	C	100
IV, or IIII .	4	XXX	30	D, or IↃ .	500
V	5	XL	40	M, or CIↃ .	1000
VI	6	L	50	MM, or $\overline{II}$.	2000
VII	7	LX	60	IↃↃ, or $\overline{V}$.	5000
VIII	8	LXX	70		

MDCCCXXIV, or CIↃIↃCCCXXIV, 1824.

SIMPLE ADDITION.

THE object of ADDITION is to find the number which is equivalent to two or more given numbers taken together.

The number which is equivalent to two or more numbers taken together, is called their SUM.

When the given numbers are all of the same denomination, as all yards, or all gallons, the operation is termed SIMPLE ADDITION.

When the numbers to be added express quantities of the same kind, but of different denominations, the operation is termed COMPOUND ADDITION.

To add Quantities of the same Denomination.

RULE (1.) Place the numbers so that units may stand under units, tens under tens, &c. (2.) Find the sum of the column of units, set down the last figure of it below that column, and carry to the next the number expressed by the remaining figure, or figures, if there be any. (3.) Proceed as before with the remaining columns, and at the last column set down the entire sum.

Thus, to add together 9468, 2956, and 79, let them be set as in the margin, and proceed thus:—9 and 6 are 15, and 8 are 23; set down 3, and carry 2 to the column of tens. Then 2 and 7 are 9, and 5 are 14, and 6 are 20; set down a cipher, and carry 2. 2 and 9 are 11, and 4 are 15; set down 5, and carry 1. 1 and 2 are 3, and 9 are 12; set down 12, and the sum, or answer required, is twelve thousand, five hundred, and three.

9468	
2956	
79	
12503,	sum

The sign +, commonly called *plus*, is employed in arithmetic and other parts of mathematics, to signify that the quantities between which it is placed, are to be added together; and the sign =, called *the sign of equality*, is used to denote, that the quantities between which it stands, are equal to one another. Thus, the expression, 12+9=21, means, that 12 and 9 added together, are equal to 21; or that the sum of 12 and 9 is 21.

Reason of the Rule.

The rule for performing addition depends on the nature of notation, and on the obvious principle, that *the whole is equal to the sum of all its parts.* By placing units under units, tens under tens, &c. we are enabled the more easily to add together the figures of the same local value; and *one* is carried for every *ten*, because, by the nature of notation, *ten* in any column is equivalent only to *one* in the column immediately to the left of it. We commence with the units merely for the convenience of carrying to the next columns. Thus, in the preceding example, the sum of the column of units is 23; and therefore, after setting down 3, we have 20 remaining. But, by the nature of notation, 2 in the next column is equal to 20 in this; and therefore we carry only 2.

Some teachers may, perhaps, consider it proper to make pupils commit the following table to memory:—

ADDITION TABLE.

2 and	2 and	3 and	4 and	5 and	6 and	7 and
2 = 4	9 = 11	7 = 10	6 = 10	6 = 11	7 = 13	9 = 16
3 = 5		8 = 11	7 = 11	7 = 12	8 = 14	
4 = 6	3 and	9 = 12	8 = 12	8 = 13	9 = 15	8 and
5 = 7	3 = 6		9 = 13	9 = 14		8 = 16
6 = 8	4 = 7	4 and			7 and	9 = 17
7 = 9	5 = 8	4 = 8	5 and	6 and	7 = 14	9 and
8 = 10	6 = 9	5 = 9	5 = 10	6 = 12	8 = 15	9 = 18

To enable the learner to acquire accuracy and despatch in addition, it is proper to train him to add in the following manner, till he can do it with facility:—Since 6 and 6 are 12, 26 and 6 are 32: (here it should be pointed out to him, that 12 and 32 end in the same figure:) since 9 and 7 are 16, 39 and 7 are 46: since 8 and 6 are 14, 88 and 6 are 94; since 6 and 9 are 15, 16 and 9 are 25, &c.

Methods of Proof.

1. Add the several columns, according to the rule, commencing at the top and proceeding downward, and if the result be the same as was obtained by adding them upward, it may be presumed that the work is right.

2. Separate the given numbers into two or more divisions.—Find the sums of these divisions, severally, and add these partial sums together. If the last result be equal to that found by the common method, the work is right.

This will appear obvious from the following example:

	37928
	93640
	23574
	75849
Entire sum,.....	230991
Sum of the 1st division,	131568
..... 2d, ...	99423
Entire sum,.....	230991, proof.

This method may also be employed with advantage in finding the sums of large columns, instead of adding the whole at a single operation. The first method is very convenient and useful, when the columns are not very large.

3. Commencing at the left hand, add the several columns without *carrying*, and set down the full sum of each column with the units in their proper place, and the tens below the figure immediately to the left. Add together the two lines thus resulting, and if the last result agree with that obtained by the common method, it may be concluded, that both are right.

Thus, in the annexed example, the sum of the left hand column is 25, which is set down in full: the sum in the next column is 30; the cipher is set in its proper place, and 3 under the 5; and so with the rest. The sum of the two lines thus obtained, is equal to the sum found by the ordinary method.*—This method of addition might be used instead of the common method; and as it requires nothing to be carried, it might be employed with advantage when the calculator is liable to interruptions.

5946	
9738	
2697	
9868	
28249,	sum.
25029	
322	
28249,	proof

* Addition may also be proved by *casting the nines* out of each of the given numbers, as will be explained in multiplication; and then by casting the nines out of the sum of the excesses, and out of the sum of the numbers. If these latter excesses be equal, the work is generally right; otherwise, it *must* be wrong. This method however, is of little use in proving addition.

Exercises in Simple Addition.

1	2	3	4	5
3789	92864	486759	17896	259111
4236	79784	537192	570937	4174456
7483	4759	468013	784947	6880921
6047	28936	16975	9678	9911604

6. 94753 + 2847 + 793688 + 9386 + 258 + 3456.
7. 8289364 + 275224 + 6875144 + 12897 + 7650368 + 94986347 + 42682 + 3749286 + 7676.
8. 294796 + 489276 + 16759284 + 4938 + 5713245 + 3348675 + 798426 + 9482 + 39867.
9. 27515436 + 8937549 + 37246375 + 48795 + 378 + 2863487 + 864937 + 3894 + 7863927 + 826957.
10. 986759 + 4976346 + 29483 + 898647 + 3984753 + 6489778 + 57893 + 2468144 + 576989 + 498653.
11. 4683795 + 248675931 + 94986473 + 2849758 + 53388336 + 7788995 + 2137485 + 6758927 + 4926431 + 27729512 + 7842634 + 949867 + 34[illegible]216 + 78934.
12. Add together 9466495, 375573735, 754547, 3789284, 29886799 992984, 293675, 2684487, 3592873, 8847599, 738873, 7849376 334486, 123845, 672849, 73554, 8674.
13. Required the sum of 978 + 749 + 4764 + 8967 + 79889294 + 7759286 + 939723 + 864937 + 99375847 + 294886 + 94623 + 924086 + 794867 + 935279423 + 9738413208 + 2468975 + 945237 + 3834975.
14. 28674 + 39257 + 3834 + 92751 + 92503 + 86759 + 394875 + 34938 + 375396 + 759394 + 267934 + 6846 + 94657835 + 1926 + 48467319 + 2488 + 9357.

15. Add together seven thousand and ninety-four; two thousand, one hundred, and nine; eight thousand, nine hundred, and sixty; eighty-seven thousand and sixty-two; three hundred and seventy-five; nine thousand and thirty; thirty thousand and forty-six; fifty-four thousand, seven hundred, and seventy-five; seven thousand, eight hundred, and fifty-four.

16. Sir Isaac Newton was born in the year 1642, and died in his eighty-fifth year. In what year did he die?

17. William the Conqueror began his reign in England in the year 1066, and reigned 21 years; William II. reigned 13 years; Henry I. 15 years; Stephen, 39 years; Henry II. 35 years; Richard I. 10 years; John, 17 years; Henry III. 56 years; Edward I. 35 years; Edward II. 20 years; Edward III. 50 years; Richard II. 22 years: in what year was this last prince dethroned?

18. In 1821, the population of the following towns in England, Scotland, Ireland, and France, (the three largest in each) were as follows:—

London,.....	1,274,800	Glasgow,.....	147,043
Manchester,.....	133,788	Edinburgh,.....	138,235
Liverpool,.....	118,972	Paisley,.....	47,003
Dublin,.....	186,276	Paris,.....	720,000
Cork,.....	100,535	Lyons,.....	115,000
Limerick,.....	66,042	Marseilles,.....	102,000

Required the number of inhabitants contained in the three largest towns, in each country.

19. In the year 1810, the weight, in pounds of the cotton wool imported into England from the rest of Europe, was 16,725,708; from the United States, 55,194,616; from the British West Indies and conquered Colonies, 17,889,184; from foreign American Colonies, 22,137,397; from the East Indies, 23,144,907; from other sources, 1,478,291. Required the entire quantity.

20. The Pyramids of Egypt are thought to have been built 337 years before the founding of Carthage, Carthage to have been founded 49 years before the destruction of Troy, and Troy to have been destroyed 431 years before Rome was founded; Carthage was destroyed 607 years after the founding of Rome, and 146 years before the commencement of the Christian era; the Western empire of Rome ended in the year 476 of the Christian era, and 590 years before the Norman conquest; Constantinople was taken by the Turks 387 years after the Norman conquest, and 348 years before the union of Great Britain and Ireland in 1801. How many years elapsed between the first and last of these events?

21. Captain Cook, in his first voyage round the world, sailed from Portsmouth to the Madeiras, a distance of 1451 British miles; thence to the Canaries, 339 miles; from these to the Cape Verd islands, 985 miles; and thence to Rio Janeiro, 3058 miles; from that to Cape Horn, 2659 miles, and thence to Otaheite, 4919 miles; from Otaheite to the most southern point of the voyage, 1619 miles; and thence to Cook's strait in New Zealand, 1988 miles; from Cook's strait to Green Cape in New Holland, 1368 miles; and thence along the eastern coast of New Holland to the most northern point of that island, 2176 miles; thence to the straits of Sunda, 2487 miles; and thence to the cape of Good Hope, 5818 miles; from that cape to St. Helena, 1884 miles; and thence to Ascension island, 822 miles; from Ascension to Corvo in the Azores, 3462 miles; and thence to Portsmouth, 1598 miles. How far did he sail in all, exclusive of numerous deviations from these courses?

22. The following is the number of barrels of porter brewed in London by the twelve principal houses, between 5th July, 1814, and 5th July, 1815: what is the entire quantity? 337,621; 182,104; 172,162; 161,618; 123,100; 119,333; 105,081; 72,080; 56,922; 51,294; 38,107; 32,256.

23. Required the entire population of England, from the fol-

lowing statement of the population of its several counties, according to the Census of 1821:—

Bedford,.....	83,716	Hertford,.....	129,714	Shropshire,..	206,153
Berks,.....	131,977	Huntingdon,	48,771	Somerset,....	355,314
Bucks,.....	134,068	Kent,.....	426,016	Southampton,	283,298
Cambridge,...	121,909	Lancashire,	1,052,859	Stafford,.....	341,040
Cheshire,.....	270,098	Leicester,....	174,571	Suffolk,.....	270,542
Cornwall,.....	257,447	Lincoln,.....	283,058	Surrey,.....	398,658
Cumberland,.	156,124	Middlesex,	1,144,531	Sussex,.....	233,019
Derby,.....	213,333	Monmouth,..	71,833	Warwick,....	274,392
Devon,.....	439,040	Norfolk,.....	344,368	Westmoreland,	51,359
Dorset,.....	144,499	Northampton,	162,483	Wilts,.....	222,157
Durham,.....	207,673	Northumberland,	198,965	Worcester,...	184,424
Essex,.....	289,424	Nottingham,.	186,873	York, E. Riding,	190,449
Gloucester,..	335,843	Oxford,.....	136,971	——, N. Riding,	183,381
Hereford,....	103,243	Rutland,.....	18,487	——, W. Riding,	799,357

24. The following are the Irish acres, and the inhabitants of the several counties of Ireland.* Required the number of acres, and the population of each province, and of the whole kingdom; Lough Neagh, in Ulster, containing 58,200 acres:

PROVINCE OF ULSTER.

	Acres.	Population.
Antrim,.....	387,200	269,856
Armagh,.....	181,450	196,577
Cavan,.....	301,000	194,330
Donegall,....	679,550	249,483
Down,.....	348,550	329,348
Fermanagh,.	283,450	130,399
L. Derry,....	318,500	194,099
Monaghan,..	179,600	178,183
Tyrone,.....	463,700	259,691

PROVINCE OF LEINSTER.

	Acres.	Population.
Carlow,.....	137,050	81,287
Dublin,.....	142,050	346,550
Kildare,.....	236,750	101,715
Kilkenny,...	300,350	180,326
King's co. ...	282,200	132,319
Longford,...	134,150	107,702
Louth,.....	110,750	119,188
Meath,.....	327,900	174,716
Queen's co..	235,300	129,391
Westmeath,.	231,550	128,042
Wexford,....	342,900	169,304
Wicklow,....	311,600	115,162

PROVINCE OF MUNSTER.

	Acres.	Population.
Clare,.....	476,200	209,595
Cork,.....	1,048,800	802,535
Kerry,.....	647,650	205,037
Limerick,....	386,750	280,328
Tipperary,...	554,950	353,402
Waterford,...	262,800	154,466

PROVINCE OF CONNAUGHT.

	Acres.	Population.
Galway,.....	989,950	314,748
Leitrim,.....	255,950	105,976
Mayo,.....	790,600	297,538
Roscommon,	346,650	207,777
Sligo,.....	247,150	127,879

* The acres are given from Beaufort, and the population from the Census of 1821. As that Census was, in some instances, in a slight degree defective, it is thought, that the population exceeded seven millions. The acres given above are Irish, 121 of which are equivalent to 193 of the standard acres now in use.

25. The number of inhabitants contained in the several counties of Scotland, in 1821, was as follows. Required the population of the whole kingdom:—

Aberdeen,...	155,387	Elgin,.....	31,162	Orkney & Shetland.	53,124
Argyle,.....	97,316	Fife,.....	114,556	Peebles,.....	10,046
Ayr,.....	127,299	Forfar,.....	113,430	Perth,.....	139,050
Banff,.....	43,561	Haddington,	35,127	Renfrew,.....	112,175
Berwick,.....	33,385	Inverness,...	90,157	Ross and Cromarty,	68,828
Bute,.....	13,797	Kincardine,..	29,118		
Caithness,...	30,238	Kinross,.....	7,762	Roxburgh,.....	40,892
Clackmannan,	13,263	Kirkcudbright,	38,903	Selkirk,.....	6,637
Dumbarton,.	27,317	Lanark,.....	244,387	Stirling,.....	65,376
Dumfries,...	70,878	Linlithgow,..	22,685	Sutherland,...	23,840
Edinburgh,..	191,514	Nairn,.....	9,006	Wigton,.....	33,240

SIMPLE SUBTRACTION.

THE object of SUBTRACTION is to find the difference between two numbers.

The number found in Subtraction is called the REMAINDER, the DIFFERENCE, or the EXCESS.*

When the given numbers are of the *same denomination*, the process is termed SIMPLE SUBTRACTION.

When the given numbers express quantities of the *same kind*, but of *different denominations*, the process is termed COMPOUND SUBTRACTION.

Rule for Simple Subtraction.

(1.) Place the less number below the greater, with units under units, tens under tens, &c. as in Addition. (2.) Beginning with the units, take, if possible, each figure in the lower line, from the figure above it, and set down the remainder. (3.) But if any figure in the lower line be greater than the figure above it, add ten to the upper; then subtract as before, and carry one to the next figure in the lower line.

The sign —, generally called *minus*, when set between two numbers, denotes, that the latter is to be taken from the former Thus, 16—9=7 denotes, that if 9 be taken from 16 the remainder is 7.

* The number to be subtracted is sometimes called the *subtrahend;* and that from which it is to be taken, the *minuend.*

Methods of Proof.

1. Add the remainder and the less of the given numbers togegether: if the sum be equal to the greater, the work is correct.

2. Subtract the number found from the greater of the given numbers; if the remainder be equal to the less, the work is correct.

Examples in Simple Subtraction.

Ex. 1. From 7854 take 4513.

Set the numbers as in the margin, and proceed thus:—3 from 4, and 1 remains; 1 from 5, and 4 remain; 5 from 8, and 3 remain; 4 from 7, and 3 remain: the remainder therefore is 3341.

	7854
	4513
Remainder,	3341
Proof,	7854

To prove the work, to the less of the given numbers add the remainder, and the sum will be 7854, the greater; or, as in the second method, subtract the remainder from the greater number, and the result will be 4513, the less.

	7854
	4513
Remainder,	3341
Proof,	4513

Ex. 2. Required the difference of 3712 and 1831.

In this example proceed thus:—1 from 2, and 1 remains; 3 from 11, and 8 remain; carry 1 to 8, and then 9 from 17, and 8 remain; carry 1, and then 2 from 3, and 1 remains.—The difference, therefore, is 1881, and the operation would be proved in the same manner as before.

From	3712
take	1831
Remainder,	1881

Reason of the Rule.

The rule for Subtraction depends on the principle, that the differences of the several parts of two numbers are, when taken together, equal to the difference of the numbers themselves. The reason of placing units under units, tens under tens, &c. is, that figures may be subtracted from others of the same local value with more facility. By carrying one to the lower figure, we increase the lower line as much as we increased the upper, and thus the difference will be the same as if neither had been increased. Thus, in the second of the above examples, when in the tens' place we subtract 3 from 11, we thus add 10 to the 1 in the upper line; then the lower line is increased by the same quantity by adding 1 to the 8; because, by the nature of notation, 1 in the third column is equivalent to 10 in the second. Thus, therefore, both the given numbers are equally increased, and consequently the difference must be the same as if they had received no increase.

As a farther illustration of Subtraction, let it be required to find the difference of 83 and 57. Here, as 7 cannot be taken from 3, we may consider 83 as equal to 70 and 13; and subtracting 7 from 13, and 5 from 7, we find the difference to be 26. In this simple and natural method, the values of the given numbers undergo no change; and, with only one exception, it might be employed with as much facility as the common method, the next figure in the upper line being always diminished by a unit, when one would be carried to the figure below it, in the common method. The exception is the case in which the next figure in the upper line is a cipher: in this case the common method is considerably preferable; and, as in practice, that method is in no case inferior, it is universally preferred.

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83
57
—
26
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Exercises in Simple Subtraction.

Ex. 1. 45079—32048	Ex. 8. 915161718—151617189
2. 33456—17748	9. 202122223—192021222
3. 65934—48566	10. 357912468—24680135
4. 90401—58270	11. 7503046571—34992884
5. 623417—32686	12. 376995145—49490718
6. 8463192—177825	13. 153425178—53845248
7 4444444—1234567	14. 100000000—10001001

15. Take four thousand and four from four millions.

16. Required the difference between three millions and three thousand.

17. Subtract nineteen thousand and nineteen, from twenty thousand and ten.

18. Required the difference between three, and three hundred thousand.

19. Subtract one million, nine thousand, and six, from two millions, twenty thousand, nine hundred, and thirty.

20. Required the excess of nine hundred and twelve thousand, and ten, above fifty thousand and eighty-two.

21. La Place, the celebrated French Mathematician and Philosopher, was born in 1749: required his age in 1824.

22. The height of Mont Blanc, the highest mountain in Europe, is 15680 feet; and the height of Chimborazo, the highest mountain in America, is 21427 feet: how much is the latter higher than the former?

23. The height of the Peak of Tibet, thought to be the highest mountain in the world is 24,235 feet; and the height of the Peak of Teneriffe is 12,340 feet: required the difference of their heights.

24. In the years 1707, 1708, 1709, and 1710, the neat revenue arising from the British Post-Office, was, at an average, £58,052; in 1722, it was £98,010; in 1783, £159,858; in 1792, £368,784; and in 1801, 755,299. Required the increase between the first of these periods and the second, between the second and the third, &c

25. The population of London in 1821, was 1,274,800; of Glasgow, 147,043; Edinburgh, 138,235; Manchester, 133,788; Liverpool, 118,972; Birmingham, 106,722; Leeds, 83,796. Required the excess of the population of the first of these cities above that of the second, of that of the second above that of the third, &c.

26. In 1821, 1822, and 1823, the imports of Ireland from Great Britain and elsewhere, were £5,197,193, £6,407,428, and £6,607,488, respectively: required the excess of the third above the second, and of the second above the first.

27. The exports of Great Britain and Ireland, in 1821, 1822, and 1823, were £46,980,565, £47,289,320, and £46,196,554, respectively: required the excesses of the second above the first and third.

28. In 1795, the quantity of tea sold by the East India Company, for home consumption, was 18,498,569 pounds; and in 1800, 20,780,724 pounds. How much more was sold in the latter year than in the former?

29. The following are the years of the Christian era in which the undermentioned events happened: required the number of years from each till the year 1824. Commencement of the Hegira, or era of the flight of Mahomet, 622; The Arabic, or modern notation in Arithmetic, introduced from Arabia into Europe by the Saracens, 991; First Crusade, 1096; Magna Charta signed by King John, 1215; Linen first made in England, 1253; Termination of the Crusades, 1291; Spectacles invented by a monk of Pisa, 1299; Gunpowder first used in Europe, 1330; University of St. Andrews founded, 1411; Algebra introduced into Europe from Arabia, 1412; Printing invented, 1440; Constantinople taken by the Turks, 1453; America discovered by Columbus, 1492; Vasquez de Gama's discovery of the route to the East Indies by the Cape of Good Hope, 1497; Commencement of the Reformation, 1517; Spinning Wheel invented, 1530; Copernicus died, 1543; Spanish Armada destroyed, 1588; Telescopes invented, 1590; University of Dublin founded, 1591; English East India Company established, 1600; Decimal Fractions invented, 1602; Thermometers invented, and Satellites of Jupiter discovered, 1610; Logarithms published by Napier, 1614; Circulation of the blood discovered by Harvey, 1619; Barometer invented, 1643; Air Pump invented, 1654; Newtonian Philosophy published, 1686; Georgium Sidus discovered, 1781; Union of Great Britain and Ireland, 1801; Battle of Trafalgar, 1805; Battle of Waterloo, 1815.

SIMPLE MULTIPLICATION.

THE object of MULTIPLICATION is to find the amount of a number repeated a certain number of times. It is, therefore, only an abridged method of performing Addition, when the numbers to be added are equal to one another.

The number to be repeated is called the MULTIPLICAND; the number which shows how often the multiplicand is to be repeated, is called the MULTIPLIER; and the number found is called the PRODUCT. Both the multiplicand and multiplier are sometimes called FACTORS, from their *making* or producing the product.

When the multiplicand expresses a quantity of only *one denomination*, the process is termed SIMPLE MULTIPLICATION.

When the multiplicand expresses a quantity of the same kind, but of *more denominations than one*, the process is termed COMPOUND MULTIPLICATION.

MULTIPLICATION TABLE.*

Twice	3 times	4 times	5 times	6 times	7 times
1 = 2	1 = 3	1 = 4	1 = 5	1 = 6	1 = 7
2 = 4	2 = 6	2 = 8	2 = 10	2 = 12	2 = 14
3 = 6	3 = 9	3 = 12	3 = 15	3 = 18	3 = 21
4 = 8	4 = 12	4 = 16	4 = 20	4 = 24	4 = 28
5 10	5 = 15	5 = 20	5 = 25	5 = 30	5 = 35
6 = 12	6 = 18	6 = 24	6 = 30	6 = 36	6 = 42
7 = 14	7 = 21	7 = 28	7 = 35	7 = 42	7 = 49
8 = 16	8 = 24	8 = 32	8 = 40	8 = 48	8 = 56
9 = 18	9 = 27	9 = 36	9 = 45	9 = 54	9 = 63
10 = 20	10 = 30	10 = 40	10 = 50	10 = 60	10 = 70
11 = 22	11 = 33	11 = 44	11 = 55	11 = 66	11 = 77
12 = 24	12 = 36	12 = 48	12 = 60	12 = 72	12 = 84

8 times	9 times	10 times	11 times	12 times
1 = 8	1 = 9	1 = 10	1 = 11	1 = 12
2 = 16	2 = 18	2 = 20	2 = 22	2 = 24
3 = 24	3 = 27	3 = 30	3 = 33	3 = 36
4 = 32	4 = 36	4 = 40	4 = 44	4 = 48
5 = 40	5 = 45	5 = 50	5 = 55	5 = 60
6 = 48	6 = 54	6 = 60	6 = 66	6 = 72
7 = 56	7 = 63	7 = 70	7 = 77	7 = 84
8 = 64	8 = 72	8 = 80	8 = 88	8 = 96
9 = 72	9 = 81	9 = 90	9 = 99	9 = 108
10 = 80	10 = 90	10 = 100	10 = 110	10 = 120
11 = 88	11 = 99	11 = 110	11 = 121	11 = 132
12 = 96	12 = 108	12 = 120	12 = 132	12 = 144

* Though the part of the Multiplication Table given in the text, is quite enough for the pupil to commit to memory at first; yet after he has made some proficiency in

It will perhaps enable the pupil to commit the preceding Table to memory with more ease, if he be caused to construct it by Addition.—Thus, to find the products by 7, let him set in one column the figure 1 seven times, in another the figure 2 seven times, in another the figure 3 as often, &c. Then, the sums of these columns will be the products by 7. He will thus be taught the construction and nature of the Table, and will perhaps find it easier and more interesting to commit the products which he has formed himself, than those which he finds without explanation in the Table. It will also assist the learner, if his attention be turned to the relations subsisting between some of the successive products in the Table. Thus he will see that the products by 10 are formed simply by the addition of a cipher; that the first nine products by 11 are formed by repeating the figure; that the products by 5 terminate in 5 and 0, alternately; that in the products by 9, the first figure generally increases, and the second decreases, by a unit, &c.

Rule for Simple Multiplication.

(1.) Place the multiplier below the multiplicand, with units under units, tens under tens, &c. (2.) If the multiplier do not exceed 12, multiply, by means of the Multiplication Table, each figure of the multiplicand by it, beginning with the units, and setting down and carrying as in Addition. The result will be the product required. (3.) But if the multiplier be greater than 12, find the products of the multiplicand by the several figures of the multiplier, successively; setting the right hand figure of each product under that figure of the multiplier which produces it. The sum of all these products will be the total product required.

If either the multiplicand, or the multiplier, or both, end in ciphers, the significant figures may be arranged and multiplied according to the rule, and as many ciphers annexed to the product, as are found at the end of both factors.—Ciphers in any other part of the multiplier, are to be neglected.

Arithmetic, it may be found of advantage to require him to commit what follows, as it will enable him, in many cases, to shorten his work in a considerable degree. The labour of committing a more extended table, would be scarcely compensated by the advantage resulting.

13 times	14 times	15 times	16 times	17 times	18 times	19 times
2 = 26	2 = 28	2 = 30	2 = 32	2 = 34	2 = 36	2 = 38
3 = 39	3 = 42	3 = 45	3 = 48	3 = 51	3 = 54	3 = 57
4 = 52	4 = 56	4 = 60	4 = 64	4 = 68	4 = 72	4 = 76
5 = 65	5 = 70	5 = 75	5 = 80	5 = 85	5 = 90	5 = 95
6 = 78	6 = 84	6 = 90	6 = 96	6 = 102	6 = 108	6 = 114
7 = 91	7 = 98	7 = 105	7 = 112	7 = 119	7 = 126	7 = 133
8 = 104	8 = 112	8 = 120	8 = 128	8 = 136	8 = 144	8 = 152
9 = 117	9 = 126	9 = 135	9 = 144	9 = 153	9 = 162	9 = 171

The sign $\times$, called *the sign of Multiplication*, is frequently used to denote the multiplication of the numbers between which it is placed. Thus, $20 \times 12 = 240$, denotes, that the product of 20 and 12 is 240.

Examples in Simple Multiplication.

Ex. 1. Multiply 324 by 8.

Set the numbers as in the margin, and proceed thus:—8 times 4 are 32; set down 2, and carry 3: 8 times 2 are 16, and 3 are 19; set down 9, and carry 1; 8 times 3 are 24, and 1 are 25; set down 25; and the product is two thousand, five hundred, and ninety-two, the same that would have been obtained by adding together eight lines, each 324: and thus it would be well to cause the pupil to work this and a few similar questions, to make him sensible of the identity of Multiplication and Addition.

Multiplicand...	324
Multiplier.....	8
Product...	2592

Ex. 2. What is the product of 29 and 365?

Set the numbers as in the margin, multiply by 9, and set 5, the first figure of the product, under the 9; then multiply by 2, and set 0, the first figure, under the 2.—The sum of these partial products will give ten thousand, five hundred, and eighty-five, the entire product required.

Multiplicand...	365
Multiplier.....	29
	3285
	730
Product...	10585

Ex. 3. Multiply 36407 by 40206.

Here the first figure of the first partial product, is set below the figure 6 in the multiplier; the first figure of the second partial product, below 2; and the first of the third, below 4; the ciphers in the multiplier being neglected.

Multiplicand...	36407
Multiplier.....	40206
	218442
	72814
	145628
Product...	1463779842

Ex. 4. Multiply 320 by 2400.

The numbers being arranged as in the margin, multiply as in the last example 32 by 24: to 768 the product, add three ciphers, and the entire product is seven hundred and sixty-eight thousand.

Multiplicand...	320
Multiplier.....	2400
	128
	64
Product...	768000

Reason of the Rule.

The principle on which this rule depends, is, that the products of the several parts of the multiplicand by the multiplier, or its parts, are, when taken together, equal to the product of the whole multiplicand by the whole multiplier. The reason for the mode of arranging the partial products, is, that figures of the same local value may stand in the same column. This will be understood from the following illustration.

In the example in the margin, we have first 7 times 7=49=4 tens and 9; the 9 is set down and the 4 tens reserved. Then 7 times 9=63, and 4 are 67, or 6 tens and 7. The 4 added to 63, is, by the nature of notation, equivalent to the 4 tens reserved, since 4 in the second place is equivalent to 40 in the first; 7 is then set down, and 6 tens, or properly 6 hundreds, reserved, and 6 added for them in the hundreds' place.

$$\begin{array}{r} 497 \\ 7 \\ \hline 3479 \end{array}$$

The annexing of one cipher to a number expressed by the modern notation, makes its value 10 *times what it was before; two ciphers* 100 *times; three ciphers,* 1000 *times,* &c. This is evident from the principles of notation, since the annexing of a cipher removes each figure one place to the left. Thus, the figure 7, with one cipher annexed, expresses 70, or 10 times 7; with two ciphers, 700, or 100 times 7, &c.

From this the operation in the margin will be understood. The multiplier is equivalent to 300, 40, and 6, taken together; and, therefore, the multiplicand is multiplied by 6, 40, and 300, successively, and the sum of the products taken. In multiplying by 40, the product by 4 is taken, and a cipher annexed; and in multiplying by 300, two ciphers are annexed to the product by 3; in both instances according to the principle just delivered. In the second, or common form of the operation, these additional ciphers are omitted, and the omission is compensated by making the figures occupy the same places which they do when the ciphers are annexed; and hence the reason of setting the figures in the manner prescribed in the rule, is evident. When the pupil is learning the reason of the rule for Multiplication, it may be well to cause him for some time to annex the ciphers, in the mode above explained.

$$\begin{array}{rl} 6758, & \text{Multiplicand} \\ 346, & \text{Multiplier.} \\ \hline 40548 = 6758 \times & 6 \\ 270320 = 6758 \times & 40 \\ 2027400 = 6758 \times & 300 \\ \hline 2338268 = 6758 \times & 346 \end{array}$$

$$\begin{array}{rl} 6758, & \text{Multiplicand} \\ 346, & \text{Multiplier} \\ \hline 40548 & \\ 27032 & \\ 20274 & \\ \hline 2338268, & \text{Product.} \end{array}$$

Methods of Proof.

1. Let the multiplier be taken as multiplicand, and the multiplicand as multiplier; and if the product thus obtained agree with the product found before, the process is right.

Thus, when 64 is multiplied by 45, the product is found to be 2880; and when 45 is multiplied by 64, the same result is obtained.

$$\begin{array}{r} 64 \\ 45 \\ \hline 320 \\ 256 \\ \hline 2880 \end{array} \qquad \begin{array}{r} 45 \\ 64 \\ \hline 180 \\ 270 \\ \hline 2880 \end{array}$$

2. Add together all the digits of the multiplicand, except 9, which is to be neglected when it occurs; reject also 9 from the successive sums of the digits, when those sums exceed 9, and reserve the final excess. Proceed in like manner with the multiplier and the product. Then multiply together the excesses found in the multiplicand and the multiplier, and, in the same manner as before, find the excess in this product. If this be the same as the excess in the product of the given factors, the work is, in most instances, correct: *if it differ, the work must be wrong.**

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* From the great brevity and facility of this mode of proof, it is very convenient in practice. In some cases, indeed, operations which are incorrect may appear by this method of proof, to be correct; an occurrence however which is extremely rare, unless it be either purposely effected, or arise from the misplacing of figures; and hence, for the purposes of the experienced arithmetician, it is perhaps preferable to any other method of proof. It is scarcely necessary to say, that the using of a cross, (×) in this method of proof, is both awkward and unnecessary.

The principle on which this process depends, is, that *if any number and the sum of its digits be both divided by* 9, *the remainders in both cases, are the same.* Thus, 1000=999+1, where the remainder must be 1, since 999 is evidently divisible by 9, without remainder. Hence, since 3000=1000×3=999×3+3, the remainder here will evidently be 3, since 999×3 is divisible by 9, without remainder. In like manner it might be shown, that in dividing by 9, the remainder in 5000 would be 5; in 400, 4; in 20000, 2, &c.; being in all cases, the same as the significant figure contained in the number. Hence if the number 487 be proposod, it is equivalent to 400+80+7; which parts, if divided successively by 9, would leave, by the above principle, the remainders 4, 8, 7; and therefore the remainder of the entire number will evidently be the same as the sum of these, diminished by the rejection of 9 as often as possible. Now, suppose it were required to multiply 112 by 48; 112=12×9+4, and 48=5×9+3; the product therefore will be equal to 12×9+4 multiplied by 5×9+3; which, by multiplying the several terms, becomes 12×9×5×9+5×9×4+12×9×3+4×3. In this product the three first terms are evidently divisible by 9, without remainder: consequently, the remainder obtained by dividing the whole product by 9, will be the same as the remainder obtained by dividing 4×3, the product of the excesses of the factors, by 9; and thus the reason of this method of proof is evident. The same property belongs to the digit 3; but it is more convenient in practice to use 9.

Another method of proof, which is nearly as easy, and is perhaps more certain than the preceding, depends on a property of the number 11 in the decimal notation. This property might be proved in a manner similar to that given above. It would be too tedious however to be introduced here; and indeed both are better and more easily given by Algebra. The following is the method referred to: *Commencing at the units of the multiplicand, add together the digits in the odd places, rejecting* 11 *as often as possible, and reserve the result; proceed in the same manner with the digits in the even places, and from the former result, increased if necessary by* 11, *take this result and place the excess opposite to the multiplicand. In the same manner find the excesses of the multiplier and the product; then multiply the excesses of the two factors together, and find in the same manner, the excess of their product: if this excess, and the excess of the product of the factors be the same, the work is in most cases correct; if they differ, it must be wrong.*

In the annexed example, (the work of which, for brevity, is omitted,) 3 and 9 are 12, which exceeds 11 by 1; 1 and 8 are 9; then 6 and 4 are 10, which being taken from 20 (=9+11,) the excess is 10. Then, in the multiplier, the sum of 5, 0, and 4, is 9, from which 1, the excess of

Multiplicand...84965.....10
Multiplier...44085..... 8

Product, 3745593855..... 3

In the annexed example, 6 and 8 are 14, which exceeds 9, by 5; 5 and 5 are 10, which exceeds 9, by 1; 1 and 7 are 8, which excess we set opposite to the multiplicand. In the multiplier, in like manner, 8 and 7 are 15; which exceeds 9, by 6, which excess is set opposite to the multiplier. In the product, 6 and 1 are 7, and 5 are 12, which exceeds 9 by 3; 3 and 3 are 6, and 1 are 7, and 5 are 12, which gives an excess of 3, to be set opposite to the product. Then the product of the two excesses, 6 and 8, is 48, the sum of the digits in which is 12, which gives an excess of 3, the same as the excess of the product; and hence we judge the work to be correct.

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   68597.....8
     897.....6
   ------
  480179
 617373
548776
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61531509.....3
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Exercises in Simple Multiplication.

Ex. 1. 144 × 3	Ex. 6. 125 × 8	Ex. 11. 40376 × 40
2. 795 × 4	7. 3046 × 9	12. 40376 × 4000
3. 729 × 5	8. 7308 × 11	13. 819200 × 700
4. 936 × 6	9. 1729 × 12	14. 7854 × 900
5. 1599 × 7	10. 1816 × 10	15. 4096 × 12000

		Answers.
Ex. 16.	148 × 53	=7844
17.	958 × 34	=32572
18.	7198 × 216	=1554768
19.	31416 × 175	=5497800
20.	8862 × 189	=1674918
21.	7071 × 556	=3931476
22.	93186 × 4455	=415143630
23.	40930 × 779	=31884470
24.	12345 × 686	=8468670
25.	46481 × 936	=43506216
26.	16734 × 708	=11847672
27.	7575 × 7575	=57380625
28.	8320900 × 1328	=11050155200
29.	17500 × 732	=12810000
30.	15607 × 3094	=48288058
31.	7422153 × 468	=3473567604
32.	9264397 × 9584	=88789980848
33.	4687319 × 1987	=9313702853

8+4 above 11, being taken, the excess is 8. In the product, 5 and 8 are 13; 2, the excess, and 9 are 11, which is rejected; 5 and 7 are 12, the excess of which is 1; then 5 and 3 are 8, and 5 are 13; 2 and 4 are 6, and 3 are 9; 9 from 12, (=1+11,) and the excess is 3. The product of the first two excesses is 80, the excess of which (found by taking 8 from 11,) is 3, the same as the excess of the product of the factors.

By applying both these modes of proof to the same operation, an almost absolute certainty of its correctness will be obtained.

Answers.

Ex. 34. 204053 × 1617000 ..... = 329953701000
35. 9507340 × 7071 ..... = 67226401140
36. 39948123 × 6007 ..... = 239968374861
37. 73885246 × 6079 ..... = 449148410434
38. 57902468 × 5008 ..... = 289975559744
39. 57902468 × 5080 ..... = 294144537440
40. 57902468 × 5800 ..... = 335834314400
41. 12481632 × 1509 ..... = 18834782688
42. 79068025 × 1386 ..... = 109588282650
43. 92948789 × 7043 ..... = 654638320927
44. 58763718 × 6754 ..... = 396890151372
45. 73084163 × 7584 ..... = 554270232192
46. 144 × 144 × 144 ..... = 2985984
47. 3851 × 3851 × 3851 ..... = 57111104051
48. 79094451 × 764095 ..... = 60435674536845
49. 79548050 × 97280 ..... = 7738434304000

50. Multiply fifty-six millions, seven thousand eight hundred, and fifty-four, by eighty millions, six hundred thousand, nine hundred and seventy-six. *Answ.* 4514287696065504.

51. Multiply five hundred and seven millions, five hundred and forty-six thousand, two hundred and fifty-six, by seven hundred and eight millions, nine hundred thousand, and fifty-eight. *Answ.* 359799570316082848.

52. Multiply eighty millions, seven thousand, six hundred, by eight millions, seven hundred, and sixty. *Answ.* 640121605776000.

53. Required the amount of seven hundred and nine millions, four hundred and sixty-five thousand, nine hundred and eight, repeated eight hundred thousand, three hundred and sixty-five times. *Answ.* 567831681456420.

54. Multiply eight hundred, and seventy-seven millions, five hundred and ten thousand, eight hundred and sixty-four, by five hundred and forty-five thousand, three hundred and fifty-seven. *Answ.* 478556692258448.

55. Multiply one hundred and twenty-three millions, four hundred and fifty-six thousand, seven hundred and eighty-nine, by nine hundred and eighty-seven millions, six hundred and fifty-four thousand, three hundred and twenty-one. *Ans.* 121932631112635269.

56. How many yards of linen are in 759 pieces, each containing 25 yards? *Answ.* 18975.

57. Sound is known to move about 1130 feet per second; how many feet will it move in 69 seconds? *Answ.* 77970.

58. It is found by microscopic observations, that in each square inch of the human skin there are about 1000 pores; and the surface of the body of a middle sized man contains about 2304 inches, (or 16 square feet.) Required the number of pores in the surface of such a body, 999 being supposed to be contained in each inch. *Answ.* 2301696.

59. An elm will live 100 years; and it has been computed that

each year, at an average, it produces three hundred and twenty-nine thousand grains or seeds, each of which, if properly treated, would produce a tree. How many trees might be produced from one elm in 67 years, and how many in 100 years? *Answ.* 22043000, *and* 32900000.

60. In London, of morning newspapers which are printed every day, except Sunday, there are sold about fifteen thousand each day; and of evening papers published the same days, about thirteen thousand; of papers published three times a-week, about ten thousand of each impression are sold. There are likewise sold of Sunday papers about twenty-six thousand, six hundred, weekly; and of other kinds about twenty thousand, weekly. How many single papers are sold in London weekly, and how many yearly, the year containing 52 weeks? *Answ.* 244600, *and* 12719200.

61. During the 50 years between 1700 and 1750, the quantity of linen exported from Ireland, each year at an average, was 4 millions of yards; during the next six years, 11,796,361 yards in each; during the next seven years, 14,511,973 yards annually; during the next seven, 17,776,862 yards per year; and during the succeeding seven years ending 1777, the average quantity was 20,252,239 yards annually. Required the whole quantity exported from 1700 till 1777. *Answ.* 638565684.

62. Rees's "Cyclopædia," consists of 39 volumes, each containing at an average, 774 pages, of two columns. In each column there are 67 lines, each containing at an average, 10 words, and in those 10 words, there are, at an average, 47 letters. Required the number of pages, lines, words, and letters, contained in the entire work. *Answ.* 30186 *pages*, 4044924 *lines*, 40449240 *words*, 1901114280 *letters*.

SIMPLE DIVISION.

DIVISION is the method of finding how often one given number, called the DIVISOR, is contained in another, called the DIVIDEND.*

The number which shows how often the divisor is con-

* Or, *Division is the method of finding a number which will be contained a given number of times, in another given number*; or, *Division is the method of finding one of the factors of a given product, when the other factor is given.*

These two definitions, and that given in the text, result from the different points of view in which Division is considered, and are easily reduced to one another. The last is applicable to numbers of every kind, whether whole or fractional, and the one before it is alike applicable to Simple and Compound Division. The one in the text, however, is the most simple for the learner, and has the advantage of showing very easily the identity of Division and Subtraction.

tained in the dividend, is called the QUOTIENT; and if any thing be left after the operation is finished, it is termed the REMAINDER.*

When the dividend expresses a *quantity of one denomination*, the process is termed SIMPLE DIVISION.

When the dividend expresses a *quantity of the same kind, but of different denominations*, the process is termed COMPOUND DIVISION.

Rule for Simple Division.

(1.) Place the divisor to the left of the dividend, with a line between them, and leave a space to the right of the dividend for containing the quotient. (2.) Find by the Multiplication Table how often the first figure of the divisor is contained in the first figure, or the two first figures of the dividend, and set the figure denoting the number of times in the quotient. (3.) Multiply the divisor by the figure thus found, set the product below the leading figures of the dividend, and subtract it from them. (4.) To the remainder annex the next figure† of the dividend, divide the result as before, and thus proceed till the operation is finished.

If any product be greater than the number which stands above it, the last figure in the quotient must be changed for one of smaller value; but if any remainder be greater than the divisor, or equal to it, the last figure of the quotient must be changed for a greater. If any of the successive dividends be less than the divisor, a cipher must be put in the quotient, and another figure, if any remain, *brought down* from the given dividend.‡

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* When one number contains another a certain number of times *exactly*, or *without remainder*, the greater is called a *multiple* of the less, or it is said to be *divisible* by the less; and the less is said to *measure* the greater, or to be a *measure*, or a *divisor*, or an *aliquot part* of the greater. Thus, 21 is a multiple of 7, and 7 is a measure, a divisor, or an aliquot part of 21.

When a number is a multiple of two or more numbers, it is called a *common multiple* of these numbers; and a number which is a measure of two or more numbers, is called a *common measure* of them. Thus, 12 is a common multiple of 2, 3, 4, and 6; and 3 is a common measure of 6, 21, and 33.

When a whole number is a multiple of any other whole number, greater than unity, it is called a *composite number*. Thus, 4, 6, 8, 9, &c. are composite numbers. But when a whole number is not divisible by any other whole number, greater than unity, it is called a *prime number*. Thus, 2, 3, 5, 7, 11, &c. are prime numbers.

† It is proper, for preventing mistakes, to put a dot beneath each figure of the given dividend, when it is *brought down*.

‡ It is carefully to be observed, that *for each figure of the dividend which is brought down, either a significant figure or a cipher must be put in the quotient, besides the first figure that was put in it.*

When the divisor is below 13, the several multiplications and subtractions may be performed mentally, and the quotient set under the dividend.*

The sign ÷, called *the sign of Division*, placed between two numbers, denotes that the former is divided by the latter. The division of one number by another, is also frequently denoted by writing the dividend above the divisor, with a line drawn between them. Thus, $144 \div 9 = 16$, or $\frac{144}{9} = 16$, denotes that 144 is divided by 9, and that the quotient is 16. When there is a remainder, after the operation is finished, the division of it by the divisor is generally expressed in the latter mode: thus, if 76 be divided by 9, the quotient is written $8\frac{4}{9}$.

Methods of Proof.

1. Find the product of the divisor and quotient, and add to it the remainder; if the sum be equal to the dividend, the work is correct.

2. Subtract the remainder from the dividend, and divide the result by the quotient; if the quotient thus found be the same as the original divisor, the work is right.

3. *Cast the nines* out of the divisor, dividend, quotient, and remainder; then, to the product of the excesses of the divisor and quotient, add the excess of the remainder, and cast the nines out of the sum; if this excess be equal to the excess of the dividend, the work is generally correct.

Examples in Simple Division.

Exam. Divide 136 by 8.

Here we say, 8 in 13 once, and 5 remain; 8 in 56, 7 times, and nothing remains. The quotient therefore is 17; and by multiplying it by the divisor 8, we obtain 136, the dividend, which proves the operation.

```
8) 136
   ---
    17, Quotient.
     8
   ---
   136, Proof.
```

Division may be regarded as a contracted mode of performing successive subtractions of the same number. Thus the preceding example might be wrought by subtracting 8 from 136, 8 from the remainder, 8 from that remainder, &c. till, after 17 subtractions, the dividend would be exhausted, there being no remainder. The number of subtractions shows how often the divisor is contained in the dividend, and agrees with the result already found. The pupil might, with advantage, be required to work, in this manner, a few examples in which the quotients would be small numbers.

* When the operation is performed in this manner, it is usually termed *Short Division*, while the other method is called *Long Division*.

Ex. 2. Divide 15967 by 57.

Let the divisor, 57, be set before the dividend, 15967, as in the margin, and proceed thus:—How often is 5 contained in 15? twice:* place 2 in the quotient, multiply the divisor by it, and set the product below 159, the leading part of the dividend. This being subtracted from 159, the remainder is 45, to which 6, the next figure of the dividend, is annexed. Again, how often 5 in 45? 8 times: place 8 in the quotient, proceed as before, and there is no remainder. Then 7, the remaining figure of the dividend, containing 57 no times, a cipher is placed in the quotient, and the remainder is written in the quotient over the divisor 57. The quotient, therefore, is $280\frac{7}{57}$.

57) 15967	($280\frac{7}{57}$
114··	57
456	1967
456	1400
7	15967, proof

If the divisor end in ciphers, cut them off for a contraction, and cut off the same number of figures from the right of the dividend; then proceed with the remaining figures, according to the rule, and to the last remainder annex the figures cut off in the dividend, to find the true remainder: thus—

Ex. 3. Divide 782967 by 3700.

In this exercise two ciphers are cut off from 3700, and two figures from the dividend; then the division by 37 proceeds in the common way, and to the last remainder, 22, the figures cut off are brought down, and the true remainder is 2267.

37|00) 7829|67	($211\frac{2267}{3700}$
74··	3700
42	2267
37	1477
59	633
37	782967, Proof.
2267, Remainder.	

Hence, when the divisor is unity, with one or more ciphers annexed the quotient will be found by cutting off from the dividend as many figures for remainder as there are ciphers in the divisor: thus, If it be required to divide 53826 by 100, the quotient is simply $538\frac{26}{100}$.

* It would seem here at first sight, that 3, and not 2, should be put in the quotient. Were the divisor multiplied by 3, however, the product, 171, would be greater than 159 The learner often finds difficulty, in this way, in discovering what figure he should place in the quotient. In this example, 5 is contained in 15 twice, with the remainder 5, which, with 9, the next figure in the dividend, makes 59. In this, 7, the second figure of the divisor, is contained 8 times, which being more than 2, the number of times 5 was found in 15, we are certain that the divisor is contained twice. Had 5 been taken 3 times in 15, however, we should have had no remainder to prefix to 9; and 7, the second figure of the divisor, is not contained 3 times in 3, and consequently, the whole divisor is not contained 3 times in the leading figures of the dividend. This method may perhaps be employed with some advantage when the divisor is large; practice, however, will soon render it unnecessary. When the second figure of the divisor is above 5, in

When the divisor can be resolved into two factors not exceeding 12, the dividend may be divided by one of them by Short Division, and the result by the other. In proceeding thus, if there be remainders, the true remainder on the whole division, is found by multiplying the last remainder by the first divisor, and adding to the product the first remainder.*

Ex. 4. Divide 947351 by 56.

Here the first remainder is 6 and the second 7, and by the rule we obtain 55 for the remainder, in dividing by 56. The reason of this is best shown by means of fractions. It is evident, however, since the first line is 7 times the second, that whatever remains in dividing, the second line is only a seventh part of what it would have been in the first, and hence the reason will be manifest.

$$\begin{array}{r} 7)\ 947351 \\ \hline 8)\ 135335\tfrac{6}{7} \\ \hline 16916\tfrac{55}{56} \end{array}$$

When the pupil has had some practice in the methods already explained, he may be taught to omit writing the products, which will at least save much room in his operations. This method will be understood from the following example:—

Ex. 5. Divide 436951 by 173.

Here the first figure put in the quotient is 2; then we say, twice 3 are 6; 6 from 6, and nothing remains; twice 7 are 14; 14 from 23, and 9 remain; twice 1 are 2, and 2 (carried,) are 4; 4 from 4, and nothing remains. We then bring down 9, and place 5 in the quotient; then 5 times three are 15; 15 from 19, and 4 remain; 5 times 7 are

$$\begin{array}{r} 173)\ 436951\ (2525\tfrac{126}{173} \\ \underline{\ldots\ldots} \\ 909 \\ \underline{\ldots} \\ 445 \\ \underline{\ldots} \\ 991 \\ \underline{\ldots} \\ 126 \end{array}$$

trying for the figure to be placed in the quotient, the first may be increased by unity. Thus, in the example before us, we might have said, how often 6 in 15? &c.

The French place the divisor to the right of the dividend, and the quotient below it, as in the example in the margin. This mode gives the work a more compact and neat appearance, and possesses the advantage of having the figures of the quotient near the divisor, by which means the practical difficulty of multiplying the divisor by a figure placed at a distance from it, is removed. This difficulty every one must have felt, particularly in long operations; and hence this method might, with much propriety, be adopted in preference to that which is employed in this country.

$$\begin{array}{r|l} 59122 & 82 \\ 574\cdot\cdot & 721 \\ \hline 172 & \\ 164 & \\ \hline 82 & \\ 82 & \\ \hline \cdot\cdot & \end{array}$$

* It may be proper to observe, that this method is not applicable when the divisor is a prime number, as the factors must be such as exactly to produce the divisor. It may be farther remarked, that when there are more than two factors, the true remainder will be found by applying the rule given in the text to the two last remainders, then to the result and the remainder before them, &c.

35, and 1 are 36; 36 from 40, and 4 remain; 5 times 1 are 5, and 4 (carried,) are 9; 9 from 9, and nothing remains, &c.

Reason of the Rule.

The principle on which the operations in Division depend, is, that *a part* of the quotient is found, and the product of it and the divisor deducted from the dividend: then *another part* of the quotient is found, and its product by the divisor deducted from the remainder found before; and thus the operation proceeds till nothing remains, or till the remainder is less than the divisor. This will be understood from the following example:

Ex. 6. Divide 31272 by 73.

Here, the first part of the quotient is 400, the product of which by 73, is 29200. This taken from the dividend, leaves 2072 to be divided by 73. The next part of the quotient is 20; the product of which by 73, is 1460, which still leaves a remainder of 612 to be divided by 73. This gives 8, with the remainder 28. Hence, it appears that 73 is contained in the dividend 400+20+8 times, or 428 times, with the remainder 28. By comparing this and the common process subjoined, it will be found that the latter is merely an abbreviation of this, the ciphers being omitted in the one and retained in the other.

```
73) 31272 (400
    29200
    ------
73) 2072  (20
    1460
    ------
73) 612   (8
    584   ---
    ---   428
     28

73) 31272 (428 28/73
    292··
    ----
     207
     146
     ---
      612
      584
      ---
       28
```

*Exercises in Simple Division.**

Ex. 1. 470850÷3	Ex. 4. 3782047÷6	Ex. 7. 74593822÷9
2. 1829765÷4	5. 7165537÷7	8. 53248675÷11
3. 4265983÷5	6. 27459332÷8	9. 49275189÷12

Answers.

Ex. 10. 51846734÷102..... $=508301\frac{38}{102}$

11. 727346489÷408..... $=1782711\frac{281}{408}$

12. 980263711÷809..... $=1211698\frac{88}{809}$

13. 536819237÷907..... $=591862\frac{48}{907}$

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* The pupil should be required to prove the work of the several exercises in Division, by Multiplication. Every operation in Long Division, thus proved, affords him in reality an exercise in each of the four fundamental rules; as, in finding the quotient he employs Division and Subtraction, and in the proof, Multiplication and Addition. Hence, perhaps the exercises here given will not seem, on due consideration, too numerous, though at first sight they might appear to be so. It is of the first importance that the pupil should have acquired both accuracy and despatch in performing the operations in the fundamental rules, before he proceeds to apply them in the more advanced parts of Arithmetic.

Answers.

Ex. 14. 1457924651 ÷ 1204 ..... $= 1210900\frac{1051}{1204}$

15. 28101418481 ÷ 1107 ..... $= 25385201\frac{874}{1107}$

16. 513513513513 ÷ 917 ..... $= 559992926\frac{571}{917}$

17. 648751624367 ÷ 731 ..... $= 886945176\frac{711}{731}$

18. 465465465465 ÷ 644 ..... $= 722772461\frac{121}{644}$

19. 347382600435 ÷ 727 ..... $= 477830261\frac{188}{727}$

20. 1700649160000 ÷ 759 ..... $= 2240644479\frac{739}{759}$

21. 571824753344 ÷ 839 ..... $= 681555129\frac{113}{839}$

22. 245379633477 ÷ 1263 ..... $= 194283161\frac{1134}{1263}$

23. 536847555555 ÷ 1138 ..... $= 471746533\frac{1991}{1138}$

24. 234516447519 ÷ 759 ..... $= 308980826\frac{685}{759}$

25. 111111111111 ÷ 854 ..... $= 130106687\frac{715}{854}$

26. 789012345676 ÷ 7384 ..... $= 106854326\frac{3492}{7384}$

27. 75843639426 ÷ 8593 ..... $= 8826211\frac{8293}{8593}$

28. 65358547823 ÷ 2789 ..... $= 23434402\frac{840}{2789}$

29. 33333333333 ÷ 5299 ..... $= 6290495\frac{338}{5299}$

30. 321987653321 ÷ 7766 ..... $= 41461196\frac{1188}{7766}$

31. 542713060315 ÷ 4444 ..... $= 122122650\frac{3715}{4444}$

32. 265210420816 ÷ 7539 ..... $= 35178461\frac{4470}{7539}$

33. 7314617334753 ÷ 6784 ..... $= 1078215998\frac{3271}{6784}$

34. 3146173847837 ÷ 9387 ..... $= 335162868\frac{8921}{9387}$

35. 5557779994447777 ÷ 891 ..... $= 6237687984778\frac{79}{891}$

36. 765809034537648 ÷ 793 ..... $= 965711266756\frac{140}{793}$

37. 58239017194511 0 ÷ 693 ..... $= 8403898585064\frac{128}{693}$

38. 58239017194511 0 ÷ 593 ..... $= 982108215759\frac{23}{593}$

39. 58239017194511 0 ÷ 493 ..... $= 118131880719 0\frac{149}{493}$

40. 58239017194511 0 ÷ 393 ..... $= 148190883446 5\frac{185}{393}$

41. 58239017194511 0 ÷ 293 ..... $= 198767967730\frac{239}{293}$

42. 58239017194511 0 ÷ 193 ..... $= 301756565774 6\frac{132}{193}$

43. 3333333333333333 ÷ 483 ..... $= 6901311249137\frac{98}{483}$

44. 3333333333333333 ÷ 484 ..... $= 6887052341597\frac{185}{484}$

45. 1000000000000000 ÷ 81 ..... $= 12345679012345\frac{55}{81}$

46. 1000000000000000 ÷ 729 ..... $= 1371742112482\frac{328}{729}$

47. 1000000000000000 ÷ 111 ..... $= 9009009009009\frac{1}{111}$

48. 1000000000000000 ÷ 1111 ..... $= 900090009000\frac{999}{1111}$

49. 1000000000000000 ÷ 11111 ..... $= 90000900009\frac{1}{11111}$

50. 555555555555 ÷ 123456 ..... $= 4500028\frac{88787}{123456}$

51. 555555555555 ÷ 654321 ..... $= 849056\frac{384879}{654321}$

52. 102030405060 ÷ 123456 ..... $= 826451\frac{79404}{123456}$

53. 908070605040 ÷ 654321 ..... $= 1387805\frac{428035}{654321}$

54. 3784926474826 ÷ 384365 ..... $= 9847219\frac{128881}{384365}$

55. 467817938473 ÷ 2100 ..... $= 222770446\frac{1873}{2100}$

56. 367817938429 ÷ 36500 ..... $= 10077203\frac{18928}{36500}$

57. 267817938473 ÷ 8760 ..... $= 30572824\frac{333}{8760}$

58. 167817938176 ÷ 957000 ..... $= 175358\frac{331178}{957000}$

59. 167817938473 ÷ 87700 ..... $= 1913545\frac{41873}{87700}$

60. 267817938115 ÷ 1360000 ..... $= 196924\frac{1281115}{1360000}$

61. 367817938473 ÷ 87000 ..... $= 4227792\frac{14478}{87000}$

62. Devis, the highest mountain in the neighbourhood of Belfast, is 1542 feet high; and Mont Blanc 15,680 feet: how many mountains, each as high as the former, must be piled one above another to equal the height of the latter? *Answ.* $10\frac{880}{1542}$.

63. In 1821, the population of Belfast and the suburbs, was about 40,000, and that of London was 1,274,800: how many towns, each containing as many inhabitants as Belfast, must be combined, to form one containing the same population as London? *Answ.* $31\frac{34800}{40000}$.

64. The linen exported from Ireland in 1809, was 43,904,382 yards: of how many pieces, each containing 25 yards, did this quantity consist? *Answ.* $1,756,175\frac{7}{25}$.

65. The entire quantity of tea sold by the East India Company in 1799, was 24,853,503 pounds: how many chests, each containing 87 pounds, would this quantity fill? *Answ.* $285,672\frac{23}{87}$.

66. If it be supposed, as in common circumstances is found to be nearly true, that as many persons die in 33 years as are equal to the entire population, it is required to find how many persons die each year, at an average, in the British dominions in Europe, the population (in 1821,) being 21,481,152. *Answ.* 650,944.

67. How many lessons of ninety-five lines each, are contained in Virgil's Æneid, the number of lines contained in that poem being nine thousand, eight hundred, and ninety-two? *Answ.* $104\frac{12}{95}$.

68. The following is the quantity, in hundreds weight, of the butter exported from Ireland, from 1800 till 1811 inclusive: required the quantity exported each year, at an average: in 1800, 263,288; in 1801, 178,496; in 1802, 304,666; in 1803, 396,353; in 1804, 334,251; in 1805, 320,155; in 1806, 294,415; in 1807, 338,508; in 1808, 333,998; in 1809, 346,856; in 1810, 385,953; in 1811, 390,833. *Answ.* 323,981.

69. The following is the quantity, in pounds, (avoirdupois,) of cotton-wool imported into England, from 1805 till 1810 inclusive: required the quantity imported each year, at an average: in 1805, 59,862,406; in 1806, 58,176,283; in 1807, 74,925,306; in 1808, 43,605,982; in 1809, 92,812,282; in 1810, 136,570,103. *Answ.* 77,658,727.

ABBREVIATIONS IN THE FUNDAMENTAL RULES.

The rules already given for performing the fundamental operations in Arithmetic, are of a nature completely general, and are fully adequate to the performance of all operations that can occur. Hence the pupil should be made substantially acquainted with them, before he proceeds to any thing else. As the operations, however, are often of a tedious and laborious nature, it is desirable to be able to employ easy and expeditious methods of performing them, when such can be obtained Some of the most useful

of these will be found in what follows. Others might have been added, but not so easy or useful, or so frequently applicable. With respect to those which are given, it will be well to proportion to the capacity and proficiency of the pupil, the number of them which he will be required to learn.

The principal abbreviation admissible in Addition, is *the adding of two or more figures at once.* This is peculiarly convenient, *when the sum of two or more figures is exactly ten.* Thus, in the annexed example, we may say 10 and 14 are 24, and 10 are 34. Then, carrying 3, we may proceed, 10 and 10 are 20, and 10 (5+5) are 30, and 8 are 38. 10 (3+6+1) and 16 (4+5+7) are 26, and 2 are 28. 14 (2+5+7) and 14 (8+6) are 28, and 4 are 32. 14 (3+4+7) and 14 (3+5+6) are 28, and 4 are 32. When, by attention and practice, facility in this mode of proceeding is acquired, it will be found to be of more value than might at first be imagined.

```
 41257
 63783
 56558
 38416
 77198
 45672
 ------
322884
```

To multiply by 5: *Add a cipher, or rather conceive it to be added, to the multiplicand, and take half the result.* This is evidently the same as multiplying by 10, and taking half the product. To multiply by 15: *Conceive a cipher to be added to the multiplicand, and to the result add half of itself.* To multiply by 25: *Conceive two ciphers to be added to the multiplicand, and take one-fourth of the result.* This is the same as multiplying by 100, and taking one-fourth of the product, which is done because 25 is one-fourth of 100. In like manner, because 125 is one-eighth of 1000; to multiply by 125: *Conceive three ciphers to be added to the multiplicand, and take one-eighth of the result.* To multiply by 75: *Conceive two ciphers to be added to the multiplicand, and from the result take one-fourth of itself.* To multiply by 35: *Conceive two ciphers to be added, take one-fourth, and set it so that it may be added to the multiplicand, with one cipher annexed.* In a similar manner, the product by 225 may be found from the product by 125. To multiply by 175: *Divide 700 times the multiplicand by 4.* To multiply by 275: *Divide 1100 times the multiplicand by 4.*

```
384729 × 15
1923645
--------
5770935

587 × 75
14675
------
44025

463 × 35
11575
------
16205
```

To multiply by 9: *Conceive a cipher to be added to the multiplicand, and from the result subtract the multiplicand;* or, simply, *conceive a cipher to be added, and from each figure of the result take the one before it.* Thus, 5 from 10 and 5 remain: and 7 are 8; 8 from 15, and 7 remain: 1 and 4 are 5; 5 from 7, and 2 remain: 6 from 14, and 8 remain: carry 1; 1 from 6,

```
6475 × 9
-------
58275
```

and 5 remain. To multiply by 11: *Conceive a cipher to be annexed to the multiplicand, and to the result add the multiplicand;* or, simply, *conceive a cipher to be annexed to the multiplicand, and to each figure of the result add the one which immediately precedes it.* Thus, 3 and 0 are 3: 8 and 3 are 11: carry 1; 1 and 4 are 5, and 8 are 13: 1 and 6 are 7, and 4 are 11: 1 and 6 are 7. To multiply by 99: *Conceive two ciphers to be annexed to the multiplicand, and from the result subtract the multiplicand;* or, simply, *conceive two ciphers to be annexed to the multiplicand, and from each figure in the result subtract the figure nearest it on the left side, except one.* Thus, 8 from 10, and 2 remain: 1 and 3 are 4; 4 from 10, and 6 remain: 1 and 4 are 5; 5 from 8, and 3 remain: 1 from 3, and 2 remain: 0 from 4, and 4 remain: 0 from 1, and 1 remains. To multiply by 101: *Conceive two ciphers to be annexed to the multiplicand, and to the result add the multiplicand;* or, simply, *conceive two ciphers to be annexed to the multiplicand, and to each figure of the result add the figure nearest it on the left side, except one.* Thus, 8 and 0 are 8; 3 and 0 are 3; 9 and 8 are 17; 1 and 5 are 6, and 3 are 9; 0 and 59 are 59. In general, to multiply by 9, 99, 999, &c.: *To the multiplicand annex as many ciphers as there are 9's, and from the result* subtract *the multiplicand:* and to multiply by 11, 101, 1001, &c: *To the multiplicand annex as many ciphers as there are digits in the multiplier, wanting one, and to the result* add *the multiplicand.*

$$\begin{array}{r} 6483 \times 11 \\ \hline 71313 \end{array}$$

$$\begin{array}{r} 1438 \times 99 \\ \hline 142362 \end{array}$$

$$\begin{array}{r} 5938 \times 101 \\ \hline 599738 \end{array}$$

The principle on which these last rules depend, may be generalized in its application in the following manner: *If the multiplier be a little less than* 100, 1000, *or any other number expressed by a unit with ciphers annexed, which number may be called the* approximate multiplier; *or if it be a little greater than one of these numbers, take the difference between it and whichever of them is nearest to it; in the former case, call this difference the* complement, *in the latter, the* excess; *then, to the multiplicand annex as many ciphers as there are in the approximate multiplier; and in the one case* subtract *from the result the product of the multiplicand and the complement; in the other,* add *to the result the product of the multiplicand and the excess.**

* It is obvious that this rule will be principally useful, when the complement or excess does not exceed 12. It may indeed be employed with considerable facility, when the complement or excess is between 12 and 20, if the pupil have committed to memory the additional part of the Multiplication Table given in the note in page 17, or by means of the method of multiplying by 13, 14, &c. illustrated in the following example: Here, 7 times 9 are 63; carry 6; 7 times 2 are 14, and 6 are 20, and 9 (the following figure,) are 29; carry 2: 7 times 3 are 21, and 2 are 23, and 2, (the following figure,) are 25; carry 2: 7 times 7 are 49, and 2 are 51, and 3 are 54; carry 5: 7 times 0 is 0, but 5 and 7 are 12. It is obvious that we may multiply by any number in

$$\begin{array}{r} 7329 \times 17 \\ 17 \\ \hline 124593 \end{array}$$

Thus, in the first of the annexed examples, the complement is 11, and, therefore, we multiply 74953 by 11, and set the product, and subtract in such a manner as if the upper line were 74953000. The remainder, 74128517, is the product required. In the second example, the excess is 12; therefore, the multiplicand is multiplied by 12, and the product set in such a manner as to be added to 793600. The sum is the product required.

```
74953    × 989
824483
-------
74128517

7936     × 112
95232
-------
888832
```

Another abbreviation, which may perhaps be more frequently useful than any of the above, consists in *deriving a product from others already found;* and in doing this, and indeed in all cases, it should be considered, that *it is a matter of indifference by which figure of the multiplier we multiply first, by which secondly, &c. provided we assign to the partial products their proper places.* In the annexed example, where the multiplier is 168428, the product by 4 is first found; this product is then multiplied by 7, to find the product by 28, because $28=4\times7$. The product thus found for 28 is then multiplied by 6, to find the product by 168, because $168=28\times6$. The sum of these partial products, properly arranged, is the product required. Even when no farther abbreviation can be employed, it sometimes renders the work less laborious to derive the product for a single digit from one found already: thus, the product by 6 will be found by doubling the product by 3; the product by 8 is 4 times the product by 2, &c. It is scarcely necessary to say, that when the same digit occurs twice, or oftener, in the multiplier, it is sufficient to multiply once by it, as the product thus found may be copied, when that digit recurs in the multiplier.

```
673849
168428
-------
2695396
18867772
113206632
----------------
113495039372
```

To divide by 5: *Double the dividend, and cut off the last figure of the result, the half of which figure will be the remainder.* This is obviously nothing else than doubling the dividend, and dividing by 10. To divide by 15, 35, 45, or 55: *Double the dividend, divide the result by* 30, 70, 90, *or* 110, *respectively, and for the true remainder take half the remainder thus found.* To divide by 25: *Multiply by* 4, *cut off two figures from the result, and take one-fourth of the number expressed by them for the remainder.* To divide by 125: *Multiply the dividend by* 8, *cut off three figures from the result, and take one-eighth*

tween 20 and 30, by multiplying by the units' figure, and adding each time, along with what is carried, *twice* the following figure of the multiplicand; and the principle may be readily extended to numbers above 30, but the operation becomes more complicated and difficult.

of the number expressed by them for the remainder. To divide by 75: *Multiply the dividend by* 4, *divide the product by* 300, *and for the true remainder take one-fourth of the remainder thus found.* To divide by 175, 225, and 275: *Multiply the dividend by* 4, *divide the result in the first case by* 700, *in the second by* 900, *and in the third by* 1100, *and take one-fourth of the remainder, in each case, for the true remainder.*

To divide by a number expressed by any number of 9's: *Divide the dividend by the number expressed by a unit, with as many ciphers annexed as there are figures in the divisor, divide the quotient thus found by the same number, and thus proceed as long as possible; then add together all the remainders and all the quotients, carrying from the sum of the remainders to the sum of the quotients, and adding also to the sum of the remainders what is carried to the quotients; the sums thus found will be the quotient and the remainder required.* Should the remainder thus found be equal to the divisor, it is evident that there is really no remainder; but that the quotient must be increased by unity. The operation will be facilitated by arranging the lines in the manner adopted in the annexed example. Here a vertical line is drawn, cutting three figures from the dividend; then 4, the last figure of the quotient, thus obtained, is set below the unit figure of the remainder, and the other figures are set in the corresponding places. The vertical line being continued, we have 5739 for the second quotient, the unit figure of which is set in the units' place of the remainder, and the other figures as before; the third quotient, 5, being set in the same manner, the lines are added together: the sum of the column of hundreds in the remainder is 16, from which 1 is carried to the units' column of the quotients, and also added to the remainder, 679: the true quotient and remainder, therefore, are 5745029, and 680.

5739284	651÷999
5739	284
5	739
	5
5745029	679
	1
	680

The second example, in which 3467581 is divided by 989, shows the mode of generalizing this principle. Here the complement, *(see page* 32,*)* is (1000—989,) 11, and the first quotient is 3467, which, instead of being simply set beneath the given dividend, is multiplied by the complement, and the product is set as in the preceding example; we have then the quotient 38, which is multiplied, in like manner, by the complement, and the product (418,) set as before. In the addition, the sum of the hundreds' column of the remainders is 11, from which one is carried to the quotient, and is also multiplied by the complement; the product thus found is also added to the remainder; hence, the true quotient and remainder are 3506 and 147.

3467	581÷989
38	137
	418
3506	136
	11
	147

The reason of these operations will be understood from the following illustration. The quotient by 100, in the annexed example, is 8053, and the remainder 93. Now, it is evident, that were this multiplied by 97, instead of 100, the product would be less than 805300 by 3 times 8053, and therefore, 3 times 8053, or 24159, still remains to be divided by 97, and the quotient to be added to 8503, the part of the quotient already found; this being divided by 100, the quotient is 241, and the remainder 59; then, as before, were 241 multiplied by 97 (100—3,) it is obvious that the product would be less than 24100 by 3 times 241; whence, the reason of multiplying 241 by 3 is evident: and thus the reason of all the similar multiplications is shown. The sum of the partial remainders is 196, from which 1 is carried for 100, because 97 is contained once in 100, with the remainder 3; this remainder being annexed to 96, the other part of the remainder, the sum is 99, which exceeding 97 by 2, one is added to the quotient. Hence, the true quotient and remainder are 8303, and 2. It is evident that for each hundred in the remainder, in the present example, 3, which is equal to the complement, must be added to the number expressed by the units and tens of the remainder; and hence the reason of completing the remainder by the product of the complement and the number carried from the remainder to the quotient.

8053	93÷97
241	59
7	23
	21
8302	96
1	3
8303	99
	97
	2

In long operations in Division, or when the same divisor is to be frequently employed, *it is often of advantage to form a table containing the several products of the divisor and the nine digits.** Thus, to divide 341949853 by 73, let a table of the products of 73 be formed, as in the margin; then, by looking in the table we find that the nearest product below 341 is 292, the product by 4; we place 4, therefore, in the quotient, and subtract 292 from 341; the remainder is 49, to which 9 is brought down: we then see in the table that 438, the product for 6, is the nearest product below 499; 6 is, therefore, put in the quotient, and 438 subtracted from 499, &c.

```
73)341949853(4684244
   292·····
   499
   438
    614
    584
     309
     292
      178
      146
       325
       292
        333
        292
         41
```

1...	73
2...	146
3...	219
4...	292
5...	365
6...	438
7...	511
8...	584
9...	657

* In Multiplication also, the multiplicand may be tabulated in like manner, when it is to be frequently used. The advantage, however, is not so great as in Division. Such a table may be readily formed by successive additions of the multiplicand in the one case, and of the divisor in the other.

In the article on Division, the method of dividing by the factors of the divisor, has been given. In most cases in which this method is useful, the factors are readily discovered by any one who is well acquainted with the Multiplication Table. In some cases, however, which may be of use, they are not so easily discovered; and, besides, the discovering of the factors of composite numbers is a problem of some interest in the theory of numbers, in fractions, &c. Hence, the following observations on this subject may be found useful.

2 is a factor of every number which ends in 0, or in any of the even* digits, 2, 4, 6, 8: thus, 2 measures 226; for, since it measures 10, and consequently the multiples of 10, it must measure 220; and measuring 220 and 6, it must measure their sum. It may be proved in the same manner, that 5 is a measure of every number that ends in 0 or 5. 3 is a measure of every number, the sum of whose digits is divisible by 3; and 9 is a measure of every number, the sum of whose digits is measured by 9.—(*See p.* 20.)

A composite number, suppose 600, is decomposed into all its prime, or simple factors in the following manner: Divide by 2, (the least prime number;) divide the quotient, 300, by 2, and the next quotient, 150, by 2, also; let the quotient, 75, which is not divisible by 2, be divided by 3, (the next prime number;) then the quotient, 25, which is not measured by 3, being divided by 5, (the prime number next in order,) and the result by 5, we have the quotient 1, and the decomposition is completed; 600 being equal to the continual product of 2, 2, 2, 3, 5, 5. If none of the prime numbers, 2, 3, 5, 7, 11, &c. measure the given number, it is prime. In this case it is unnecessary to carry the divison any farther, when the quotients begin to be less than the divisors which produce them. The decomposition will be often facilitated by the remarks in the preceding paragraph: thus, 245 has evidently the factor 5; and dividing by it, we have 49, the factors of which are 7 and 7; and therefore the simple, or prime factors of 245 are 5, 7, and 7.

2	600
2	300
3	150
5	75
5	25
	5
	1

Having found the simple divisors of a number, we may thence find all the divisors which it admits. Thus, to resume the same number, 600, let all its simple factors, together with unity, (which is a divisor of every whole number,) be set as in the margin; then set unity as the first divisor, and, for the second, multiply it by 2, the second simple divisor, and the result is 2 for another divisor. Then, the next simple factor being 2, we multiply 1 and 2 (the divi-

1	2	2	2	3	5	5
1	2	4	8	3	5	25
				6	10	50
				12	20	100
				24	40	200
					15	75
					30	150
					60	300
					120	600

* *Even* numbers are those which are divisible by 2, without remainder; all others are called *odd* numbers.

sors already found,) by it; the products are 2 and 4, the former of which is rejected, as it is the same as one already found. We next multiply the divisors, 1, 2, and 4, by the next simple factor 2, and reject the products 2 and 4, as they are the same as two divisors already found. The next divisor is 3, and by this the divisors 1, 2, 4, 8, are multiplied: the products, 3, 6, 12, 24, are other divisors. Proceeding thus with the two remaining factors, 5 and 5, we find the divisors of 600 to be, 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, &c. being twenty-four in number; and these are the only divisors which it admits.

TABLES OF MONEY, WEIGHTS, MEASURES, &c.*

TABLE OF MONEY.

2 farthings	= 1 halfpenny	
2 halfpence, or 4 farthings	= 1 penny.....	*d.*†
12 pence	= 1 shilling ...	*s.* or /
20 shillings	= 1 pound.....	£
21 shillings	= 1 guinea.‡	

Hence, a pound contains 240 pence, 480 halfpence, or 960 farthings; and a guinea, 252 pence. A half-guinea is 10/6.

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* Before the pupil proceeds to Reduction, the Compound Rules, &c. he should be acquainted with the divisions of the money, weights, and measures, which are most generally used. The teacher will exercise his own judgment in determining which of the Tables here given should be committed to memory: perhaps the parts that are printed in the largest type may be sufficient. The Tables fixed on for this purpose should be severally committed immediately before the pupil commences those parts of Reduction in which they are respectively employed; and when he commences the corresponding parts of the Compound Rules, it will have a good effect to require him to revise them.

† This character, and others similarly placed, in this and some of the following Tables, are used as abbreviations for the names that precede them. The marks of abbreviation are generally omitted, when they are sufficiently understood without explanation.

Farthings were formerly denoted by *q.*; but now ¼, annexed to pence, denotes a farthing; ½, a halfpenny, or 2 farthings; and ¾, three farthings.—*£. s. d.* and *q.* are the initial letters of the Latin words, *libra*, *solidus*, *denarius*, and *quadrans;* which denote *pound*, *shilling*, *penny*, and *farthing*, or *quarter*, respectively. The character / is a corruption of the long *f*, arising from rapidity in making it.

‡ Other coins (some of them mentioned chiefly in old authors,) are the groat, value 4*d.*; the crown, 5*s.*; the noble, 6*s.* 8*d.*; the angel, 10*s.*; the mark, or merk, 13*s.* 4*d.*; the pistole, about 16*s.* 10*d.*; the moidore, 27*s.*

The guinea (so called because the gold of which the first guineas were made, was procured from Guinea, in Africa) originally bore the figure of an elephant, as an emblem of the place from which the gold was brought. It passed at first for 20*s.* British; but, in consequence of a scarcity of gold, it afterwards rose to 21*s* 6*d.*: at a subsequent period it fell to its present value. 44½ guineas weigh a pound troy, and hence the weight of each is a little more than 5 penny-weights, 9 grains. The pound is so called, because anciently the silver for it weighed a pound troy.

In the year 1825, the currency of Ireland was assimilated to that of Great Britain. The

TABLE OF TROY WEIGHT.*

24 grains..... = 1 penny-weight, dwt.
20 penny-weights..... = 1 ounce,..... oz.
12 ounces..... = 1 pound..... ℔.

Hence, an ounce contains 480 grains; and a pound, 240 penny-weights, or 5760 grains.

This weight was formerly used for weighing articles of every kind: it is now used in weighing gold, silver, jewels, and liquors; and in philosophical experiments.

It is also employed by apothecaries in mixing their medicines, though they buy and sell them by avoirdupois weight. When troy weight is thus used, it is called APOTHECARIES' WEIGHT; but in this case the ounce (℥) is divided into 8 *drams*, (ʒ) the dram into 3 *scruples*, (℈) and the scruple into 20 *grains*.

TABLE OF AVOIRDUPOIS WEIGHT.

16 drams..... = 1 ounce,..... oz.
16 ounces..... = 1 pound,..... ℔.
28 pounds..... = 1 quarter of a hundred, q. or qr.
4 quarters, or 112 ℔s..... = 1 hundred weight,..... cwt. or c.
20 hundreds..... = 1 ton
Also, 14 pounds..... = 1 stone
8 stone..... = 1 cwt.

The hundred weight here mentioned is sometimes called the *great* hundred, or the *standard* hundred, to distinguish it from hundreds of different magnitudes, which are used in particular places. One of the most general of these is the *long* hundred, or the hundred in *long* weight, which contains 120 pounds: hence,

In Long Weight, 30 pounds avoirdupois ... = 1 quarter;
4 quarters, or 120 pounds = 1 hundred weight.

The use of this weight, however, is now illegal.

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that time, though accounts were kept in both countries in the same denominations. the Irish pounds, shillings, pence, and farthings, were of less value than those of Britain, 13 of each denomination in the former country being equivalent only to 12 of the corresponding denomination in the latter. Thus, the British shilling was equivalent to 13 pence Irish; and, while the guinea was £1 1 0 British, it was equivalent to £1 2 9 Irish.

* Troy weight was introduced into Europe from Cairo in Egypt, about the time of the Crusades, and was first adopted in Troyes, a city in France, where great fairs were held, and whence it has its name.

The weights and measures used in the British Empire, as well as in almost all other places, were derived originally from very imperfect standards. As the origin of weights, a grain of wheat was taken from the middle of the ear, and being well dried, was used as a weight, and called by its original name, a *grain*. A weight equal to 32 grains was called a *penny-weight*, from its being the weight of the silver penny then in currency. A weight equal to 20 penny-weights, was called an *ounce*,—a word of the same origin (the Latin word, *uncia*,) and import as the word *inch*, each signifying a *twelfth-part*, the one appropriated to denote the twelfth part of a pound, and the other the twelfth part of a foot. At a later period the penny-weight came to be divided, not into 32, but into 24

The stone, in the greater number of places, is 14 pounds, which alone is the legal one; but in different parts of England it is of various magnitudes, from 8 to 16 pounds. In Ireland, also, in the sale of some articles, the stone of 16 pounds, or one-seventh of the standard hundred, is used.* A ton of stones is 21 hundreds, *long* weight.

TABLE OF LONG MEASURE.

12 lines	=1 inch†	40 perches	=1 furlong
12 inches	=1 foot	8 furlongs	=1 mile
3 feet	=1 yard	3 miles	=1 league.
$5\frac{1}{2}$ yards	=1 perch‡		

A *fathom* is 2 yards, or 6 feet; a *hand*, (used in measuring horses,) is 4 inches; a *span*, 9 inches.

From this Table, by an easy reduction, it will appear that a mile contains 320 perches, 1760 yards, or 5280 feet.

Till the year 1826, the perch in Ireland contained 7 yards instead of $5\frac{1}{2}$, so that 11 Irish miles were equivalent to 14 British ones, and the Irish mile contained 2240 yards, or 6720 feet.

TABLE OF CLOTH MEASURE.

4 nails =1 quarter	4 quarters =1 yard.

A Flemish ell is 3 quarters of a yard; an English ell, 5 quarters, or a yard and a quarter; and a French ell, 6 quarters, or a yard and a half.

Cloth measure is a species of long measure, and the yard is the same in both. Hence, a quarter of a yard is 9 inches; and a nail, 2 inches and a quarter.

TABLE OF SQUARE MEASURE, OR OF THE MEASURES OF SURFACES.

144 square ‖ inches	=1 square foot
9 square feet	=1 square yard
$30\frac{1}{4}$ square yards	=1 square perch.

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equal parts: each of which, however, was called a grain, though really a third part greater than the original grain.

* In England, 14 pounds of wool=1 stone, 2 stone=1 tod, 6 tods and a half=1 wey, 2 weys=1 sack, 12 sacks=1 last: and a pack of wool=240 pounds.

† 3 *barleycorns* make an inch. The barleycorn, however, is never employed now as a measure. Instead, also, of being divided into lines, the inch is now generally divided into tenths.

‡ The perch is also sometimes called a *pole* or *rod*. Each of the names given to this measure is expressive of the instrument by which it was measured—a *rod*, a *pole*, or *perche*, a French word of the same import. In some counties of England the perch is 6 yards, in some 7 yards, and in others 8 yards. In *Cunningham measure* it is $6\frac{1}{2}$ yards; in *forest measure*, 8 yards: and in *woodland*, or *Burleigh measure*, 6 yards. None of these, however, is now legal. The yard is said to have been taken from the length of the arm of Henry I. of England, and seems to be the origin of long measure.

‖ A *Square* is a figure which has four equal sides, each perpendicular to the adjacent ones. A *square inch* is a square each of whose sides is an inch in length; a *square yard*, a square, each of whose sides is a yard in length, &c. The table of square measure is

TABLE OF LAND MEASURE.

40 square perches..... =1 rood | 4 roods..... =1 acre.*

This is obviously a continuation of the Table of square measure.

TABLE OF CUBIC† OR SOLID MEASURE, OR OF THE MEASURES OF CAPACITY.

1728 cubic inches. =1 cubic foot | 27 cubic feet..... =1 cubic yard.

LIQUID MEASURE.

4 naggins, or gills..... =1 pint | 2 pints..... =1 quart
4 quarts, or 8 pints..... =1 gallon.‡

This is evidently a species of cubic measure.

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formed from the table of long measure, by multiplying each lineal dimension by itself; thus, a square foot is=12×12=144 square inches, &c.

* In measuring land, surveyors use a *chain*, which is 4 perches in length, and is divided into 100 equal parts, called *links*. They also compute by chains and links, but exhibit the result in acres, roods, and perches. 10 square chains, or 100,000 square links, are an acre. It may be observed, also, that 640 acres are a *square mile*; and that a *hide* of land, mentioned by old writers, is 100 acres.

In *Irish measure*, or, as it is often called, *Irish plantation measure*, 64 acres are equivalent to 49 acres in *Forest measure*; 625 to 784 in *Cunningham measure*; 36 to 49 in *Woodland, or Burleigh measure*; 121 to 196 in *English Statute measure*; and 1369 to 1764 *Scotch acres*. These numbers are found by multiplying each of the numbers expressing the length of the perches, in the different kinds of measure, by itself. Thus, the English perch being 11 half yards, and the Irish 14 half yards, we have 121=11×11, and 196=14×14. Hence, to prevent a common mistake, it may be proper to remark, that the difference in the comparative magnitudes of the *acres* is much greater than that of the *perches* or *miles*. The chain in Scotland, prior to 1826, was fixed at 74 feet and hence, the chain in Ireland being 84 feet, we find the equivalent numbers for Irish and Scotch acres, by multiplying the halves of 74 and 84, respectively, by themselves.

From these principles the following Table is constructed, which is useful in reducing Irish measure to any other. The first line shows the quantities equivalent to 1 Irish acre, the second those equivalent to 10, &c.; and by means of Compound Multiplication, and Compound Addition, the values of other numbers of acres may be readily found. Thus, for 729 Irish acres, add together 7 times the numbers in the third line, twice those in the second, and 9 times those in the first. It may be observed that, now, the only legal measure is the *English Statute measure*

Irish.	*Forest.*	*Cunningham.*	*Scotch.*	*Woodland.*	*English.*
a.	*a. r. p.10ths.*	*a. r. p.10ths.*	*a. r. p.10ths.*	*a. r. p.10ths.*	*a. r. p.10ths.*
1	0 3 2 5	1 1 0 7	1 1 6 2	1 1 17 8	1 2 19 2
10	7 2 25 0	12 2 7 0	12 3 21 6	13 2 17 8	16 0 31 7
100	76 2 10 0	125 1 30 4	128 3 16 5	136 0 17 8	161 3 37 4
1000	765 2 20 0	1254 1 24 0	1288 2 5 1	1361 0 17 8	1619 3 13 6

† A *cube* is a figure contained by six equal squares. (Dice afford a familiar instance of this figure.) A *cubic inch* is a cube whose sides are each a square inch; a *cubic foot*, a cube whose sides are each a square foot, &c. It may be remarked, that 1728 is equal to 12×12×12, and 27=3×3×3.

‡ The English hogshead, in wine measure, contains 63 gallons; the pipe, 2 hogsheads,

TABLE OF DRY MEASURE.

2 pints	=1 quart	4 pecks, or 8 gallons	=1 bushel
2 quarts	=1 pottle	8 bushels	=1 quarter
2 pottles, or 4 quarts	=1 gallon	5 quarters	=1 wey
2 gallons	=1 peck	2 weys, or 10 qrs.	=1 last.

This measure, which is another species of cubic measure, is used in measuring grain, seeds, and various kinds of dry articles. In many places, however, these are bought and sold by weight.

TABLE OF TIME.

60 seconds	=1 minute
60 minutes	=1 hour
24 hours	=1 day
7 days	=1 week
52 weeks and 1 day, or 365 days	=1 common year
52 weeks and 2 days, or 366 days	=1 leap year.

The year is divided into 12 portions, called *calendar months*, the names of which are January, February, March, April, May, June, July, August, September, October, November, December. Of these, April, June, September, and November, have 30 days each; and the rest, except February, have 31 days each. In leap years* February has 29 days; in common years, 28 days; † so that a leap year contains 366 days, and any other 365. The precise length of the year is found to be 365 days, 5 hours, 48 minutes, 48 seconds: it is, therefore, 365 days, 6 hours, nearly.

---

or 126 gallons; and the tun, 2 pipes, or 4 hogsheads, or 252 gallons. In the measure of foreign wines, however, there are great varieties. A tierce (the third part of a pipe) is 42 gallons, and a puncheon is 2 tierces, or 84 gallons.

In *beer measure*, 9 gallons=1 firkin; 2 firkins, or 18 gallons=1 kilderkin; 2 kilderkins, or 36 gallons=1 barrel 54 gallons=1 hogshead 2 hogsheads, or 108 gallons=1 butt.

The hogshead of ale, in London, contains 48 gallons; the barrel, 32; the kilderkin, 16; and the firkin, 8.

In England, in the country, both in ale and beer measure, the hogshead contains 51 gallons; the barrel, 34; the kilderkin, 17; the firkin, 8½.

* Leap years occur at intervals of 4 years, and may be known by dividing by 4 the number expressed by the last two figures in the number of the year, according to the Christian era: if there be no remainder, it is leap year; otherwise, the remainder shows how many years it is after leap year. To this there is one exception, as the exact centuries are not leap years, except when the number of centuries is divisible by 4, without remainder. Thus, the year 1820 was leap year, because 20 is divisible by 4: but 1823 was the third year after leap year, because 3 remain when 23 is divided by 4. Also, the year 2000 will be a leap year, but 1900 not, as 20 is divisible by 4, but 19 not.

† Learners may easily remember the number of days contained in each month, by recollecting that *the months are long and short alternately*, with the exception of August, which is *long*, while the months after it follow the rule.

TABLE OF THE DIVISION OF THE CIRCLE.

The circumference of every circle is supposed to be divided into 360 equal parts, called *degrees;** each degree is subdivided into 60 equal parts, called *minutes;* and each minute into 60 equal parts, called *seconds.*

MISCELLANEOUS TABLE.

12 articles.....	=1 dozen	6 score or 120..	=1 great, or long hundred
12 dozen.....	=1 gross	24 sheets of paper	=1 quire
20 articles.....	=1 score	20 quires.....	=1 ream.
5 score.....	=1 hundred		

An Act of Parliament, already referred to, "for Ascertaining and Establishing Uniformity of Weights and Measures," came into operation in January, 1826. By this act, "the standard yard of 1760," in custody of the Clerk of the House of Commons, is to be the standard yard, when the temperature is at 62° of Fahrenheit's thermometer; and this is to be the origin of all other measures of length. The thirty-sixth part of this yard is an inch; and the length of a pendulum vibrating seconds in the latitude of London is found to be 39·1393 such inches; that is, according to the notation of decimal fractions, 39 inches and 1393 ten thousandths of another. This affords the means of recovering the standard yard, should it be lost.

It is enacted, in like manner, that "the Troy pound of 1758," in custody of the same officer, shall continue to be the standard unit of weight; and, this pound containing 5760 grains, the avoirdupois shall contain 7000 grains. Hence, 144 pounds avoirdupois are equivalent to 175 pounds troy; and 1 lb. avoirdupois is equal to 1 lb. 2 oz. 11 dwts. 16 grs. troy; but the ounce troy exceeds the ounce avoirdupois by 42½ grains. The weight of a cubic inch of distilled water is 252·458 grains troy, the barometer being at 30 inches, and the thermometer at 62°; and thus there is a means of recovering the standard pound, if it should be lost.

It is farther enacted, that the standard measure of capacity for all liquids, and for dry goods not measured by heaping, shall be a gallon containing 10 pounds avoirdupois of distilled water in the same state

* Degrees, minutes, and seconds are marked thus: °, ′, ″. Hence, the expression, 41° 24′ 54″, is read, 41 degrees, 24 minutes, 54 seconds. The reason of these marks being employed will appear evident from the consideration, that minutes and seconds are only abbreviated expressions for *first* minutes, or minutes of the *first order*, and *second* minutes, or minutes of the second order; *minutes*, in each instance, signifying *small parts.* It may be proper to remark, that the *circumference* of a circle is the line which contains it; that all straight lines drawn from the centre to the circumference are equal; that any of these lines is called a *radius;* and that a line drawn through the centre, and terminated both ways by the circumference, is called a *diameter.* It may be farther remarked, that seconds, both in time and in the circle, were formerly divided each into 60 *thirds*, but that they are now divided into tenths or hundreds.

of the barometer and thermometer. This gallon, therefore, contains 277·274 cubic inches; and the bushel weighs 80 pounds, and contains 2218·192 cubic inches. It is a curious coincidence, that the cube of one-sixth of the length of the seconds' pendulum is so nearly equal to the gallon, as to exceed it by only about three-tenths of an inch.

In Heaped Measure, the diameter is to be at least double the depth, and the height of the cone or heap is to be three-fourths of the depth. The measures are to be made cylindrical, with a plain and even bottom; and the outside of the measure is to be the extremity of the base of the cone. The internal capacity is to be the same as that of the corresponding liquid measure. The bushel is to be the standard of this measure, and its diameter is to be 19½ inches from outside to outside.*

REDUCTION.

When a given quantity is expressed in any denomination, Reduction shows the method of expressing it in another.

Reduction consists of two branches, or problems; one in which quantities are reduced to a *lower* denomination; the other in which they are reduced to a *higher*. The former is often called Reduction Descending, and the latter Reduction Ascending. In the former *Multiplication* is employed; in the latter *Division*, according to the following rules:

Problem 1. *To reduce a quantity to* a lower denomination.

Rule.—*Multiply* the number which expresses the quantity by the number which shows how many of the lower denomination make *one* of the higher; and if any part of the given quantity be already of the lower denomination, add to it the product.

Problem 2. *To reduce a quantity to* a higher denomination.

Rule.—*Divide* the number expressing the quantity, by the number which shows how many of the denomination in which it is, make *one* of the higher denomination: the quotient will be of the higher denomination, and if there be any remainder, it will be of the lower.

When there are intermediate denominations between that in which the quantity is given, and that to which it is to

* It may be useful for the more advanced pupil to know, that the o'd French foot is 12·78933 English inches, or 12 inches, 9¼ lines, nearly; the toise, or fathom, or 6 French feet, 6 feet, 4·736 inches, English, or 6 feet, 4 inches, 9 lines, nearly; and the metre, 39·3702, or 1 yard, 3 inches, and 3 eighths, nearly.

be reduced, it is generally better to reduce it by successive steps; first, to one or more of the intermediate denominations, and then to the required denomination.

Methods of Proof.

1. To the answer found by either of the preceding rules, apply the other rule, and if the result be the same as the given number, the work is correct.

2. The operation may be proved by the methods of proving Multiplication or Division.

The reasons of the rules will be explained in the illustrations of the examples.

REDUCTION OF MONEY.

Exam. 1. Reduce £59 to farthings.

In this example the pounds are multiplied by 20, to reduce them to shillings, because there are 20 shillings in each pound. The shillings, in like manner, are multiplied by 12, to reduce them to pence, and the pence by 4, to reduce them to farthings, because there are 12 pence in each shilling, and 4 farthings in each penny. Hence, it appears, that in £59 there 1180 shillings, 14160 pence, or 56640 farthings. The operation is proved by dividing successively by 4, 12, and 20, the former multipliers, in a reversed order; and the work is correct, since the final quotient, £59, is the same as the quantity given in the question.

```
£
59
20
1180 shillings.
12
14160 pence.
4
4)56640 farthings.
12)14160 pence.
2|0) 118|0 shillings.
£59, proof.
```

Exam. 2. Reduce £94 12*s.* 8¼*d.* to farthings.

In this example in the multiplication by 20, 12 shillings are added to the product; in the multiplication by 12, 8 pence are added, and in the multiplication by 4, 1 farthing is taken in. Hence, the answer is 90849 farthings.

```
£  s.  d.
94 12  8¼
20
1892 shillings
12
22712 pence
4
90849 farthings.
```

```
4)90849
12)22712¼
2|0) 189|2 8¼
£94 12 8¼, proof
```

Exam. 3. Reduce 83918 farthings to pounds, &c.

In this example, the farthings are divided by 4, because in the same sum there are 4 times as many farthings as there are pence: for a similar reason, the pence are divided by 12, to reduce them to shillings, and the shillings thus found, by 20, to reduce them to pounds. Hence, it appears that 83918 farthings are equivalent

```
4) 83918 farthings.
12) 20979½
2|0) 174|8 3½.
£87 8 3½, Ans
```

to 20979 pence, with two farthings, or a halfpenny; to 1748 shillings, and 3 pence halfpenny; or finally, to £87 8 3½. The operation would be proved by reducing the answer to farthings, in the manner exhibited in the foregoing example. By this means we should obtain 83918, which being the same as the given number of farthings, it follows that the answer is correct.

	Exercises.	*Answers.*
1.	Reduce £341 0 4 to pence	81844
2.	——— £97 17 3½ to halfpence	46975
3.	——— £783 2 3¼ to farthings	751789
4.	——— £481 0 0 to pence	115440
5.	——— 33333 pence to pounds	£138 17 9
6.	——— £1023 16 0 to shillings	20476
7.	——— £113 15 3 to pence	27303
8.	——— £1 2 9 to pence	273
9.	——— 2300 pence to pounds	£9 11 8
10.	——— £1 9 3 to pence	351
11.	——— £463 19 7½ to farthings	445422
12.	——— £1 14 1½ to farthings	1638
13.	——— 95283 halfpence to pounds	£198 10 1½
14.	——— £38 14 0 to pence	9288
15.	——— £133 6 8 to farthings	128000
16.	——— £7853 19 11¾ to farthings	7539839
17.	——— 47589 pence to pounds	£198 5 9
18.	——— 1234567 farthings to pounds	£1286 0 1¾
19.	——— £53 14 0 to farthings	51552
20.	——— 75396 shillings to pounds	£3769 16 0
21.	——— 12/4 to pence	160
22.	——— 47474747 farthings to pounds	£49452 17 2¾
23.	——— 348 guineas to pence	87696
24.	——— 967 guineas to pounds*	£1015 7
25.	——— £497 12 8 to farthings	477728
26.	——— 69173 pence to guineas	274 gs. and 10/5
27.	——— £1000 to guineas	952 gs. and 8/0
28.	——— £371 14 10 to halfpence	178436
29.	——— 823903 farthings to guineas	817 gs. and 7/7¾

* To reduce guineas to pounds, multiply them by 21: the product is shillings, which reduce to pounds. To reduce pounds to guineas, reduce them to shillings, and divide those shillings by 21.

REDUCTION OF TROY WEIGHT.

Exam. 4. Reduce 1 lb. 5 oz. 12 dwts. 13 grains, to grains.

In this example the multipliers are 12, 20, and 24, because in each pound there are 12 ounces, in each ounce 20 penny-weights, in each penny-weight 24 grains; and the 5 ounces, 12 penny-weights, and 13 grains, are taken in, as in the former examples.

```
 lb oz. dwt. grs.
  1  5  12  13
 12
 —
 17
 20
 —
352
 24
——
1421
704
——
8461 grains, Answ.
```

```
24)8461
——
20)35|2 13
——
12)17 12 13
——
lb 1 5 12 13,
Proof.
```

Exam. 5. Reduce 111111 grains to pounds.

In this example the grains are divided by 24, (or by 6 × 4,) and the result is 4629 penny-weights, 15 grains; these penny-weights are again divided by 20, and the result is 231 oz. 9 dwts. 15 grs.; and these ounces being divided by 12, the final result is 19 lbs. 3 oz. 9 dwts. 15 grs. the answer required. The operation would be proved in the manner in which the last example was wrought.

```
 grains
6)111111
——
4) 18518 3
——
2|0) 462|9 15*
——
12)231 9 15
——
lbs. 19 3 9 15,
Answ.
```

	Exercises.	*Answers.*
30.	Reduce 11 oz. 12 dwts. 12 grs. to grains.	5580
31.	——— 3 lbs. 7 oz. to penny-weights.	860
32.	——— 1785 dwts. to pounds	7 lbs. 5 oz. 5 dwts.
33.	——— 1785 grains to ounces	3 oz. 14 dwts. 9 grs.
34.	——— 785398 grains to pounds	136 lbs. 4 oz. 4 dwts. 22 grs.
35.	——— 29 pounds to grains	167040

* For the mode of finding the true remainder here, see page 27.

REDUCTION OF AVOIRDUPOIS WEIGHT.

Exam. 6. Reduce 27 cwt. 2 qrs. 22 lbs. to pounds.

In this example, the hundreds being multiplied by 4, the product is quarters, because there are 4 quarters in each hundred; and the quarters being multiplied by 28, the product is pounds, because in each quarter there are 28 pounds. The odd quarters and pounds are taken in, as the operation proceeds.

```
cwt. qrs. lbs.
 27   2   22        4)3102
  4                 7) 775  2
 110                4) 110  22
  28               cwt. 27  2  22,
 902                    proof.
220
3102 pounds, Answ.
```

Hundreds may be very easily reduced to pounds by the following rule, derived from the principles explained in pages 32 and 33:

Multiply the hundreds by 12, without writing the multiplier, and, as the work proceeds, take in the odd pounds, and 28 pounds if there be 1 quarter, 56 pounds if there be 2, or 84 pounds if there be 3: set the product below the hundreds, two places to the right hand, and add the two lines together.

Thus, in working the preceding example in this manner, 27 is multiplied by 12, and the product increased by 22 and 56, and the result, 402, being added to 2700, (which is evidently done in consequence of the way in which the numbers are set,) the sum, 3102 lbs. is the answer, the same as before.

```
cwt. qrs. lbs.
 27   2   22
 402      56
3102 pounds, Answ.
```

Exam. 7. Reduce 591241 pounds to tons.

In this example, as in the proof of the last, the pounds are divided by 28, (or, which is equivalent, by 4, and the quotient by 7,) to reduce them to quarters; the quarters by 4 to reduce them to hundreds; and the hundreds by 20 to reduce them to tons. From the operation, it appears, that 591241 pounds are equivalent to 21115 qrs. 21 lbs.; or to 5278 cwt. 3 qrs. 21 lbs.; or to 263 tons, 18 cwt. 3 qrs. 21 lbs.

```
lbs.                          263  18  3  21
4)591241                       20
7)147810  1                  5278
4) 21115  21                    4
2|0) 527|8  3  21            21115
                                28
Tons 263  18  3  21,        168941
        Answ.                42230
                          lbs. 591241, proof.
```

Exercises.	*Answers.*
36. Reduce 137 tons to hundreds	2740
37. ..... 47 cwt. 3 qrs. 24 lbs. to pounds	5372
38. ..... 135 cwt. 3 qrs. 11 lbs. to pounds...	15215
39. ..... 484 cwt. 1 qr. 20 lbs. to pounds ...	54256
40. ..... 13 lbs. 4 oz. 5 drams, to drams	3397
41. ..... 35 cwt. 2 qrs. 19 lbs. to pounds ...	3995
42. ..... 313 cwt. 1 qr. 25 lbs. to pounds ...	35109
43. ..... 1 ton to ounces	35840
44. ..... 214 cwt. 3 qrs. to pounds	24052
45. ..... 94 cwt. 1 qr. 11 lbs. to pounds	10567
46. ..... 59 tons, 11 cwt. 8 lbs. to pounds...	133400
47. ..... 57386 pounds to hundreds	512 cwt. 1 q. 14 lbs.
48. ..... 57386 quarters to tons	717 t. 6 cwt. 2 qrs.
49. ..... 57386 drams to hundreds	2 c. 2 oz. 10 drams.
50. ..... 1000000 pounds to tons.....	446 t. 8 c. 2 q. 8 lbs.
51. ..... 1000000 ounces to hundreds.....	558 cwt. 0 qr. 4 lbs.
52. ..... 38489 lbs. to cwt. *long weight*.....	320 cwt. 2 q. 29 lbs.
53. ..... 40865 pounds to hundreds.....	364 cwt. 3 q. 13 lbs.
54. ..... 92950 pounds to tons.....	41 t. 9 c. 3 q. 18 lbs.
55. ..... 238 c. 2 q. 10 lbs. *long wt.* to lbs. .	28630
56. ..... 783914 pounds to tons	349 t. 19 c. 26 lbs.

Exam. 8. Reduce 53 miles, 3 furlongs, 12 perches, 4 yards, to yards.

In this example, because the English perch contains $5\frac{1}{2}$ yards, the perches are multiplied by 5, and half of them taken, and the 4 yards are added in with the two results. It may be observed, that if the last figure be rejected from the product by 5, there will remain half the number of perches; and if the figure rejected be 5, it will be half a yard, or a foot and a half. This example would be proved in the manner in which the next is wrought.

M.	F.	P.	Y.
53	3	12	4
8			

427 furlongs.
40

17092 perches.
$5\frac{1}{2}$

85460
8546

94010 yards, *Answ.*

Exam. 9. Reduce 231278 yards to miles.

In this example, the yards are multiplied by 2, to reduce them to half yards, and the result divided by 11, the half yards in a perch, to reduce it to perches. The remainder is 6 half yards, or 3 yards; and the answer is found to be 131 miles, 3 furlongs, 10 perches, and 3 yards.—The proof would be performed in the same manner in which the last example was wrought. The pupil should be required to perform it. Had the remainder been 7 half yards, instead of 6, it would have been equivalent to 3 yards, and 1 foot, 6 inches. Hence, instead of 3 yards at the end of the answer, we should have had 3 yards, 1 foot, and 6 inches.

Yards.	
231278	
2	
11)462556	half yards.
4|0)4205|0	6 half yards, or 3 yds.
8)1051	10 perches.
Miles 131	3 10 3, *Answ.*

Exercises.	*Answers.*
57. Reduce 51 feet 4 inches to inches ...	616
58. ..... 3 miles 3 furlongs to yards...	5940
59. ..... 94 miles, 1 fur. 6 per. to per.	30126
60. ..... 751823 feet to leagues	47l. 1m. 3f. 4p. 5y. 2f.
61. ..... 758246 inches to miles	11m. 7f. 29p. 2y. 2f. 8i
62. ..... 571 leagues 2 miles to miles	1715
63. ..... 573911 yards to miles	326m. 0f. 27p. 2y. 1f. 6i.
64. ..... 23456 feet to miles	4m. 3f. 21p. 2y. 3f. 6i.
65. ..... 25 miles and 34 per. to feet...	132561
66. ..... 1000000 inches to miles	15m. 6f. 10p. 2y. 2f. 4i.
67. ..... 100 miles to inches	6336000

REDUCTION OF CLOTH MEASURE.

Exercises.	*Answers.*
68. Reduce 28 yards, 3 qrs. 2 nails to nails* ...	462
69. ..... 5247 nails to English ells	262 e. 1 q. 3 n.
70. ..... 58 yards to nails	928
71. ..... 285 nails to yards	17 y. 3 q. 1 n.
72. ..... 1241 English ells, 2 qrs. 3 nails, to nails	24831.
73. ..... 468 yards to English ells	374 e. 2 q.
74. ..... 1560 Flemish ells to yards	1170

* Multiply the yards by 4, and take in the quarters; and multiply the result by 4, and take in the nails. To work the next exercise, divide the nails by 4, the quotient is quarters, which is to be divided by 5 to find the answer.

D

Exercises.	*Answers.*
75. Reduce 1123 English ells to French ells	935 e. 5 q.
76. ..... 985321 inches to yards	27370 y. 1 in.

REDUCTION OF SQUARE MEASURE.

Exercises.	*Answers.*
77. Reduce 245 square perches to square inches ..	9604980
78. ..... 1325419 square inches to sq. yds.	1022 y. 6 f. 43 i.

REDUCTION OF LAND MEASURE.

Exam. 10. Reduce 37 acres, 3 roods, 12 perches, to square yards.

```
   A. R. P.
   37 3 12
    4
   ---
   151 roods.
    40
   ----
  6052 perches.
    30¼
  ------
 181560
   1513
 ------
 183073 yards, Answ.
```

Exam. 11. Reduce 111111 square yards to acres.

```
        yards.
        111111
             4
        ------
121 { 11)444444 qrs. of yards.
      ---------
      11)40404
      ---------
    4|0)367|3 p. 11 qrs. or 2¾ yards.
      ---------
      4)91 r. 33 p. 2¾ yards.
      ---
```

Answ. 22 a. 3 r. 33 p. 2¾ yards.

In this example, to find the acres, the yards are reduced to quarters, and divided by 121, the quarters in a perch, or 30¼ yards. The remainder is 11 quarters of a yard, or 2 yards and 3 quarters.

Exercises.	*Answers.*
79. Reduce 234 acres, 1 rood, 13 per. to per.....	37493
80. ..... 93827 perches to acres	586 a. 1 r. 27 p.

Exercises.	*Answers.*
81. Reduce five millions, five hundred yds. to acres	1033 a. 0 r. 25 p. 23¾ y.
82. ..... 256 acres, 15 perches, to yds..	1239493¾

REDUCTION OF TIME.*

Reduce *Exercises.*	*Answers.*
83. 17 days to minutes.....	24480
84. 1 day, 4 hours, 12 seconds, to seconds.....	100812
85. 12345678 seconds to days.....	142d.21h.21m.18s.
86. 34652 hours to weeks.....	206 w. 1 d. 20 h.
87. Eleven millions one hundred seconds to days...	127 d. 7 h. 35 m.

Exam. 12. How many days are there from the 8th of May till the 23d of July?†

31 days in May	
8	
23 May	
30 June	
23 July	
76 days, *Answ.*	

88. How many days are there from the 12th of August, 1817, till the 24th of April, 1818? *Answ.* 255.

89. How many days are there from the 8th of January, 1816, till the 12th of December in the same year? *Answ.* 339.

90. How many days are there between the 17th of March, and the 25th of December?‡ *Answ.* 283.

91. In the mint in London there are eight coining presses, which, with a child to supply each, strike 19000 coins in an hour. Now, if these were employed 12 hours each day, for 313 days, in coining halfpence, what would be the number and the value of all the pieces coined during that time? *Answ. Number*, 71364000; *value*, £148675.

---

* Reduction of cubic measure, of liquid measure, and of dry measure, is so easy, after what has gone before, that it is unnecessary to give particular sets of exercises for each kind.

† This, and the similar questions which follow, though not strictly of the same kind as the others in Reduction, are inserted on account of their utility.

‡ Questions of the following kind are frequently useful:

1. If the 8th of August be on Saturday, on what day of the week will the 1st of November be?

The number of days between these dates is found to be 85, which being divided by 7, becomes 12 weeks and 1 day; counting one day, therefore, after Saturday, we find that the first of November must be on Sunday.

2. If a common year begin on Friday, on what day will the 18th of June, the anniversary of the battle of Waterloo, happen? *Answ.* Friday.

3. The 9th of July, 1824, was on Friday; on what day did the year commence. *Answ.* On Thursday.

4. If a leap year commence on Wednesday, what day of September will be the first Monday of that month? *Answ.* 7th.

Here, let the day be found on which September will commence, and the rest is easy.

92. The quantity of linen exported from Ireland to the United States in 1806, was two millions, six hundred and seventy-five thousand, six hundred and nineteen yards. How many miles in length was the whole? *Answ.* 1520 m. 1 f. 36 p. 1 y.

93. Required the amount of a collection of one halfpenny each, from the inhabitants of the counties of Down and Antrim.—(*See exercise* 24, *page* 11.) *Answ.* £1248 6 10.

94. "In the year 1815, Catharine Woods, of Dunmore, near Ballynahinch, then about thirteen years of age, spun a hank of linen yarn, of 12 *cuts*, each cut 120 threads, each thread two yards and a half, which weighed ten grains," being at the rate of 700 hanks to the pound avoirdupois.* It is required to determine the length into which, at this rate, a pound of flax would have been extended.—(*See Stuart's "Historical Memoirs of the City of Armagh," page* 424.) *Answ.* 1431 m. 6 f. 21 p. $4\frac{1}{2}$ y.

95. In the salmon fishery on the Bann, near Coleraine, 320 tons of salmon were taken in 1760. How many stones of 14 pounds, are in this quantity? *Answ.* 51200.

96. The quantity of cotton wool imported into Ireland, in 1811, was 53,133 cwts. of which 16,148 cwts. were exported. Required the number of pounds in each of these quantities, and in the quantity consumed in the country. *Answ.* 5950896 lbs.; 1808576 lbs.; and 4142320 lbs.

97. Suppose one person to lie in bed nine hours each day at an average, and another only six hours and a half; and suppose the latter to employ the time thus gained in reading and study, for forty years; to how many years' study, of 12 hours each day, would the entire time gained be equivalent? *Answ.* 8 years, 121 days, 8 hours.

98. The whole surface of the terraqueous globe contains 196,649,494 square miles; and of these, Europe is supposed to contain 4,456,065. Required the number of acres contained in each. *Answ.* 125855676160, and 2851881600.

99. In what time would a body move from the earth to the moon, at the rate of thirty-one miles per day; the mean distance being 238,545 miles? *Answ.* 21 years, 30 days.

100. In what time would a body, moving with the velocity of sound, pass from the earth to the sun, the distance being 95 millions of miles? (*See Ex.* 57, *page* 22.) *Answ.* 14 y. 27 d. 15 h. 50 m. $5\frac{330}{1150}$ s.

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* For this extraordinary, and perhaps unequalled performance, a premium of fifteen guineas was awarded by the Linen Board of Ireland. $17\frac{1}{2}$ lbs. of such yarn would contain a thread more than equal to the circumference of the earth.—*Stuart's Memoirs.*

The same year, another premium, of the same amount, was awarded to Elizabeth M'Cance, of the same neighbourhood, for having spun a hank which weighed 12 grains, and which was consequently at the rate of (7000 grains÷12) 583 hanks 4 cuts in the pound.

101. In how long time would a cannon ball, with the velocity of 1960 feet per second, move from the sun to the Georgium Sidus? (*See Rx.* 25, *page* 5.) *Answ.* 154 years, 224 days, 5 hours, 46 min. $7\frac{690}{1960}$ sec.

102. By the latest measurements, the earth's mean diameter is found to be 7911 miles and three quarters, nearly. How many yards, feet, and inches, are contained in it? *Answ.* 13924680 yds.; 41774040 feet; 501288480 in.

103. In England there are 50,535 square miles; and in Wales, 7425 such miles. Required the number of acres contained in both. *Answ.* 37,094,400.

104. In the city of Pekin there are said to be seven bells, each weighing 120,000 lbs; and in Moscow there is one bell which weighs 127,836 lbs.; another which weighs 288,000 lbs.; and a third which weighs 432,000 lbs. Required the weight of each in tons. *Answ.* 53 t. 11 c. 1 q. 20 lbs.;—57 t. 1 c. 1 q. 16 lbs.;—128 t. 11 c. 1 q. 20 lbs.;—and 192 t. 17 c. 16 lbs.

COMPOUND ADDITION.*

RULE. (1.) Arrange the given quantities so that those in each column may be of the same denomination. (2.) Add the numbers of the lowest denomination together; reduce their sum to the next higher denomination; set the remainder below the column added, and carry the quotient to the next. (3.) Proceed thus with all the other denominations, except the highest, which is to be added in the same manner as numbers in Simple Addition.

Either of the two first methods of proof given in Simple Addition, may be employed in Compound Addition.

In this example, the sum of the farthings is 14, which being divided by 4, the farthings in a penny, the quotient is 3 pence; and the remainder, 2 farthings, or a halfpenny, is set down. The quotient, 3, is then added with the pence; the sum, 54, being divided by 12, the pence in a shilling, the quotient is 4 shillings, with a remainder of 6 pence, which is set down. The quotient,

Exam. 1.

£.	s.	d.
39	18	$7\frac{3}{4}$
51	12	$4\frac{1}{2}$
79	19	$10\frac{1}{2}$
8	7	$11\frac{1}{4}$
43	13	$9\frac{3}{4}$
375	16	$10\frac{3}{4}$
Sum, £599	9	$6\frac{1}{2}$†

* For the definitions of the several Compound Rules, see the corresponding Simple Rules, pages 6, 12, 16, and 24.

† The proof is left to exercise the learner; and it will be proper to require him to perform it, not only in this example, but in all the exercises in this rule.

4, is then added with the units of the shillings: the sum is 39, of which the latter figure is set down, and the tens being carried to the tens of the shillings, the sum is 8, which being divided by 2, because 2 tens, or 20 shillings, make a pound, the quotient is 4, which is added with the pounds, as in Simple Addition.*

In this example the halfpence amount to 5, or 2 pence halfpenny. In such examples, where all the fractional parts are halfpence, it is easier to call each a halfpenny, than 2 farthings.

	£.	s.	d.
Exam. 2.	15	3	8½
	31	16	7½
	94	13	8½
	55	12	11½
	37	11	9½
Sum,	£234	18	9½

When the columns are very long, the work becomes heavy and laborious; and therefore, in such a case, the given quantities may be separated into two or more divisions, as is suggested in page 8.

Exercises in Compound Addition

1			2			3			4		
£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
485	12	7¾	3	14	8½	413	13	10¼	4	13	6¼
49	16	3½	0	19	7½	1245	10	9	3	15	7¾
186	13	11¾	5	17	9¾	7085	15	11¾	7	10	11
787	10	8¼	2	12	6	8519	6	4½	1	12	9¼
239	9	9½	2	16	10½	3456	14	10	2	8	7½
843	11	4½	1	5	8¼	90	12	5½	4	0	3
374	16	7	3	8	3	69	15	2¾	6	16	8¾
285	4	9¾	5	2	4½	179	18	11½	7	6	2
599	19	8	10	14	5¼	788	9	9	9	15	10½

5. Add together £59 12 7¾, £95 14 2½, £345 5 9¼, £88 16 2¾, £186 17 4½, £347 7 6, £3 2 9½, £7 14 7¾, £52 8 6¾, £59 3 4, £42 18 10½, £187 10 10¼, £954 16 5¼.

6. Add together £324 14 10½, £518 5 9½, £39 15 6, £54 11 11½, £49 1 8½, £9 7 11½, £1000, £86 6 3½, £324, £79 11 6½, £5 13 9, £611 4 2½, £186 13 1½, £476 8 5.

*PENCE TABLE.

d.	s.	d.	d.	s.	d.	d.	s.	d.	d.	s.	d.	d.	s.	d.	d.	s.	d.
12=1		0	40=3		4	72=6		0	100=	8	4	132=11		0	160=13		4
20=1		8	48=4		0	80=6		8	108=	9	0	140=11		8	168=14		0
24=2		0	50=4		2	84=7		0	110=	9	2	144=12		0	170=14		2
30=2		6	60=5		0	90=7		6	120=10		0	150=12		6	180=15		0
36=3		0	70=5		10	96=8		0	130=10		10	156=13		0	200=16		8

This Table has been inserted, lest some teachers should consider the want of it an imperfection. It seems better, however, not to impose upon the learner the labour of committing it to memory, except perhaps a small part at the beginning. If, instead of using it, he divide the amounts of the pence in his operations by 12, he will soon acquire a readiness in this species of Addition, which will be highly valuable; and by practice he will soon make himself acquainted with the Table, without formally committing it, especially if his attention be occasionally directed to the subject by the teacher.

7. Required the sum of £21 0 10½, £73 18 9, £22 3 7½, £64 16 9, £19 18 1½, £78 9 9, £16 9 10½, £250 9 6, £17 12 7½, £54 0 7½, £797 12 9, £1 14 1½, £60 5 9, £113 15, £170 12 6, £98 19 3.

8. Add together one thousand and six pounds, fifteen shillings, and three farthings; three hundred pounds, seventeen shillings, and a halfpenny; four thousand and ninety-six pounds, eight shillings, and three farthings; seven thousand pounds, eighteen shillings, and eleven pence; two pounds, and three halfpence; five hundred and eleven pounds, and sixpence halfpenny; eighty-six pounds; five hundred and eight pounds, seven shillings, and a farthing; five thousand pounds; two hundred and ninety pounds, ten shillings; two hundred and nine pounds, and ten pence.

9. Required the sum of £15 13 8½, £53 16 7¾, £199 6 3¾, £548 8, £458 0 8, £1 2 9, £2 5 6, £3 8 3, £1 14 1½, £2 16 10½, £3 19 7½, £180 18 8, £31 13 4¾.

10. Add together 15/8½, 13/10¼, 16/6¾, 19/3, 16/7½, 5/8¼, 13/9, 12/7¼, 15/1¾, 11/4½, 9/1¼, 10/9½, 17/4¼, 18/4, 4/5½, 3/10¾.

11. £513 15 10¼+£34 2 10½+£183 17 3½+£548 13 8¼+£8 15 9½+£88 15 9½+£176 15 6¾+£79 13 11+£5 0 6½+£189 13 8¼+£195 8 10¾+£179 16 6+£348 17 9¾+£59.

12. Add together £18 14 8½, £12 13 9½, £21 12 10, £32 9 10½, £63 13 9½, £16 4 8½, £35 14 9½, £17 16 7½, £23 15 9½, £35 17 2½, £8 19 8, £12 10 0½, £13 8 8½, £17 18 4.

13. Find the sum of one hundred and two pounds, and ten pence; fifty-eight pounds, and three farthings; forty pounds, seventeen shillings, and five pence halfpenny, nine pounds, fifteen shillings, and eleven pence; seventy-three pounds, twelve shillings; thirty-nine pounds, fourteen shillings, and a farthing fifty-nine pounds; eighty-two pounds, and nine pence halfpenny; fifty pounds, and a penny.

14. £918 12 7¼+£51 16 8½+£519 14 4½+£83 16 10½+£55 12 8¾+£183 7 9+£188 18 2¼+£375 12 3½+£491 11 4½+£318 15 9¾+£780 10 4+£10 19 9¾+£508 16 9+£374 19 5½+£348 6 8¼.

15. £459 16 11+£583 11 11¼+£15 14 8½+£191 14 8¾+£99 13 6½+£195 12 6½+£17 18 4¾+£473 18 0½+£18 16 7½+£5 8 9½+£31 12 2½+£63 3 11+£5 11 8+£1 2 9+£1005 11 9.

16. Add together £3 4 7¾, £41 12 4, £186 17 9½, £3 8 10½, £67 18 9½, £112 16 6¾, £73 15 5¼, £139 7 4½, £510 6 7½, £706 7 7½, £581 4 4¾, £608 19 6½, £519 12 8¼, £588 6 11, £137 1 2¼.

17. Required the sum of £13 6 8, £88 14 7½, £175 16 4½, £1245 13 5¾, £512 16 6, £91 12, £78 14 5, £475 19 9¼, £839 16 6¾, £512 11 11¼, £447 16 0¾, £43, £74 7 11, £691 14 7¼, £9 4 7¾, £907 18 6, £10 3 0¾.

18. Add together £118 16 10¾, £514 4 11, £880 17 4½, £644 15 4½, £359 16 6, £346 3 10¼, £435 10 3¼, £634 17 6, £364 16 8½, £463 14 8, £643 16 2½, £5 13 9, £3018 15 6, £998 4 10½, £580 0 6¼, 14/9½, £533 16 7¼, £85 18 9½, £375 15 8, £90 14 8¾, £187 16 6, £57 3 11½, £348 6 10, £4048 14 9¾, £798 16 5, £8 15 6, £89 19 8¾, £175 18 8¼, £311 17 10¼, £73 14 9¾, £864 13 7¼, £364 14 10, £819 14 3½, £59 17 8, £1044 5 8¼.

In this example the sums of the pounds, quarters, and hundreds, are respectively divided by 28, 4, and 20, the remainders set below their respective columns, and the quotients carried to the next columns, respectively. The hundreds may be added as shillings in Addition of Money, the divisor being the same in both cases.

Exam. 3.

	tons.	*cwt.*	*q.*	*lbs*
	35	16	0	20
	42	14	2	18
	18	9	1	16
	17	18	3	7
	31	5	3	19
	45	12	2	5
Sum,	191	17	2	1

Ex. 19.			20.			21.			22.	
cwt.	*qrs.*	*lbs.*	*lbs.*	*oz.*	*drs.*	*t.*	*cwt.*	*qrs.*	*lbs.*	*oz.*
53	2	12	16	12	13	75	13	1	7	11
17	1	15	5	15	3	83	17	2	53	14
16	3	19	12	12	5	17	8	0	47	10
19	0	18	3	11	9	16	16	1	86	9
25	3	18	19	1	11	61	15	2	94	7
48	3	6	14	4	8	39	9	3	29	3
42	2	7	24	7	14	88	7	3	42	10

23. Add together 55 c. 3 q. 18 lbs.; 34 c. 2 q. 22 lbs.; 63 c. 1 q. 23 lbs.; 71 c. 0 q. 19 lbs.; 16 c. 3 q. 20 lbs.; 3 c. 3 q. 26 lbs.; 27 c. 2 q. 23 lbs.; 41 c. 3 q. 9 lbs.; 35 c. 1 q. 18 lbs.; 43 c. 2 q. 24 lbs.; 95 c. 0 q. 10 lbs.; 29 c. 2 q. 17 lbs.; 32 c. 2 q.; 59 c. 3 q. 24 lbs.; 43 c. 1 q. 21 lbs.; 37 c. 3 q. 1 lb.

24. Add together 17 c. 2 q. 17 lbs.; 22 c. 1 q. 27 lbs.; 55 c. 3 q. 19 lbs.; 13 c. 1 q. 15 lbs.; 73 c. 2 q. 13 lbs.; 88 c. 2 q.; 48 c. 0 q 23 lbs.; 32 c. 3 q. 19 lbs.; 52 c. 2 q. 22 lbs.; 43 c. 3 q. 20 lbs.; 53 c. 1 q. 18 lbs.; 31 c. 2 q. 26 lbs.; 45 c. 1 q. 14 lbs.; 36 c. 3 q.

25. Required the sum of 2 lbs. 3 oz.; 10 lbs. 5 oz.; 1 lb. 14 oz.; 3 lbs. 13 oz.; 5 lbs.; 7 lbs. 7 oz.; 8 lbs. 12 oz.; 7 lbs. 15 oz.; 3 lbs. 6 oz.; 1 lb. 11 oz.; 12 lbs. 10 oz.

26. Required the sum of 58 tons, 12 c. 3 q. 21 lbs.; 32 t. 11 c. 2 q. 20 lbs.; 19 t. 15 c. 1 q. 12 lbs.; 17 t. 17 c. 0 q. 17 lbs.; 5 t. 3 c. 1 q. 25 lbs.; 73 t. 15 c. 1 q. 12 lbs.; 98 t. 16 c. 2 q. 22 lbs.; 35 t. 16 c. 3 q. 19 lbs.; 16 t. 14 c. 2 q. 16 lbs.; 70 t. 13 c. 1 q. 24 lbs.

27. Add together the following quantities, in long weight: 57 c. 3 q. 11 lbs.; 39 c. 3 q. 26 lbs.; 51 c. 3 q. 18 lbs.; 26 c. 2 q. 24 lbs.; 67 c. 1 q. 18 lbs.; 18 c. 2 q. 21 lbs; 132 c. 2 q. 13 lbs.; 73 c. 3 q.

28. Add together 13 acres, 3 roods, 27 perches; 45 a. 1 r. 27 p.; 63 a. 2 r. 17 p.; 26 a. 2 r. 26 p.; 16 a. 3 r. 34 p.; 21 a. 0 r. 8 p.; 55 a. 2 r. 31 p.; 37 a. 2 r. 18 p.; 44 a. 2 r. 20 p.; 57 a. 0 r. 19 p.; 61 a. 3 r. 18 p.; 39 a. 2 r.; 5 a. 1 r. 30 p.; 27 a. 2 r. 14 p.*

29. 19 a. 2 r. 28 p.; 10 a. 0 r. 20 p.; 11 a. 3 r. 16 p.; 19 a. 2 r. 18 p.; 14 a. 2 r. 25 p.; 17 a. 3 r. 22 p.; 15 a. 2 r. 19 p.; 4 a. 3 r. 27 p.; 24 a. 2 r. 24 p.; 149 a. 1 r. 14 p.; 719 a. 2 r. 16 p.; 107 a. 1 r. 23 p.; 149 a. 2 r. 17 p.; 6 a. 3 r. 15 p.†

30. The following are the quantities and values of the linen exported from Ireland to America in the undermentioned years:—required the value of the whole, and the number of miles contained in the entire number of yards:

		Yards.	£	*s.*	*d.*
In 1798,	...	2,361,483	183,343	4	0
1799,	...	1,376,382	106,818	16	1
1800,	...	1,156,467	89,948	11	11
1801,	...	2,519,575	248,762	2	6
1802,	...	1,089,223	106,671	18	6
1803,	...	1,873,423	184,180	18	6
1804,	...	2,258,176	223,034	5	9
1805,	...	2,221,606	209,404	6	2
1806,	...	2,675,619	309,525	9	9
1807,	...	1,657,446	191,413	3	6

Answ. Value, £1,853,102 16 8; *length,* 10903 m. 0 f. 21 p. 4½ y.

31. The amount of the duties paid on goods imported into Limerick in 1764, was £19,869 15 9; in 1765, £21,332 4 8; in 1766, £16,729 8 2; in 1767, £16,316 10; in 1768, £16,571 12 8; in 1769, £20,237 12 7; in 1770, £22,138 0 4; in 1771, £20,213 12 6; in 1772, £22,003 2 0; in 1773, £20,606 15 7; in 1774, £17,317 0 9; in 1775, £16,979 10 6. What was the entire amount during these twelve years? *Answ.* £230,315 5 6.

32. The following are the several quantities of land, in English measure, contained in the Royal Forests in England: New Forest, 66,942 a. 3 r. 26 p.; Dean Forest, 23,015 a. 3 r. 29 p.; Aliceholt and Woolmer, 8,694 a. 1 r. 31 p.; Whittlewood, 4,850 a. 3 r. 32 p.; Whichwood, 3,709 a. 3 r. 5 p.; Waltham, 3,278 a. 3 r. 2 p.; Salcey, 1,847 a. 0 r. 23 p.; Sherwood, 1,466 a. 3 r. 10 p.; Bere, 926 a. 2 r. 13 p.; Rockingham, 860 a. 3 r. 23 p. Required the sum. *Answ.* 115594 a. 0 r. 34 p.

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* The units of perches may be added as in Simple Addition, and the sum of the tens divided by 4, the tens in 40. In like manner the units of minutes, seconds, thirds, &c. may be added as in Simple Addition, and the sum of the tens divided by 6, the tens in 60.

† Several parts of Compound Addition usually delivered in books on Arithmetic, are omitted, as they are in general of no use; and if they should at any time be required, they can be performed without difficulty by the application of the same principles on which the others depend.

COMPOUND SUBTRACTION.

RULE. (1.) Place the less number below the greater,* so that the numbers in each column may be of the same denomination. (2.) Then, beginning with the lowest denomination, subtract, if possible, each number in the lower line from that which stands above it. (3.) But when this cannot be done, subtract the number in the lower line from a unit of the next higher denomination; to the remainder add the upper number, for the true remainder, and carry one to the next higher number in the lower line. (4.) Proceed thus with all the denominations, except the highest, in which the work is to be performed as in Simple Subtraction.

Exam. 1. From £33 17 10¾ take £18 8 4¼.

£	s.	d.	
33	17	10¾	
18	8	4¼	
£15	9	6½,	*Answ.*

Exam. 2. Required the difference between £159 9 4¼ and £86 17 8½.

In this example, as a halfpenny is greater than a farthing, it is taken from a penny, and the remainder being added to the farthing, the sum, three farthings, is set down; a penny is then carried to 8 pence, and the sum being taken from one shilling, the remainder 3 is added to 4 pence, and the amount set down. We then proceed thus: 1 and 7 are 8; 8 from 9 and 1 remains; 1 from 2, (the tens' figure in the shillings in a pound,) and 1 remains; 1 and 6 are 7; 7 from 9 and 2 remain, &c.

£	s.	d.	
159	9	4¼	
86	17	8½	
£72	11	7¾,	*Ans.*

Exercises in Compound Subtraction.

		£	s.	d.		£	s.	d.
1.	From	19	3	10	take	8	15	3¾
2.	.....	575	15	1½	.....	124	13	4
3.	.....	192	11	4½	.....	88	16	9½
4.	.....	511	3	2¼	.....	247	10	6½
5.	.....	12	4	9	.....	5	2	4½

* It is more usual, and generally more convenient, to set the less number below the greater, in Subtraction: it is by no means essential, however; and the pupil should be accustomed, both in Simple and Compound Subtraction, to subtract *downward* as well as *upward*, that he may be able to do so, when it may happen in complex operations, that the numbers are so arranged.

The methods of proof, and the principles on which the operations depend, are the same as in Simple Subtraction.

		£	s.	d.		£	s.	d.
Ex. 6.	From	100	0	0	take	1	2	9
7.	.....	513	5	$8\frac{1}{4}$	.....	188	17	$4\frac{3}{4}$
8.	.....	1516	19	$2\frac{3}{4}$	.....	847	12	$9\frac{1}{2}$
		cwt.	q.	lbs.		cwt.	q.	lbs.
Ex. 9.	From	13	3	20	take	9	2	12
10.	.....	23	1	5	.....	17	3	22
11.	.....	105	0	0	.....	79	1	13

		tons.	cwt.	q.	lbs.		tons.	cwt.	q.	lbs.
Ex. 12.	From	15	7	0	24	take	5	12	1	10

		days.	h.	min.	sec.		days.	h.	min.	sec.
Ex. 13.	From	5	10	27	15	take	2	4	13	29
14.	.....	83	17	3	0	.....	59	7	12	30

15. The latitude of Rome, (St. Peter's,) is 41° 53′ 54″ North; of Paris, (Observatory of the Military School,) 48° 51′ 6″ N.; of London, (St. Paul's,) 51° 30′ 49″ N.; of Dublin, 53° 21′ N.; of Edinburgh, (Observatory,) 55° 57′ 57″ N.; and of Petersburg, 59° 56′ 23″ N. Required the difference of latitude of the first and second, second and third, &c. of these capitals.

16. The latitude of Gibraltar is 36° 6′ 30″ N. and that of the North Cape in Lapland, 71° 10′ N. Required the difference.

17. The latitude of the Cape of Good Hope is 33° 55′ 15″ S. and that of Cape Horn, 55° 58′ 30″ S. Required their difference.

18. The latitudes of Belfast and Glasgow, are 54° 36′ N. and 55° 51′ 32″ N. respectively. Required their difference.

19. The following are the times in which the principal planets perform their revolutions about the sun; required the differences of the first and second, of the second and third, &c.:—

	Days.	h.	m.		Days.	h.	m.
Mercury,.....	87	23	16	Jupiter,.....	4332	14	27
Venus,.....	224	16	49	Saturn,.....	10759	1	51
Earth,.....	365	6	9	Georgium Sidus, ...	30737	18	0
Mars,.....	686	23	31				

COMPOUND MULTIPLICATION.

Problem 1. *To multiply a number of more denominations than one, by a number not exceeding* 12.*

RULE. Commencing with the lowest denomination, multiply successively the several numbers in the multiplicand by the multiplier, dividing, setting down, and carrying as in Compound Addition.

* Or *not exceeding* 19, if the learner have committed to memory the supplement to the Multiplication Table, page 17. The same circumstance will also modify the succeeding problems, in a similar manner.

Exam. 1. Multiply £1 14 7¾ by 9.

In this example the farthings, pence, and shillings are multiplied successively by 9; the products, as they are found, are respectively divided by 4, 12, 20, (or the tens of the shillings by 2;) the several remainders are written down, and the quotients carried. The pounds are multiplied as in Simple Multiplication; and the product is found to be £15 11 9¾.

£	s.	d.
1	14	7¾
		9
£15	11	9¾

Exercises in Compound Multiplication.

	£	s.	d.		£	s.	d.		£	s.	d.
Ex.1.	3	14	9½ × 2	5.	3	15	10¼ × 6	9.	0	15	10¼ × 10
2.	1	17	8¾ × 3	6.	1	2	9 × 7	10.	9	7	8 × 11
3.	0	18	11¼ × 4	7.	2	18	9 × 8	11.	3	19	7½ × 12
4.	1	10	4 × 5	8.	6	13	4 × 9	12.	1	16	7 × 12

Problem 2. *To multiply by a number which exceeds* 12, *but is the product of two or more factors, each less than* 13.*

RULE. (1.) By the preceding problem, multiply the given multiplicand by one of the factors. (2.) Multiply the result by another. (3.) Multiply this last result by another, if there be so many; and thus proceed, whatever is their number.

Exam. 2. Multiply 18/3¼ by 42.

In this example the multiplicand is multiplied by 6, and the product is £5 9 7½. This again is multiplied by 7 and the product is £38 7 4½. The reason of the operation is sufficiently obvious, since 42 is the product of 6 and 7. The work might be proved by multiplying the multiplicand by 7, and the result by 6.

£	s.	d.
0	18	3¼
		6
5	9	7½
		7
£38	7	4½

When the multiplicand contains one or more farthings, if one of the factors be *even*, it is better to use it first, as the farthings may thus disappear, and the rest of the work be easier. But if the multiplicand end in even pence, without farthings, and one of the factors be 3, 6, or 9, it is better to use that factor first, as the pence may thus disappear: and in all cases in multiplication of money, when 12 is one of the factors, it should be used first, as part of the operation may be performed by inspection, by setting down 3 pence for each farthing, and carrying to the shillings 1 for every penny in the multiplicand. Thus, in multiplying £1 13 7¼, by 12, set down 3 pence, and carry 7 to the shillings, saying, 12 times 3 are 36, and 7 are 43, &c.

£1	13	7¼
		12
£20	3	3

* For the method of finding the factors in the more difficult cases, see page 36. Pupil, however, will soon learn to find them by inspection, in all useful cases.

	Exercises.				*Answers.*		
	£	s.	d.		£	s.	d.
Ex. 13.	1	17	9½	× 14	26	9	1
14.	0	13	4½	× 15	10	0	7½
15.	0	2	8½	× 16	2	3	4
16.	3	14	0¼	× 18	66	12	4½
17.	1	5	3	× 21	26	10	3
18.	0	14	1	× 22	15	9	10
19.	0	13	2	× 24	15	16	0
20.	0	6	11¾	× 25	8	14	5¾
21.	0	12	7¼	× 27	17	0	3¾
22.	2	1	0½	× 30	61	11	3
23.	2	15	5	× 32	88	13	4
24.	1	16	6¼	× 33	60	5	2¼
25.	1	19	4	× 36	70	16	0
26.	0	8	6½	× 40	17	1	8
27.	1	11	3	× 42	65	12	6
28.	2	2	6½	× 44	93	11	10
29.	0	8	4½	× 45	18	16	10½
30.	2	12	10½	× 48	126	18	0
31.	0	16	4¾	× 49	40	3	4¾
32.	3	1	3¾	× 55	168	12	2¼
33.	0	3	10¼	× 56	10	15	10
34.	0	6	11¾	× 60	20	16	3
35.	2	10	5	× 63	158	16	3
36.	1	14	4	× 66	113	6	0
37.	0	4	5¼	× 72	15	19	6
38.	1	6	9¾	× 80	107	5	0
39.	0	5	3¼	× 81	21	6	11¼
40.	1	6	7	× 84	111	13	0
41.	4	0	9¼	× 88	355	7	10
42.	1	10	8	× 90	[illegible]	0	0
43.	1	15	11¾	× 96	172	4	0
44.	0	8	7¾	× 99	42	15	11¼
45.	1	17	2	× 100	185	16	8
46.	0	16	11½	× 108	91	11	6
47.	0	18	9	× 110	103	2	6
48.	0	4	8¼	× 120	28	2	6
49.	2	19	6	× 121	359	19	6
50.	1	16	7½	× 132	241	14	6
51.	2	19	2¼	× 144	426	3	0
52.	1	12	5	× *75	121	11	3
53.	1	18	8¾	× 112	216	17	8
54.	0	14	2¾	× 128	91	1	4

* In this and each of the six following exercises, the multiplier is the product of *three* factors; 75 being=3×5×5; 112=8×2×7, or 4×4×7; 128=8×8×2, or 4×4×8; &c. In like manner, 140=5×7×10; 192=12×4×4; 240=12×2×10; 1728=12×12×12, &c. &c.

	Exercises.				*Answers.*		
	£	s.	d.		£	s.	d.
Ex. 55.	1	2	$7\frac{1}{4}$	× 147	166	2	$9\frac{3}{4}$
56.	0	11	$4\frac{1}{2}$	× 168	95	11	0
57.	1	16	8	× 175	320	16	8
58.	2	15	$2\frac{1}{4}$	× 196	540	16	9

Problem 3. To multiply by a number which exceeds 12 *but ot produced by factors below* 13.

ULE. (1.) Use those factors whose product is nearly al to the multiplier. (2.) Increase or diminish the re-, as the case may require, by the product of the multi- and and the difference between the multiplier and the duct of the factors employed.

Exam. 3. Multiply 14/10½ by 38.

£	s.	d.		
0	14	$10\frac{1}{2}$		
		12		
8	18	6,	prod. by 12.	
		3		
26	15	6,	...	36.
1	9	9,	...	2.
£28	5	3,	...	38.

1 this example, 38 not being the pro- t of any two factors not exceeding 12, multiply by 36, as before, and to the luct we add twice the multiplicand, nd the product by 38. The answer ld have been obtained with nearly the e facility, had we multiplied by 40, 10,) and subtracted twice the mul- cand; and thus the operation might roved.

Exam. 4. Multiply £1 4 10 by 61.

£	s.	d.		
1	4	10		
		12		
14	18	0,	prod. by 12.	
		5		
£75	14	10,	...	61.

he operation in this example is con- ted by taking in the multiplicand, in multiplication by 5. This contrac- may always be employed when he luct of the factors is *one* less t an the tiplier. A similar contraction might mployed, though not with the same ity, when the excess is 2, 3, &c.

Exam. 5. Multiply £2 7 8½ by 79.

£	s.	d.		
2	7	$8\frac{1}{2}$		
		8		
19	1	8,	prod. by 8.	
		10		
190	16	8,	...	80.
2	7	$8\frac{1}{2}$,	to be subtr.	
£188	8	$11\frac{1}{2}$,	prod. by 79	

1 this example it is better to find the luct by 80, and subtract the multipli- l from it, than to multiply by 77, and twice the multiplicand to the product, 1 the one way there is one multiplica- fewer than in the other. A like con- ion may be applied with advantage very case in which factors can be d, whose product is *one* more than given multiplier, while factors can- be found whose product is only one than it.

	Exercises.				*Answers.*		
	£	s.	d.		£	s.	d.
Ex. 59.	5	3	2	× 13	67	1	2
60.	2	8	7¾	× 23	55	18	10¼
61.	1	10	1	× 29	43	12	5
62.	0	17	3½	× 31	26	16	0½
63.	0	11	3¾	× 39	22	1	2¼
64.	0	18	8	× 46	42	18	8
65.	1	11	0¼	× 47	72	17	11¾
66.	2	6	5	× 52	120	13	8
67.	0	13	0¼	× 53	34	10	1¼
68.	2	5	4	× 58	131	9	4
69.	1	12	1¼	× 65	104	6	9¼
70.	0	19	9¾	× 68	67	7	3
71.	0	12	8½	× 69	43	16	10½
72.	3	7	11	× 76	258	1	8
73.	1	0	9½	× 78	81	1	9
74.	0	14	5	× 82	59	2	2
75.	1	13	6	× 89	149	1	6
76.	1	3	2¾	× 91	105	13	10¼
77.	1	16	4	× 94	170	15	4
78.	2	2	11¼	× 95	203	19	0¾
79.	3	9	7	× 98	340	19	2
80.	0	15	6¾	× 103	80	2	11¼
81.	0	11	4½	× 104	59	3	0
82.	1	10	6¼	× 105	160	4	8¼
83.	2	5	10	× 111	254	7	6
84.	3	15	3¾	× 117	440	11	6¾
85.	3	17	8¾	× 122	474	2	11½
86.	2	13	1¼	× 125	331	18	0¼
87.	1	10	8	× 130	199	6	8
88.	0	6	9½	× 139	47	4	0½
89.	0	17	4¾	× 145	126	2	4¾
90.	0	15	7	× 150	116	17	6
91.	2	4	8½	× 155	346	9	9½
92.	2	11	5¼	× 156†	401	4	3

* In multiplying by 13, if the pupil have committed the Multiplication Table no further than 12 times 12, he may multiply by 12, and take in the multiplicand as he proceeds, as in the annexed example. In this way the multiplication by any multiple of 13, not exceeding 13 times 13, may be performed in two lines, as also by any number which is greater by unity than any of these multiples. Hence, only two lines are necessary in multiplying by 26, 39, 52, 53, 65, 78, 79, 91, 92, 104, 105, 117, 118, 130, 131, 143, 156, 157, 169, 170.

£	s.	d.	
1	15	3¾	×13
		12	
£22	19	0¾,	*Answ.*

† The subjoined example will show how Compound Multiplication may be performed, however great the multiplier is. This method, however, will scarcely be employed by any one who has learned the modes of computation usually taught in the Rule of *Practice*. It is therefore given in a note, and not in the text.

Exam. 6. Multiply 33 c. 3 q. 22 lbs. by 7.*

	cwt.	*q.*	*lbs.*	
	33	3	22	
			7	
	237	2	14,	*Answ.*

In this example the pounds, when multiplied, are divided by 28, and the quarters by 4. In long weight the divisors are 30 and 4.

Exercises.	*cwt.*	*q.*	*lbs.*		*Answers.* *cwt.*	*q.*	*lbs.*
Ex. 93.	1	2	17	× 27	44	2	11
94.	0	3	22	× 86	81	1	16
95.	2	2	19	*(long weight,)* × 59	156	3	11
96.	1	2	27	*(long weight,)* × 96	165	2	12

97. Required the cost of a chest of tea, containing 97 lbs. at 6/10½ ℔ lb. *Answ.* £33 6 10½.

98. Required the amount of a box of linen cloth, containing as under:—

piec.	*yd.*		*per*	*yd.*	*piec.*	*yd.*		*per*	*yd.*	*piec.*	*yd.*		*per*	*yd.*
2	49	at	2*s.*	1*d.*	3	75	at	2*s.*	11*d.*	3	75	at	3*s.*	7*d.*
2	49		2	3	3	75		3	1	3	75		3	9
2	48		2	5	3	75		3	3	3	68		3	11
3	73		2	7	3	75		3	5	3	75		4	2
3	75		2	9									*Answ.* £140	2 0.

99. Required the amount of a hogshead of rum containing 66 gallons, at 4/10 ℔ gallon, and a puncheon containing 116 gallons, at 4/9 ℔ gallon. *Answ.* £15 19 0, and £27 11 0.

100. What cost a hundred weight of indigo, at 11/4½ ℔ pound? *Answ.* £63 14 0.

Exam. What cost 2485 yards of broadcloth, at 15/7½ per yard.

£	s.	d.			
0	15	7½	=	price of 1 yard.	
		10			
7	16	3	=	...	10 yards.
		10			
78	2	6	=	...	100 ...
		10			
781	5	0	=	...	1000 ...
		2			
1562	10	0	=	...	2000 ...
312	10	0	=	...	400 ...
62	10	0	=	...	80 ...
3	18	1½	=	...	5 ...
£1941	8	1½	=	...	2485 ...

In this example we find successively the prices of 10, 100, 1000. We then multiply the price of 1000 by 2; of 100 by 4; of 10 by 8; and of 1 by 5. We have thus the prices of 2000, of 400, of 80, and of 5, the sum of which is £1941 8 1½, the answer.

The following exercises may be wrought in a similar manner:

Exercises.	£	*s.*	*d.*		*Answers.* £	*s.*	*d.*
Ex. 1.	3	6	5¼	× 3178	10556	18	4½
2.	1	11	3¼	× 15934	24913	9	5½
3.	2	6	9½	× 938	2194	10	7
4.	0	17	1¾	× 63491	54430	6	1½

* Compound Multiplication is seldom employed, except in relation to money; but if it should be necessary to use it in cases not illustrated here, no difficulty can arise, as the method is similar in all cases

101. If a carpenter receive 18/4½ ℔ week, what is his salary in 52 weeks? *Answ.* £47 15 6.

102. How many pounds are there in 91 guineas? *Answ.* £95 11 0.

103. What is the amount of the duty on 100 gallons of brandy, at 13/7 ℔ gallon? *Answ.* £67 18 4.

104. What is the duty on 63 gallons of rum, at 10/10 ℔ gallon? *Answ.* £34 2 6.

105. What is the duty on 58 cwt. of raw sugar, at £1 12 6 ℔ cwt.? *Answ.* £94 5 0.

106. What is the amount of the duty on 149 lbs. of West India coffee, at 7¾d. ℔ lb.? *Answ.* £4 16 2¾.

107. From 1783 to 1793, both inclusive, the money paid for slaves, imported into the West Indies in Liverpool vessels, was, at an average, £1,380,622 16 4½ each year. What was the entire amount? *Answ.* £15,186,851 0 1½.

It may perhaps be proper to caution learners against the absurdity of attempting *to multiply money by money*. This caution will not appear unnecessary, if it be considered that whole pages have been filled with instructions how to perform this problem; and it has been attempted to be shown, even with the semblance of *geometrical demonstration*, that if 2/6 be multiplied by 2/6, the product may be either 3¾d. or 6/3! Let it be considered, however, that in Multiplication a quantity is simply *repeated* a given number of times: thus, if 2/6 be repeated 4 times, the amount is 10/; if 5 times, 12/6 &c. To talk, therefore, of multiplying 2/6 by 2/6, or, which is precisely the same, *of repeating* 2/6 2/6 *times*, is absolute nonsense. In the Rule of Proportion, indeed, we sometimes *appear* to multiply such quantities. Thus, in finding the interest of a sum at a given rate, for a year, we multiply by the rate, and divide by 100. In this case, however, both 100 and the rate are divested of their characters as expressions for money, and are merely to be regarded as abstract numbers, used as the terms of a ratio. By multiplying by the rate, suppose 5, we merely *repeat* the principal 5 times, or find a principal 5 times as great; and then, as there must be one pound of interest for each hundred pounds in this increased principal, we try by Division how often it contains £100; and we thus find the pounds of the interest.—We see from the nature of Division, that there is no absurdity in *dividing* money by money; that is, in finding how often one sum is contained in another.

COMPOUND DIVISION.

Problem 1. *To divide a number of more denominations than one, by a number not exceeding* 12.

RULE. (1.) Divide the highest denomination by the given divisor, by Short Division (2.) Reduce the remainder, if

there be any, to the denomination next lower, and add to the result what was given of that denomination. (3.) Divide the sum by the divisor; and thus proceed to the lowest denomination, or till nothing remains.

Exam. 1. Divide £14 16 7¼ by 10.

£	s.	d.	
10)14	16	7¼	
1	9	7¾	...7 far.
		10	[or 1¾d.
£14	16	7¼,	proof.

In this example, after dividing £14 by 10, we have remaining £4, or 80 shillings; which, increased by 16, becomes 96 shillings. Hence, we find the next part of the quotient to be 9 shillings, and the remainder is 6 shillings, or 72 pence, which, increased by 7, becomes 79. This being divided by 10, we have the remainder 9 pence, or 36 farthings, to which the odd farthing is annexed; and continuing the division, we find the entire quotient to be £1 9 7¾, and the remainder 7 farthings, or 1¾d. The proof is performed as in Simple Division.

Exercises in Compound Division.

	£	s.	d.		£	s.	d.		£	s.	d.
Ex. 1.	10	11	8½ ÷ 2	6.	1	19	5½ ÷ 7	11.	20	1	3 ÷ 12
2.	2	16	7 ÷ 3	7.	3	15	7 ÷ 8	12.	1	0	0 ÷ 6
3.	4	5	9¾ ÷ 4	8.	5	17	11½ ÷ 9	13.	1	0	0 ÷ 7
4.	9	9	6¼ ÷ 5	9.	7	13	6 ÷ 10	14.	1	10	0 ÷ 8
5.	8	13	4 ÷ 6	10.	5	12	9 ÷ 11	15.	1	9	3 ÷ 9

Problem 2. To divide by a number which is greater than 12, but is the product of two or more factors, each less than 13.

RULE. (1.) Divide the given number, by Short Division, by one of the factors. (2.) Divide the quotient by another factor. (3.) Divide the result thus obtained by another factor, if there be so many: and thus proceed, whatever may be their number.

Exam. 2. Divide £59 13 3½ by 66.

£	s.	d.	
6)59	13	3½	
11)9	18	10½	...2
£0	18	0¾	...56far
		6	[or 1/2.
5	8	4½	
		11	
59	12	1½	add.
	1	2	
£59	13	3½,	proof

In this example the factors are 6 and 11. In the division by 6 the quotient is £9 18 10½, and the remainder 2 farthings; and in the division of this quotient by 11, the quotient resulting is 18/0¾, and the remainder 9. This remainder being multiplied by 6, the first divisor, and the product increased by the former remainder 2, (see page 27,) the true remainder is found to be 56 farthings, or 1/2.—In the proof by Multiplication, this remainder must be added to the final product.

In the use of this Rule, if there be no pence in the dividend, or if it end in 3, 6, or 9 pence, and if one of the factors be 3, 6, or 12, it is better to use that factor first, as in the division by it there will be no remainder. If one factor be 2, 4, or 8, and there be no farthings in the dividend, it is generally better to begin with that factor. If the shillings and pence of the dividend be any multiple of 10 pence, (as 1/8, 2/6, 3/4, 15/10, &c.) and one of the factors be 5 or 10, it is best to begin with that factor. The same may be observed in relation to multiples of 5d. and 2½d. (and, when 5 is a factor, in relation to multiples of 1¼d. ;) but these are not so easily discovered.

	Exercises.					*Answers.*			
	£.	s.	d.			£	s.	d.	*Rem.*
Ex. 16.	73	18	2	÷	14	5	5	7	
17.	93	11	8	÷	42	2	4	6¾	½d.
18.	8	3	6	÷	18	0	9	1	
19.	101	12	2¾	÷	45	2	5	1¾	8d.
20.	40	10	0	÷	49	0	16	6¼	5¾d.
21.	36	11	8	÷	16	2	5	8¾	
22.	58	7	0	÷	36	1	12	5	
23.	59	11	11¼	÷	27	2	4	1¾	
24.	45	10	0	÷	55	0	16	6½	2½d.
25.	48	8	2½	÷	50	0	19	4¼	6d.
26.	9	14	8	÷	32	0	6	1	
27.	42	18	4½	÷	21	2	0	10½	
28.	31	2	8½	÷	48	0	12	11½	8½d.
29.	50	0	0	÷	63	0	15	10¼	1/2¼
30.	122	19	1	÷	66	1	17	3	7d.
31.	55	8	0¾	÷	77	0	14	4½	1/2¼
32.	53	12	7½	÷	81	0	13	2¾	1/0¾
33.	113	14	9	÷	96	1	3	8¼	9d.
34.	106	4	4½	÷	110	0	19	3¾	
35.	53	10	6	÷	120	0	8	11	6d.
36.	31	10	4	÷	70	0	9	0	4d.
37.	38	10	0	÷	88	0	8	9	
38.	124	16	6	÷	84	1	9	8½	1/0
39.	100	0	0	÷	121	0	16	6¼	11¾d.
40.	79	14	8½	÷	132	0	12	0¾	2/5½
41.	375	10	6	÷	144	2	12	1¾	1/6
42.	138	15	0	÷	150	0	18	6	
43.	111	2	6	÷	180	0	12	4	2/6
44.	255	18	9	÷	225	1	2	9	
45.	746	0	0	÷	275	2	14	3	1/3

Problem 3. To divide by a number which is greater than 12, and is not produced by factors below 13.

RULE. The process is to be conducted as in problem 1, except that Long Division is to be employed instead of Short.

Exam. 3. Divide £2074 6 9¼ by 597.

Here, the first part of the quotient is £3, and the remainder £283 6, or 5666 shillings, from which, as in Simple Division, we obtain for quotient 9 shillings, and a remainder of 293 shillings and 9 pence, or 3525 pence. Dividing this by the divisor, we obtain for quotient 5 pence, and a remainder of 540¼ pence, or 2161 farthings, which gives a quotient of 3 farthings, and a remainder of 370 farthings, or 7/8½. The work is proved by multiplying the quotient by 600 (6 × 10 × 10,) and subtracting three times the quotient, to obtain the product by 597, and finally, by adding the remainder, 7/8½, to the result.

£	s.	d.	£	s.	d.	
597)2074	6	9¼	(3	9	5¾,	*Answ.*
1791					6	
283			20	16	10½	
20					10	
5666			208	8	9	
5373					10	
293			2084	7	6	
12			10	8	5¼	
3525			2073	19	0¾	
2985				7	8½	
540			£2074	6	9¼,	
4						Proof.
2161						
1791						
370 farthings, or 7/8½.						

Exercises. *Answers.*

	£	s.	d.			£	s.	d.		Rem.
Ex. 46.	61	0	0	÷	13	4	13	10		2d.
47.	9	3	7	÷	17	0	10	9½		1½d.
48.	50	4	2	÷	19	2	12	10		4d.
49.	99	0	0	÷	26	3	16	1¾		2½d.
50.	45	10	0¼	÷	29	1	11	4½		1¾d.
51.	115	12	6	÷	37	3	2	6		
52.	93	9	4½	÷	43	2	3	5½		8d.
53.	82	6	11	÷	51	1	12	3½		½d.
54.	67	3	10¾	÷	57	1	3	6¾		10d.
55.	54	13	2	÷	65	0	16	9¾		4½d.
56.	108	18	1½	÷	71	1	10	8		9½d.
57.	329	4	0	÷	78	4	4	4¾		1/1½
58.	167	16	0	÷	85	1	19	5¾		3½d.
59.	53	12	0	÷	91	0	11	9¼		10½d
60.	288	5	0	÷	97	2	19	5		1/7
61.	344	14	0	÷	103	3	6	11		1/7
62.	179	6	0	÷	133	1	6	11½		6½d.
63.	599	2	4	÷	201	2	19	7¼		1/10¾
64.	342	11	4½	÷	313	1	1	10½		4/6
65.	400	0	0	÷	365	1	1	11		5d.
66.	9846	12	4	÷	5275	1	17	4		
67.	2045	16	5¼	÷	4083	0	10	0½		1/4½
68.	3982	2	8½	÷	1347	2	19	1[illegible]		1/4

70. If the duty on a pipe of port wine, containing 138 gallons, be £52 6 6, how much is the duty on each gallon? *Answ.* 7/7.

71. If a chest of tea, containing 96 lbs. cost £33, what cost 1 lb.? *Answ.* 6/10½.

72. If a hundred weight of sugar cost £4 8 8, what are the costs of a pound and of a stone? *Answ.* 9½d. and 11/1.

73. If a contribution of £354 12 is to be made up in equal shares by 26 persons, how much must each contribute? *Answ.* £13 12 9, *rem.* 6d.

74. If prize-money to the amount of £495 5, is to be divided equally between 75 seamen, how much will each receive? *Answ.* £6 12 0¾, *rem.* 3¾d.

75. If a person spend £200 a-year, how much does he spend, at an average, each day, and how much each week? *Answ.* 10/11½, *rem.* 2½d.; and £3 16 11, *rem.* 4d.

76. The following are the amounts of the duties paid at the custom-house of Belfast, during the undermentioned years: required the annual sum, at an average.

In 1800,	£62,668	In 1806,	£207,382	In 1812,	£395,254
1801,	182,314	1807,	320,981	1813,	450,498
1802,	270,434	1808,	318,121	1814,	373,721
1803,	201,180	1809,	425,174	1820,	306,263
1804,	207,402	1810,	321,325	1821,	386,709
1805,	228,645	1811,	344,449		

Answ. £294,265 17 7$\frac{3}{17}$.

77. In 1821, the population of Great Britain and Ireland was 21,238,580, and the net amount of the public revenue of the United Kingdom was £58,108,855 2 2¾. What was the quota of this amount paid by each individual, at an average? *Answ.* £2 14 8½, *rem.* £12489 8 0¾.

78. In the year 1821, there were coined in the British mint 203,761 pounds of gold, value £9,520,732 14 6. Required the value of each pound. *Answ.* £46 14 6.

79. How much land is there, at an average, for each individual in England? *(See page* 11, *Ex.* 23, *and page* 53, *Ex.* 103.*) Answ.* 2 a. 3 r. 19$\frac{5784417}{11181437}$ p.

80. How much land is there, at an average, for each individual in Ireland? *(See page* 11, *Ex.* 24.*) Answ.* 1 a. 2 r. 39$\frac{881330}{8846349}$ p. *late Irish measure.*

It may not be improper here to show the method of multiplying and dividing, when the multiplier or divisor contains a fraction, though these operations really belong to the doctrine of fractions.

In the annexed example, to multiply by $22\frac{3}{4}$, we first multiply by 22, in the way already explained. Then, for $\frac{3}{4}$, we multiply in a separate place £1 16 6 by 3, and divide the product by 4: the result is £1 7 $4\frac{1}{2}$, which being added to the product by 22, the sum is £41 10 $4\frac{1}{2}$, the product required. The work for the fractional part is left to the learner to perform. If the multiplier had been $22\frac{1}{4}$ or $22\frac{1}{2}$, we should have found the product by 22, as before; and in the one case, have added to it one-fourth, in the other one-half of the multiplicand.

£	s.	d.	
1	16	6	$\times 22\frac{3}{4}$
		2	
3	13	0	
		11	
40	3	0	
1	7	$4\frac{1}{2}$	
£41	10	$4\frac{1}{2}$	

As an example in this kind of division, let it be required to divide £30 2 $10\frac{1}{2}$ by $22\frac{3}{4}$. Here we multiply 22 by 4, the lower term of the fraction, and to the product 88, we add 3, the upper term.—We multiply the given dividend also by 4. Then, dividing this product by 91, we find £1 6 6 for the required quotient.—It may be observed, that these methods are equally applicable in Simple and Compound Multiplication and Division.

$22\frac{3}{4}$)30	2	$10\frac{1}{2}$				
4		4	£	s.	d.	
91)120	11	6 (	1	6	6	
91						
29						
20						
591						
546						
45						
12						
546						
546						
...						

SIMPLE PROPORTION.

In comparing the magnitudes of two quantities of the same kind, the quotient, which is obtained by dividing the first by the second, is called the RATIO of the first to the second. The two magnitudes are called the TERMS of the ratio, the first the ANTECEDENT, the second the CONSEQUENT.

Thus, the ratio of 42 to 14 is 3, or is that of 3 to 1; and the ratio of 7 to 4 is $1\frac{3}{4}$, or is that of $1\frac{3}{4}$ to 1. In like manner, if the second term be divided by the first, the quotient will be the ratio of the second to the first.

Equality of ratios constitutes PROPORTION or ANALOGY; and the terms of equal ratios are called PROPORTIONALS:—the first and last the EXTREMES, the rest the MEANS.

Thus, since 8 is one-half of 16, and 6 one-half of 12, the four numbers, 8, 16, 6, and 12, are proportionals, and constitute a proportion or analogy. When considered in this relation, they are usually written thus, 8 : 16 : : 6 : 12, and are read, 8 *is to* 16, *as* 6 *is to* 12; or, they are written, *as* 8 : 16 : : 6 : 12, and read, *as* 8 *is to* 16, *so is* 6 *to* 12.

SIMPLE PROPORTION is the equality of the ratio of two quantities, to that of two other quantities.

COMPOUND PROPORTION is the equality of the ratio of two quantities to another ratio, the antecedent and consequent of which are respectively the products of the antecedents and consequents of two or more ratios.

The object of that part of Simple Proportion which is usually taught in courses of Arithmetic, is to find the number which has the same ratio to one of three given numbers, that there is between the other two; or, to find a fourth proportional to three given numbers.

*Three numbers being given, to find a fourth proportional.**

RULE. (1.) Arrange the three given terms in the same line, in succession, placing the one which is of the same kind with the required term, the third in order; and if, by the nature of the question, the required term is to be greater than the third term, put the greater of the other two terms in the second place; otherwise, put the less in that place. (2.) Then, if the two first terms be not of the same simple denomination, reduce them to the same denomination, usually the lowest mentioned in either. (3.) Find the product of the second and third terms, and divide it by the first; the quotient is the fourth proportional in the same denomination as the third term, and may be reduced to a higher denomination, if necessary.

Reason of the Rule.

By the definitions of ratio and proportion, if the first of four proportionals be divided by the second, and the third by the fourth the quotients are equal: and if the first of these quotients be multiplied by the second and fourth terms, and the second of them by the fourth and second terms, the first result will be the pro-

* The resolution of this problem is unquestionably the most important result of the doctrine of proportion. On account of its great utility and very general application, it is often called in the older works on Arithmetic, the *Golden Rule*. From the circumstance of *three* terms being given to find a fourth, it has also been very generally called the *Rule of Three*,—a name which should be disused, as it conveys no idea of the nature or object of the problem.

duct of the extremes, and the second that of the means. But these products must be equal, since it is evident, that if equals be multiplied by equals, the products are equal. Hence, it follows that *the product of the extremes is equal to the product of the means:* and therefore if the product of the means (the second and third terms,) be divided by one of the extremes, the quotient must be the other.*

Thus, in the proportionals 9 : 3 : : 6 : 2, if the quotient of 9 by 3, be multiplied by 3, the product is 9, which must be equal to three times the quotient of 6 by 2, or to the quotient of 3 times 6 by 2: and multiplying these equals by 2, we have $2\times9=3\times6$; that is, the products of the extremes and means are equal. This will be better understood if the pupil has learned fractions; since, if the two equal fractions $\frac{9}{3}$ and $\frac{6}{2}$, be multiplied successively by 3 and 2, the products are 2×9 and 3×6, which must evidently be equal.

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* It is of the utmost importance, that the pupil should, as early as possible, acquire an accurate idea of proportion. The definitions of ratio and proportion given in the text, are the same in substance as those adopted by Lacroix and others, and will perhaps convey to the mind of the learner, a clearer idea of proportion than any other, particularly after he has acquired some knowledge of fractions. It will be found to be of advantage in enabling him to understand the subject, to require him, after having wrought an exercise, to arrange in succession the three given terms, and the fourth, when found, and to examine them according to the definitions. In a short time he will thus acquire a correct idea of this interesting subject, and be able to apply it with ease and success, not only in Arithmetic, but in any other part of the Mathematics with the principles of which he may be acquainted.

It is scarcely necessary to state to the arithmetical reader, that while all writers are perfectly agreed with respect to the nature, operations, and uses of proportion, they differ very considerably in their modes of defining it, and establishing its principles.—Those who wish for farther information on this subject considered as a branch of mathematical science, may have recourse to the Elements of Euclid, and the comments of the later editors of that work, particularly Simson, Playfair, and Ekrington.—The last of these writers substitutes, instead of Euclid's definition, one which is perhaps more convenient than any other, as a basis for establishing the doctrine of proportion on strict principles.

If four magnitudes a, b, c, d, be proportional, according to the definition given in the text, we have $\frac{a}{b}=\frac{c}{d}$; and hence, multiplying by b and d, we find $ad=bc$, which affords a proof to the reader, who is acquainted with Algebra, that the product of the extremes is equal to that of the means.

In the method of arranging the terms which is delivered in almost all the books on Arithmetic, and which is very generally employed in practice, the term which is of the same kind with the answer, is put in the second place. This method is entirely subversive of the principles of proportion, and is calculated to prevent the learner from acquiring just views of this subject, as in it a ratio is, in many cases, instituted between quantities entirely heterogeneous. The method above explained, which has been adopted by Mr. Walker, and some other late writers, is founded on principles strictly Mathematical; and besides being very simple and easy in practice, it possesses the advantage of training the mind to accurate thinking, and of preparing it for the subsequent study of Mathematics. It also precludes the necessity of what is generally called *the Rule of Three Inverse*, as all the questions usually solved under that head, may be solved by the rule above delivered

When the first and second terms are in the same denomination, they are evidently in the same ratio as the numbers which express them; and therefore they are to be reduced to the same denomination, if they be not so already.

Exam. 1. If 12 yards of broadcloth cost £15, what would 8 yards cost at the same rate?

The answer to this question must evidently be in money, and therefore £15, the money given in the question, must occupy the third place: and, as 8 yards will cost less than 12 yards, 8 yards, the less of these two terms, must be put in the second place. Hence, the terms will be arranged, and the operation performed, as in the margin; and it appears, that at the rate specified in the question, 8 yards would cost £10.

yds.	*yds.*	£.
As 12 :	8 : :	15 :
		8
		—
		12)120
		—
		Answ. £10,

In this example, it is evident that as often as 8 yards are contained in 12 yards, so often would the answer be contained in £15. Hence the quantities are proportionals, and the reason of the process is evident from what goes before.

The reason of the operation will also be plain if it be considered, that the price of one yard would be found by dividing £15 by 12, and that the price of 8 yards would be found by multiplying the quotient by 8; and the result will evidently be the same that would be obtained by multiplying £15 by 8 and dividing the product by 12. The method according to the rule is in general preferable however, as it has the advantage of freeing the operation, as much as possible, from fractional quantities.

The operation may also be illustrated very simply in the following manner: since 12 yards cost £15, by multiplying by 8 we obtain £120, which must be the price of 96 (8×12) yards; and therefore dividing this by 12, we get £10, the price of 8 yards.

Ex. 1. If 57 cwt. of sugar cost £216, what would 95 cwt. cost at the same rate? *Answ.* £360.

2. If 385 yards of linen cost £63, how much might be bought for £18 at the same rate? *Answ.* 110 yards.

3. How much wine may be bought for £396, if 90 gallons cost £72? *Answ.* 495 gallons.

4. If the yearly rent of a farm of 182 acres be £273, what is the rent of a part of it containing 42 acres? *Answ.* £63.

5. If 275 reams of paper cost £330, what would 990 reams cost? *Answ.* £1188.

6. If 96 men reap 40 acres of grain in a week, how many would reap 65 acres in the same time? *Answ.* 156.

7. How many men would perform, in 168 days, a piece of work which 108 men can perform in 266 days?* *Answ.* 171.

* This question belongs to what is commonly called the *Rule of Three Inverse* as the

8. If 84 sheep can be grazed in a field for 12 days, how long might 112 sheep have been grazed equally well in the same field *Answ.* 9 days.

Exam. 2. If 12 cwt. of wheat cost £6 10 8, how much might be bought for £11 8 8?

In this example, according to the second part of the rule, the first and second terms are reduced to the same denomination, (pence,) and the rest of the work proceeds as before.

```
     £  s.  d.   £  s.  d.  cwt.
As   6  10  8 : 11  8  8 : : 12 :
    20          20
   ---         ---
   130         228
    12          12
  ----        ----
  1568        2744
                12
              -----
        1568)32928(21 cwt.
             3136  Answ.
             ----
              1568
              1568
              ----
               ...
```

Ex. 9. If 148 gallons of rum cost £119 10, how much may be bought for £89 12 6? *Answ.* 111 gallons.

10. If £114 be paid for 52 cwt. 1 q. 4 lbs. of flour, what would 122 cwt. cost at the same rate? *Answ.* £266.

11. What is the rent of 21 acres, 3 roods, 20 perches of ground, if the rent of 36 acres, 3 roods be £42? *Answ.* £25.

12. If a person walk 17 miles in 5 hours, 12 minutes, 31 seconds, how far would he walk at the same rate in 3 hours. 40 minutes, 36 seconds? *Answ.* 12 miles.

13. If £51 pay for 10 cwt. 2 qrs. 14 lbs. of sugar, what would 4 cwt. 1 qr. 14 lbs. cost at the same rate? *Answ.* £21.

14. If 19 cwt. 3 qrs. 21 lbs. of beef *(long weight,)* cost £36, what would 46 cwt. 1 qr. 29 lbs. cost at the same rate? *Answ.* £84.

15. If the fare of a coach for 88 miles be £1 9 4, for what distance would £2 3 4 pay at the same rate? *Answ.* 130 miles.

16. If the earth move 69000 miles in its orbit in an hour, through what space does it move at the same rate in 16 minutes, 48 seconds? *Answ.* 19320 miles.

If there be a remainder after the division by the first term, it is of the same denomination as the third term; and if it admit, it may be reduced lower, and the operation continued as in Compound Division.

---

greater the number of the men is, the less must be the number of days. In this case, 266 is said to have to 168 the *reciprocal* or *inverse ratio* of 108 to 171, and the numbers are said to be *reciprocally proportional.* The direct analogy is 168 days : 266 days : : 108 men : 171 men. It is obvious, that the terms must be thus arranged: since 266×108 would be the number of days in which the work would be performed by one man; and if this be divided by 168, the quotient must be the number of days required: consequently, 266 and 108 must be made the second and third terms, that their product may be taken in the operation.—The next question is of the same kind.

Exam. 3. If a web of linen containing 26 yards cost £3, what would 732 yards cost at the same rate?

Here, after we have found £84, the first part of the answer, there is a remainder of £12, which being reduced to shillings, the rest of the operation proceeds by Compound Division, and there is a final remainder of 2 farthings.

In the next exercise, the first and second terms are first reduced to the same denomination, and then the work proceeds as in this example.—When the third term is a unit, the second is simply to be divided by the first.

```
      yds. yds.   £
As 26 : 732 : : 3 :
            3 £   s.  d.
   26)2196(84   9   2¾
      208·          Answ.
       116
       104
        12
        20
       240 shillings.
       234
         6
        12
        72 pence.
        52
        20
         4
        80 farthings.
        78
         2
```

Ex. 17. If the rent of 5 acres be £4 13 4, how much land could be rented at the same rate for £70 10 6? *Answ.* 75 a. 2 r. 10p.

18. How much wheat, at 18/3 ℔ cwt. may be purchased for £149 16 4? *Answ.* 164 c. 0 q. 20$\frac{100}{219}$ lbs.

19. How much oats, at 8/8 ℔ cwt. may be bought for £64 3 2½? *Answ.* 148 c. 0 q. 7 lbs.

20. If 6 c. 3 q. 12 lbs. of flour cost £9, what would 4 c. 2 qrs. cost at the same rate? *Answ.* £5 18 1½.

21. If 39 c. 1 q. 10 lbs. *(long weight,)* of pork cost £59 0 0, what cost 13 cwt.? *Answ.* £19 10.

When the third term is of more denominations than one, it is generally necessary, or at least proper, to reduce it to the lowest denomination mentioned in it. When the first and second terms are not large, however, it is often better not to perform this reduction, but to employ Compound Multiplication and Division, especially if the third term be money.

The work is often much abbreviated by dividing the first and second, or the first and third terms, (but never the second and third,) by any number which exactly measures them, and employing the quotients instead of the numbers themselves.

Exam. 4. What is the yearly rent of 47 a. 3 r. 21 p. at £1 4 6 ℔ acre?

In this example, the first and second terms are reduced to perches, the lowest denomination mentioned in either, and the third to pence, the lowest mentioned in it.—Then, by the usual process, the answer is found to be 14077 pence, with 14 remaining; and this, by proper reduction, becomes £58 13 $1\frac{14}{160}$. In the division by 160, a cipher is cut from the divisor, and a figure from the dividend; and then the work is contracted, by using the factors of 16, instead of itself. With respect to the reason of the operation, it is evident, that were the third term, 294d. divided by the first term, 160 perches, the quotient would be the rent of one perch; and were this multiplied by 7661, the perches in the second term, the product would be the rent of 47 a. 3 r. 21 p. But as in other cases, the same result is found, and more easily, by dividing the product of the second and third terms by the first.

```
   A.     A.   R.  P.    £  s.  d.
As 1  :  47   3   21 : : 1   4   6
 ——       4              20
 160     ——              ——
         191              24
          40              12
         ——              ——
         7661             294
          294
         ——
        30644
       68949
      15322
      ————
   4)225233|4
      ————
    4)56308...14
      ————
   12)14077...14
      ————
   2|0)117|3  1
      ————
      £58  13  1 14/160, Answ.
```

Exam. 5. If 15 yards of linen cost £2 1 10½, what will 55 yards cost at the same rate?

In this example, after the terms are arranged according to the rule, the last is brought to farthings, the lowest denomination mentioned in it; these farthings are then multiplied by 55, the second term, and the product divided by 15, the first term; the quotient is 7370, which is farthings, because the third term was reduced to farthings; and these farthings, by reduction, become £7 13 6½, the price of 55 yards, as was required.

```
     yds.  yds.     £   s.  d.
As 15 : 55 : :   2   1  10½
                20
                ——
                41 shillings.
                12
                ——
               502 pence.
                 4
               ——
              2010 farthings.
                55
              ——
             10050
            10050
        ⎧ 5)110550
   15   ⎨ 3)22110
        ⎩ ——————
         4)7370 farthings.
        12)1842½
        2|0)15|3  6½
            ————
            £7  13  6½, Ans.
```

This operation may be much contracted in the following manner:—let the numbers be arranged as before, and let the first and second terms be divided by 5, which is a common measure; the quotients are 3 and 11, which are to be used instead of 15 and 55; and the operation is still farther abbreviated by employing Compound Multiplication and Division.

	yds.		yds.		£	s.	d.
As	15	:	55	: :	2	1	10½
	3	:	11				11
				3)	23	0	7½
					£7	13	6½,

Answ.

The reason of the contraction employed in this example is, that if the divisor and dividend be multiplied or divided by the same number, the quotient is not changed. Thus, if 12 be divided by 4, the quotient is the same as if 24 be divided by 8, or 3 by 1: and hence the ratio of two numbers is the same as the ratio of two numbers which are obtained by dividing or multiplying them by the same number. This principle, and indeed the whole doctrine of Proportion, will be better understood by the pupil who is acquainted with fractions.

Ex. 22. If 63 gallons of wine cost £41 10 6½, what cost 10 gallons?* *Answ.* £6 11 9¾, *rem.* 1/2¾.

23. What cost 4 tons of logwood, if 36 tons cost £316 7 3? *Answ.* £35 3 0¼, *rem.* ⅓ f.

24. If a lot of sugar, containing 56 cwt. cost £214 18 4, what does it cost ℔ ton? *Answ.* £76 15 1¼, *rem.* ⅘ f.

25. If the yearly rent of 18 acres be £24 18 6, what is the rent of 29 acres at the same rate? *Answ.* £40 3 1½, *rem.* ⅔ f.

---

* In working this exercise, and the nine following, the third term in the analogy in each may be reduced to the lowest denomination mentioned in it, and the product of the result and the second term divided by the first. In all of them, however, and in several others that follow, the reduction may be omitted, and the work performed by Compound Multiplication and Division. In exercise 23, the first and second terms may be divided by 4; and then as the second term will be 1, the answer will be found by dividing the third term by 9. In exercise 24, divide the first and second terms, in hundreds weight, by 4; and in exercise 26, divide them by 2. In like manner, in exercise 1, the first and second terms may be divided by 19; in exercise 2, by 9; in exercise 3, the first and second, or the first and third terms by 18, &c.

In this and the following exercises, the remainders, when there are any, are annexed to the answers, and are expressed in their simplest form by dividing each remainder, and the divisor which produces it, by any number that is contained in them without remainder. *(See the preceding paragraph.)* It is generally sufficient, however, to require the learner to find the answer, if it be money, *true to the nearest farthing.* To require all answers to be brought out true to the smallest fractional parts, would occupy a great deal of time which may be more advantageously spent; and such a degree of accuracy is scarcely ever aimed at, and indeed can scarcely ever be attained, in any practical application of Arithmetic. In viewing the matter in this light, when farthings are wrought for, if the remainder be more than half the divisor, it may be rejected and one added to the farthings. Thus, in exercise 22, the answer might be taken £6 11 10; in exercise 24, £76 15 1¼; in exercise 25, £40 3 1¼; in exercise 27, £96 18 1¼, &c.

26. If 10 cwt. of oatmeal cost £9 15, what will 252 cwt. cost at the same rate? *Answ.* £245 14 0.

27. What is the value of 110 gallons of Spanish brandy, at the rate of £6 3 4 for 7 gallons? *Answ.* £96 18 1, *rem.* ¼d.

28. What cost 126 cwt. of oatmeal, if 5 cwt. cost £4 17 6? *Answ.* £122 17 0.

29. If 19 gallons of whiskey cost £7 16 9, what will 37 gallons cost at the same rate? *Answ.* £15 5 3.

30. What cost 79 cwt. of flour, if 19 cwt. cost £24 12 6? *Answ.* £102 7 9, *rem.* $\frac{3}{19}$d.

31. If 23 yards of broadcloth cost £25 18 6, what cost 98 yards? *Answ.* £110 9 3, *rem.* $\frac{8}{23}$d.

32. How much cambric may be bought for £186 2 4½, if 13 yards cost £8 6 3½? *Answ.* 291 yards.

33. At 1/8 ℔ pound, how much coffee may be bought for £17 12 7½? *Answ.* 1 c. 3 q. 15$\frac{13}{16}$ lbs.

34. How much wine, at the rate of £31 15 4 for 46 gallons, may be bought for £117 16 8? *Answ.* 170$\frac{22}{23}$ gallons.

35. How much linen may be bought for £41 12 6, if 405 yards cost £69 7 6? *Answ.* 243 yards.

36. How much cotton-wool, at 2/10½ ℔ pound, may be bought for £39 14 8? *Answ.* 276$\frac{8}{21}$ lbs.

37. How much barley, at 13/9 ℔ cwt. may be bought for £62 7 9¾? *Answ.* 90 c. 3 q.

38. If 6½ yards of muslin cost £1 6 6½, what will 79 yards cost at the same rate? *Answ.* £16 2 7.

39. What cost 78 yards, 3 quarters, and 2 nails of broadcloth at 17/4 ℔ yard? *Answ.* £68 7 2.

40. If 17 c. 3 q. 19 lbs. of barley cost £8 18 9, how much may be bought for £5 12 6, at the same rate? *Answ.* 11 c. 1 q. 3$\frac{21}{148}$ lbs.

41. How much tea, at 7/6 ℔ lb. may be bought for £37 2 6? *Answ.* 99 lbs.

42. If a farm containing 396 a. 3 r. 10 p. be let at the yearly rent of £740 14 4, what is the rent of each acre? *Answ.* £1 17 4.

43. What cost 4 cwt. 2 qrs. 20 lbs. *(long weight,)* of pork, at £8 14 6 ℔ cwt.? *Answ.* £17 7 8.

44. If 172 cwt. 2 qrs. 18 lbs. of rye cost £87 6 3, how much may be bought at the same rate for £3 19 4½? *Answ.* 7 c. 3 q. 11 lbs.

45. What cost 124 cwt. 2 qrs. 14 lbs. of rice, at £2 1 6 ℔ cwt.? *Answ.* £258 11 11¼.

46. If the prime cost of 247 cwt. 3 qrs. 14 lbs. of flour, together with the charges, amount to £423 9 0¾, what does it stand ℔ cwt.? *Answ.* £1 14 2.

47. How much wheat, at 15/2 ℔ cwt. may be bought for £113 15? *Answ.* 150 cwt.

48. If 2 cwt. 3 qrs. 21 lbs. of sugar cost £12 3 4, what is the value of 17 cwt. 2 qrs. 14 lbs. at the same rate? *Answ.* £73.

49. At £9 2 ℔ cwt. what cost 3 lbs. of raisins? *Answ.* 4/10½.

50. If 6 cwt. 0 qrs. 29 lbs. *(long weight,)* of beef, cost £13 7 6, how much may be bought for £17 12 6? *Answ.* 8 c. 0 q. 27 lbs.

51. What cost 39 yards, 3 quarters, 3 nails of broadcloth, if 3 yards, 2 quarters, 1 nail cost £3 2 10? *Answ.* £35 4 4¾, nearly.

52. If the rent of 175 a. 2 r. 30 p. be £278 3 5¼, how much land can be rented at the same rate for £1 11 8? *Answ.* 1 acre.

53. What cost 365 bottles of wine at £2 13 6 ℔ dozen? *Answ.* £81 7 3½.

54. What cost 311 sheep, at £37 12 6 ℔ score? *Answ.* £585 1 4½.

55. How much sugar may be bought for £113 13 4, if 7 c. 3 q. 14 lbs. cost £31 6 8? *Answ.* 28 c. 2 q. 7$\frac{24}{47}$ lbs.

56. If the rent of 12 a. 2 r. 30 p. be £28 8 9, what is the rent of 59 a. 3 r. 20 p. at the same rate? *Answ.* £134 4 0½, *rem.* $\frac{98}{403}$ f.

57. How much muslin, at 2/8½ ℔ yard, is equal in value to 169 yards of cambric, at 7/8½ ℔ yard? *Answ.* 481 yards.

58. If the assessment on a townland, containing 445 acres, be £14 14 9¾, how much is it on each score of acres? *Answ.* 13/3.

59. If a clerk have a salary of £89 12 6 ℔ year, commencing at the first of May, how much has he to receive on leaving his situation on the 18th of December? *Answ.* £56 14 5$\frac{13}{73}$.

60. If a person save 8/1½ ℔ week, in what time will he save £112 10? *Answ.* 5 years, 16$\frac{12}{13}$ weeks.

61. If the shilling loaf weigh 3 lbs. 6 oz. when flour sells at £1 13 6 ℔ cwt. how much should it weigh when flour sells at £1 7 6 ℔ cwt.? *Answ.* 4 lbs. 1$\frac{43}{55}$ oz.

62. If the digging of a British mile of a canal cost £1347 7 6, what cost the digging of an Irish mile at the same rate? *Answ.* £1714 16 9¾, *rem.* $\frac{3}{11}$ f.

63. When wheat sells at 54/7½ ℔ quarter (of 456 lbs.), how much is it ℔ cwt.? *Answ.* 13/5.

64. What is the value of a ton of gold, the guinea weighing 5 dwts. 9 grs.? *Answ.* £127,627 18 1½; *rem.* $\frac{30}{48}$ f.

65. At 7/6 ℔ ounce, what is the value of a silver bowl weighing 9 oz. 13 dwts. 8 grs.? *Answ.* £3 12 6

66. If a person travelling 14 hours ℔ day, finish the first half of a journey in 9 days, in what time will he finish the remaining half, travelling 10 hours ℔ day at the same rate each hour? *Answ.* 12⅗ days.

67. If a person walk from Belfast to Newry, a distance of 30 miles, in 10 hours, 42 minutes; in what time will he walk at the same rate from Belfast to Dublin, the distance being 80 miles? *Answ.* 28 h. 32 m.

68. How many British miles are equivalent to 128 Irish, 11 Irish miles being equivalent to 14 British? *Answ.* 162 m. 7 fur. 10 p. 5 yds.

69. The mean length of a degree on the earth's surface, is 69

British miles and 80 yards, nearly: how much is it in Irish miles? *Answ.* 54 m. 2 fur.

70. It is found, that the diameter of every circle is to its circumference very nearly in the ratio of 113 to 355: what then is the earth's circumference, its diameter being $7911\frac{3}{4}$ miles, nearly? *Answ.* $24855\frac{1}{2}$ miles, nearly.

71. If the girt of a round tree be 6 feet 10 inches, what is its diameter? *Answ.* 2 feet, $2\frac{36}{355}$ inches.

72. The earth revolves round the sun in an orbit, the mean diameter of which is one hundred and ninety millions of miles. Were this orbit an exact circle, as it nearly is, what would be its circumference? *Answ.* $596,902,654\frac{99}{113}$ miles.

73. If a farmer lend his neighbour, for 34 days, a cart horse which draws 12 c. 3 q. how long should he have in return a horse that draws 10 c. 3 q.? *Answ* $40\frac{14}{43}$ days.

74. After A has travelled 51 miles on a journey, B sets out to overtake him, and for 16 miles travelled by A, B constantly travels 19 miles. How far will each have travelled before A will be overtaken? *Answ.* 323 miles.

75. In a copy of Milton's Paradise Lost, containing 304 pages, the combat of Michael and Satan commences at the end of the 139th page; at what page may it be expected to commence in a copy containing 328 pages? *Answ.* The fourth proportional is $149\frac{37}{38}$, and hence the passage may be expected to commence near the bottom of the 150th page.

76. A pound, Troy weight, of silver is coined into 66 shillings: what is the value of a pound, avoirdupois weight? *(See page 42.)* *Answ.* £4 0 $2\frac{1}{2}$.

77. How much should a gold coin, worth one pound, weigh, the weight of the guinea being 5 dwts. 9 grs.? *Answ.* 5 dwts. $2\frac{6}{7}$ grs.

78. The length of a wall, being tried by a measuring line, appears to be 1287 feet, 4 inches; but on examination, the line is found to be 50 feet, $10\frac{1}{2}$ inches in length, instead of 50 feet, its supposed length. Required the true length of the wall. *Answ.* 1309 feet, $10\frac{17}{50}$ inches.

79. If a shopkeeper use a false weight of $14\frac{3}{4}$ oz. for a pound, how many pounds will 112 lbs. of just weight appear to be, when weighed by his weight? *Answ.* $121\frac{28}{59}$ lbs.

80. If a dealer in spirits use, instead of a gallon, a measure which is deficient by half a pint, what will be the true measure of 100 of these false gallons? *Answ.* $93\frac{3}{4}$ gallons.

81. If the port wine contained in a vessel, weigh 15 c. 3 qrs. 24 lbs. how much weight of pure water, and how much of cow's milk, would the same vessel contain, the weight of equal bulks of port wine, of water, and of milk, being as the numbers 997, 1000, and 1032? *Answ.* 16 c. 0 q. $1\frac{379}{997}$ lbs. and 16 c. 2 q. $2\frac{768}{997}$ lbs.

82. If a stone column weigh 69 cwt. 3 qrs. 10 lbs. what will a pillar of oak of the same dimensions weigh, the weight of equal bulks of the stone and oak being as the numbers 2484 and 1170? *Answ.* 32 c. 3 q. $16\frac{18}{81}$ lbs.

83. A cubic foot of fresh water weighs 1000 ounces avoirdupois nearly: what is the weight of a vessel of such water, containing $217\frac{1}{2}$ cubic inches? *Answ.* 7 lbs. $13\frac{13}{144}$ oz.

84. The quantity of rum imported into England from the British West India Islands, in 1798, was four millions, one hundred and ninety-six thousand, one hundred and ninety-three gallons; the duty on which amounted to £95,996 5 5. Of this quantity, 333,093 gallons were exported. What was the amount of the duty on the quantity consumed in England? *Answ.* £88,376 2 $1\frac{1}{4}$, nearly.

85. In 1798, the quantity of sugar imported into England from the West Indies, was 2,361,715 cwt. 0 qr. 8 lbs.; the duty paid on which was £2,070,377 2 7. What was the duty ℔ cwt.? *Answ.* 17/6 $\frac{1380}{3,062,011}$.

86. In what time would wind move from the pole to the equator, at the rate of $2\frac{3}{4}$ miles ℔ hour, the distance being 6214 miles? *Answ.* 94 days, 3 hours, 38 minutes, $10\frac{10}{11}$ seconds.

87. The earth describes its orbit round the sun in 365 days, 5 hours, 48 minutes, and 48 seconds: through what space does it move each hour, at an average, the circumference of the orbit being 596,902,655 miles? *Answ.* $68094\frac{262141}{657426}$.

88. The ecliptic contains 360 degrees, and is apparently described by the sun in 365 days, 5 hours, 48 min. and 48 sec. through what part of it does he appear to move at an average each day, and each hour? *Answ.* 59′ 8″ 19‴, and 2′ 27″ 51‴, nearly.

89. The value of British manufactures and produce exported from Britain in 1808, was £40,881,871; and that of Irish manufactures and produce exported from Ireland the same year was £12,577,517 10 11. It is hence required to determine how much the latter sum affords for each £100 contained in the former. *Answ.* £30 15 $3\frac{3}{4}$, nearly.

The pupil having been thus presented with illustrations and exercises in the practical application of the doctrine of Proportion, what follows respecting the changes in the order and magnitude of proportionals, may be found useful, especially to such pupils as may afterwards study Mathematics.

It has already been shown, that if four numbers be proportional, the product of the extremes, is equal to the product of the means. From the converse of this principle, it follows that whatever changes are made in the order or magnitude of proportionals, the results will be proportionals, if the product of the extremes be equal to that of the means. If four magnitudes be proportional, other proportionals will be obtained, if either the antecedents, or consequents, or both, be respectively increased or diminished by like parts of themselves; or if the

two terms of either, or of both ratios be respectively increased or diminished by quantities in the same ratio with themselves. This will be understood from the following view of the most common and useful changes which proportionals admit; and from this view also, the technical terms used to denote these changes will be understood.

Let the four numbers 8, 6, 20, and 15, be proposed, which are evidently proportionals; that is,

8 : 6 : : 20 : 15. Then,

Alternately, ... 8 : 20 : : 6 : 15;
By *Inversion*,..... 6 : 8 : : 15 : 20;
By *Composition*,..... 8+6 : 6 : : 20+15 : 15, or 14 : 6 : : 35 : 15;
By *Division*, ... 8—6 : 6 : : 20—15 : 15, or 2 : 6 : : 5 : 15;
By *Conversion*, ... 8 : 8—6 : : 20 : 20—15, or 8 : 2 : : 20 : 5;

To these changes, the following, out of many to which no specific names have been applied, may be added:

Let 8 : 6 : : 20 : 15; then
8 : 8+6 : : 20 : 20+15, or 8 : 14 : : 20 : 35;
8+6 : 8—6 : : 20+15 : 20—15, or 14 : 2 : : 35 : 5;
8+20 : 8 : : 6+15 : 6, or 28 : 8 : : 21 : 6;
20+8 : 20—8 : : 15+6 : 15—6, or 28 : 12 : : 21 : 9.

Also, if 8 : 6 : : 20 : 15 : : 12 : 9 : : 4 : 3; then 8+20+12+4 : 6+15+9+3 : : 8 : 6 : : 20 : 15, &c.; that is, *the sum of all the antecedents of any number of equal ratios, is to the sum of all the consequents, as any one of the antecedents is to its consequent.*

Let, again, 8 : 6 : : 20 : 15,
and 14 : 4 : : 21 : 6;
then 8×14 : 6×4 : : 20×21 : 15×6, or 112 : 24 : : 420 : 90.

It may be observed, in conclusion, that all these changes and modifications hold, without exception, in respect to *abstract numbers*; but when the given numbers express *real quantities*, the terms of every ratio, and also the numbers whose sum or difference is taken, must always be of the same kind. Thus, in the analogy, 8 yards : 6 yards : : 20 shillings : 15 shillings, we are not at liberty to take the terms alternately; for if we did, we should thus fall into the absurdity of instituting a ratio between 8 yards and 20 shillings, and between 6 yards and 15 shillings.

COMPOUND PROPORTION.

Rule I. (1.) By the rule for Simple Proportion, find a fourth proportional to two given terms of the same kind with one another, and to the term which is of the same kind as the answer. (2.) To two other terms of the same kind, and to the last obtained find a fourth proportional; and thus proceed if there be more terms: the final result will be the answer.

Rule II. (1.) Place the term which is of the same kind

as the required term, in the last place. (2.) Comparing the other given terms by pairs, place each as antecedent or consequent, according to the rule for Simple Proportion. (3.) Divide the continual product of all the consequents and the last term, by the continual product of all the antecedents; the quotient will be the answer.

As a contraction in the use of this latter rule, divide an antecedent and any consequent by any number which will measure them, and employ the results instead of those terms: or, if an antecedent and any consequent, or an antecedent and the last term be the same, reject them.

Exam. 1. If 40 gallons of ale serve 17 persons 5 days, how many gallons will serve 9 persons for a year, at the same rate?

I. As 5 days : 365 days : : 40 gallons : 2920 gallons.
As 17 persons : 9 persons : : 2920 gallons : $1545\frac{13}{17}$ gallons.

In this operation, we first proceed as if the number of persons in both cases were 17, and on this supposition we find that 2920 gallons would be used by those persons in a year. But the number of persons being 9, instead of 17, we find by the second analogy, that if 17 persons would use 2920 gallons in a year, 9 persons would use $1545\frac{13}{17}$ gallons in the same time.

II. As 5 days : 365 days
17 persons : 9 persons } : : 40 gallons : $1545\frac{13}{17}$ gallons.

In the second method that term is placed last which is of the same kind as the required term. Then, were the number of persons the same, it is evident that the answer would be greater than 40 gallons; and therefore we put 365 days in the second, and 5 days in the first place: but were the number of days the same, it is obvious that the required term would be less than 40 gallons; and we therefore put 9 persons in the second place, and 17 in the first. We then multiply the product of the consequents by the common third term, 40, and divide the result by the product of the antecedents, and the same answer is found as before. This operation is in effect the same as that in the first method; for the result of the first single analogy, without the actual multiplication and division, is $\frac{365 \times 40}{5}$, and, consequently, the second analogy becomes $17 : 9 : : \frac{365 \times 40}{5} : \frac{9 \times 365 \times 40}{5 \times 17}$; whence it appears that in both methods the same multiplications and divisions are in reality performed, and consequently the one is only a modification of the other. In this method 5 and 365, or 5 and 40, might be divided by 5 as a contraction.

The principles on which the operations in Compound and in Simple Proportion depend are the same.

It may be remarked, that one very considerable advantage which the second rule possesses, is, that by it the operation is kept entirely free from fractions till the conclusion; while in the other mode, fractions often arise from the first analogy, and render the remaining part of the work more intricate and difficult.

Exam. 2. If the carriage of 66 standard hundreds, for 42 British miles, be £6 5, what is the carriage of 36 hundreds, *long weight*, for 90 Irish miles, at the same rate?

$$\left.\begin{array}{ll} \text{As 66 stand. cwt.} & : \text{36 long cwt.} \\ \text{112 lbs.} & : \text{120 lbs.} \\ \text{42 miles, Brit.} & : \text{90 miles, Irish,} \\ \text{11} & : \text{14} \end{array}\right\} :: £6\ 5 : £9\ 19\ 2\tfrac{870}{847}.$$

In this example, since the answer is to be in money, £6 5 is placed as the common third term. Then it is evident, that were all things alike, except the number of hundreds, the answer would be less than £6 5; and therefore, 36 is put as consequent, and 66 as antecedent. But had the number of hundreds been equal, the answer must evidently have been greater than £6 5, on account of the difference in their magnitudes; and therefore, 112 is made antecedent, and 120 consequent. The arrangement of the remaining terms proceeds on similar principles, and the answer is found by dividing the continual product of £6 5 and the consequents, by the continual product of the antecedents. The operation is much abbreviated, as in the margin, by dividing the terms of the first and third ratios by 6, and those of the second by 8, and neglecting in the work 14, which occurs as an antecedent in one ratio, and a consequent in another.

$$\left.\begin{array}{r} 11 : 6 \\ (14) : 15 \\ 7 : 15 \\ 11 : (14) \end{array}\right\} :: £6\ 5 : £9\ 19\ 2\tfrac{870}{847}.$$

This question might also have been wrought by four analogies in Simple Proportion. Every one of these, however, would have given origin to fractions, the neglecting of which would have prevented the precise result from being obtained. The work might also have been modified by reducing the hundreds and the miles, respectively, to the same kind. The using of 112, indeed, as an antecedent, and 120 as a consequent, serves this purpose, as the hundreds are thus reduced to pounds: and the multiplication by 11 and 14 serves a similar purpose, in relation to the miles.

Ex. 1. If the carriage of 59 cwt. 19 miles, cost £2 16, how far may 43 cwt. be carried at the same rate for £2 4? *Answ.* $20\tfrac{221}{602}$ m.

2. If the carriage of 13 cwt. 65 miles, cost £2 5, how many hundreds may be carried 40 miles at the same rate for £3 15? *Answ.* $35\tfrac{5}{24}$ cwt.

3. If 12 horses in 5 days plough 11 acres, how many horses would plough 33 acres in 18 days? *Answ.* 10.

4. If a person walking 12 hours each day, perform a journey of

250 miles in 9 days, in how many days, walking 10 hours each at the same rate, would he complete a journey of 400 miles? *Answ.* $17\frac{7}{25}$ days.

5. If the expenses of a family of 8 persons amount to £42, in 16 weeks, how long will £100 support a family of 6 persons, at the same rate? *Answ.* $50\frac{50}{63}$ weeks.

6. If 29 men, in 5 days of 12 hours each, reap 32 acres, in how many days of 13 hours each, will 20 men, working equally, reap 40 acres? *Answ.* $8\frac{19}{52}$ days.

7. If £15 12 pay 16 labourers for 18 days, how many labourers, at the same rate, will £35 2 pay for 24 days? *Answ.* 27.

8. If 36 yards of cloth, 7 quarters wide, cost £25 4, what cost 120 yards of the same quality, but only 5 quarters wide? *Answ.* £60.

9. If a tradesman earn 16 guineas in 108 days, how many sovereigns would he earn at the same rate, in 270 days; 20 guineas being equivalent to 21 sovereigns? *Answ.* 42.

10. If the rent of a farm containing 26 a. 2 r. 23 p. be £50 8 9, what would be the rent of another farm containing 17 a. 3 r. 2 p. if 6 acres of the latter be worth 7 of the former? *Answ.* £39 4 7.

11. If a puncheon of rum containing 85 gallons, cost £58 8 9, what would be the value of a hogshead containing 63 gallons, and composed of four parts of the same rum, and one part of water? *Answ.* £34 13 0.

12. In what time would 23 men reap a field which 40 women would reap in 6 days, if 7 men can reap as much as 9 women? *Answ.* $8\frac{6}{69}$ days.

13. If a person walking 13 hours each day, travel 191 miles in 7 days; in how many days of 10 hours, will he complete the remainder of a journey of 500 miles, at the same rate each hour? *Answ.* $14\frac{1379}{1910}$.

14. If 63 lbs. of tea cost £20 10 6, what cost 70 lbs. of a different quality, 9 lbs. of the former being equal in value to 10 lbs. of the latter? *Answ.* £20 10 6.*

FRACTIONS.†

If a quantity, considered as a whole, be divided into any number of equal parts, a FRACTION in respect to that quan-

* Many of the questions usually proposed under the head of Compound Proportion are quite misplaced, as some of them are merely anticipations of what is afterwards delivered in *Interest*, and others belong to *Mensuration*, or other subjects, with the principles of which the pupil is supposed to be unacquainted. Hence, the number and variety of exercises here given, are purposely less than in several other works on Arithmetic.

† With respect to the position which the doctrine of Fractions should occupy in a

tity, signifies one or more such parts: and the quantity divided is called, in respect to the fraction, the INTEGER, or UNIT.

A fraction is expressed by two numbers, or terms, called the *numerator* and the *denominator*.

The DENOMINATOR is written below the numerator, and expresses the number of equal parts into which the integer is divided; and the NUMERATOR expresses the number of such parts denoted by the fraction.

Thus, $\frac{4}{5}$, which is read *four-fifths*, is a fraction, and signifies that a unit is divided into five equal parts, and that four of these parts are taken.

If the numerator of a fraction be taken as an integer, and divided into as many equal parts as there are units in the denominator, the fraction may also be regarded as expressing *one* of these parts.

Thus, if 4 be divided into 5 equal parts, the fraction $\frac{4}{5}$ expresses one of these parts, and is therefore the same as *one-fifth of* 4.

A PROPER FRACTION is that whose numerator is less than its denominator.

An IMPROPER FRACTION is that whose numerator is *not* less than its denominator.

Thus, $\frac{3}{8}$ and $\frac{1}{7}$ are proper fractions; and $\frac{8}{3}$, $\frac{4}{3}$, and $\frac{100}{7}$ are improper fractions.

A number consisting of a whole number, with a fraction annexed, as $5\frac{3}{4}$, is termed a MIXED NUMBER.

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course of Arithmetic, writers seem to be much divided in their opinions; some introducing it immediately after Simple Division, some after the doctrine of Proportion, and others at a more advanced part of the course. The first of these positions is certainly the natural one, both because Fractions for the most part arise from the remainders in Division, and because on this account they are of the same nature with the quantities treated of in the Compound Rules. The principal objection to this arrangement is, that the study of Fractions is too difficult and intricate to be prosecuted with success by the majority of pupils, when they have learned only the fundamental rules, without having acquired that ease and readiness in managing numbers, which are derived from more varied and extended practice. The following article is so composed, that the pupil may study it in the place where it stands; or if the teacher should think it better, after Simple Division, after the mercantile rules, or indeed at almost any other part of the course before the extraction of roots. It may be observed, in general, that the several parts of Arithmetic tend to illustrate each other. Thus, if the pupil have learned the doctrine of Proportion, he will more easily and more successfully study Fractions; and if he have studied Fractions before Proportion, he will be able to acquire a more extended and adequate knowledge of the latter. For this reason his attention should be frequently recalled to the rules he has passed; and in this case he should be made to avail himself of such illustrations, as the subsequent rules which he has learned may afford.

A SIMPLE FRACTION expresses one or more of the equal parts into which a unit is divided, without reference to any other fraction.

A COMPOUND FRACTION expresses one or more of the equal parts into which another fraction, or a mixed number, is divided.

Thus, $\frac{5}{8}$ is a simple fraction; but $\frac{7}{8}$ of $\frac{10}{11}$, and $\frac{1}{3}$ of $\frac{4}{5}$ of $1\frac{4}{7}$, are compound fractions. Compound fractions have the word *of* interposed between the simple fractions of which they are composed.

A COMPLEX FRACTION is that which has a fraction either in its numerator or denominator, or in each of them.

Thus, $\frac{5\frac{1}{2}}{9}$, $\frac{8}{9\frac{2}{3}}$, and $\frac{5\frac{1}{2}}{6\frac{2}{3}}$, are complex fractions.

For the definitions of several other terms that will be employed in what follows, see the first Note in page 24.

It is of the greatest consequence that the pupil should acquire a correct idea of the nature of fractions, on commencing the study of this important part of Arithmetic The following illustrations may perhaps be found useful in assisting him in this respect.

Fractions generally have their origin from the division of a number by another which does not measure it. Thus, if £13 be divided by 5, the quotient is £2, with a remainder of £3 to be divided by 5. To have an accurate idea of the nature and value of the latter part, we are to conceive £3 divided into 5 equal parts; and, by the nature of division, any of these parts will be its correct value. Each of these parts is evidently less than £1, the unit under consideration, and is therefore called a *fraction*, from its being, as it were, a *broken part* of the unit or integer. Arithmeticians have agreed to denote it, as has been already remarked in Division, by the expression £$\frac{3}{5}$, which may be read, *one-fifth of £3*. In like manner, the quotient of 4 cwt. by 7, is expressed $\frac{4}{7}$ cwt. and may be read *one-seventh of 4 cwt.*

Resuming the former fraction, if we suppose each pound to be divided into 5 equal parts, 5 being the number of units in the divisor, each part will be one-fifth of £1, and in £3 there will be 5×3, or 15 such parts. Dividing these by 5, we obtain 3 of these parts, or three-fifths of £1, for the value of the fraction: whence it appears, that *one-fifth of three pounds is equivalent to three-fifths of one pound;* and in a similar manner it would be shown, that whatever may be the integer, one-fifth of three is equivalent to three-fifths of one, or three times the fifth part of one; one-seventh of four to four-sevenths of one, &c. When fractions are considered in the abstract, without reference to any particular integer, we say briefly four-sevenths, instead of four-sevenths of one; the former expression being *understood* to be equivalent to the latter. Hence we see, that the two definitions of a fraction given in the text, though they suggest to the mind ideas which are apparently different, are in effect the same. Agreeably to either, the lower number may properly be called the *denominator*, as it gives *name* (such as *fifth*, *seventh*, &c.) to the parts into which the integer is divided; and

agreeably to the first definition in the text, the upper number is called the *numerator*, as it *numbers* the parts expressed by the fraction.

According to these views, every fraction would have its numerator less than its denominator, and would consequently be less than its unit, as its name imports. Such a view is too limited however to answer the general purposes of calculation; and it is often necessary to consider as fractions, quantities which equal or exceed the integer; or which *indicate* the quotients of numbers not only by greater, but also by equal or by smaller numbers, without the division being actually performed. Quantities of the latter kind are therefore called *improper fractions*; and, for the sake of distinction, others, in respect to them, are called *proper fractions*. It is evident, from either view of the nature of a fraction, that it is less or greater than a unit accordingly as its numerator is less or greater than its denominator, and that it is equal to a unit, if its numerator and denominator be equal.

In addition to the preceding illustrations, it may be proper to state, that fractions are of the same nature as the subordinate quotients in Compound Division. Thus, if £27 be divided by 20, the quotient is £1 7, or £$1\frac{7}{20}$, each expressing precisely the same quantity, and each of the 7 remaining pounds being reduced into 20 equal parts in each case.

From these principles we may readily derive some of the chief properties of fractions. For this purpose, let us compare the fractions $\frac{2}{5}$ and $\frac{2}{15}$, the denominator of the second of which is treble of the denominator of the first, while their numerators are the same. Now it is easy to see that the former is three times as great as the latter: for in the former, the integer 2 is divided into 5 equal parts, while in the other it is divided into three times as many equal parts; and consequently, each part in the latter case, is only one-third of each part in the former; that is, the fraction $\frac{2}{15}$ is one-third of the fraction $\frac{2}{5}$. Hence, it appears, that by multiplying the denominator of the fraction $\frac{2}{5}$ by 3, we obtain a fraction equal to one-third of $\frac{2}{5}$; and that by dividing the denominator of the fraction $\frac{2}{15}$ by 3, we find a fraction equal to three times $\frac{2}{15}$. Now, since, by the preceding illustrations, the quotient of 2 by 15 is the same as two-fifteenths, the product of $\frac{2}{15}$ by 3 is six-fifteenths, in the same manner as the product of 2 shillings by 3, is 6 shillings; and since $\frac{2}{15}$ is one-third of $\frac{2}{5}$, and also one-third of $\frac{6}{15}$, it follows, that $\frac{6}{15}$ must be equal to $\frac{2}{5}$. Hence, by multiplying each term of the fraction $\frac{2}{5}$ by 3, we obtain an equal fraction $\frac{6}{15}$; and by dividing each term of the fraction $\frac{6}{15}$ by 3, we get an equal fraction $\frac{2}{5}$. We see also, that by multiplying the numerator $\frac{2}{15}$ by 3, we obtain a fraction 3 times as great as $\frac{2}{15}$; and by dividing the numerator of the fraction $\frac{6}{15}$ by 3, we find a fraction which is equal to one-third of $\frac{6}{15}$. It is evident, that similar properties might be proved in a similar manner to hold respecting any other fractions; and hence we have the following important properties which belong to all fractions:

1. *If the terms of any fraction be both multiplied, or both divided by the same number, the value of the fraction is not changed.*

2. *A fraction is multiplied by any number either by multiplying its numerator, or dividing its denominator, by that number.*

3. *A fraction is divided by any number either by dividing its numerator, or multiplying its denominator, by that number.*

4. It appears also, from the principles here explained, and from the principles of proportion, that *if two fractions be equal, the numerator of the one is to its denominator, as the numerator of the other is to its denominator; or,* by inversion, *that the denominator of the one is to its numerator, as the denominator of the other is to its numerator.*

The same operations can be performed on fractional as on integral quantities. Hence, the doctrine of Fractions comprises the *Addition, Multiplication, Subtraction,* and *Division,* of Fractions. Before entering on these operations, it is proper to show how such quantities may be modified, without changing their value, so as to fit them for the several operations to be performed on them, and for the different uses to which they are to be applied. This will constitute *Reduction of Fractions.**

REDUCTION OF FRACTIONS.†

Problem I. *To find the greatest common measure of two given numbers.*

RULE. (1.) Divide the greater number by the less. (2.) If there be a remainder, divide the less by it; and thus proceed, always dividing the last divisor by the last remainder, till nothing remains. The divisor which leaves no remainder is the common measure required.

If in the operation any divisor be a prime number, and leave a remainder, it is unnecessary to proceed farther, as there is no common measure greater than unity.

Exam. 1. Required the greatest common measure of 247 and 323.

In this example, the first remainder is 76; and the less number, 247, being divided by this, the remainder is 19. Dividing 76, the last divisor, by this, we find that there is no remainder. Hence, 19 is the common measure required, and would be found by division to be contained 13 times in 247, and 17 times in 323, both without remainder.

```
247)323(1
    247
    ---
     76)247(3
        228
        ---
         19)76(4
            76
            --
            ..
```

* One great class of fractional quantities are called *decimal fractions* from the nature of their denominators. Hence, the doctrine of fractions is generally divided into two branches; one treating of fractions in general, and the other of the peculiar management and properties of decimal fractions. Fractions which are not decimals, are usually called *vulgar*, or *common fractions.*

† Reduction of Fractions consists of two branches: one which treats of the changes that may be made in the forms of abstract fractional quantities without changing their values; and the other which treats of fractions as referred to particular units, such as the denominations of money, weights, and measures. To the former branch belong problems II. IV V. and VI.; and to the other, problem VII. &c.

With respect to the *reason of the rule*, since 19 is a measure of the last dividend, 76, it must also be a measure of the preceding dividend, 247, because $247=3\times76+19$; and 247 is one of the given numbers. Then, since 19 measures 76 and 247, it must measure their sum, 323, which is the other given number: and a similar illustration is applicable in all cases. 19 is also the *greatest* common measure. For suppose, if possible, a greater number, as 20, to measure 247 and 323; then it is evident, it must measure their difference, 76, and also 3 times 76, or 228; and since it is supposed also to measure 247, it must measure 19, the difference of 228 and 247: that is, if 20 measure 247 and 323, it must measure 19, a number less than itself, which is evidently impossible: therefore 20 does not measure both the given numbers; and in the same manner it may be shown, that no other number greater than 19 can measure them; therefore 19 is the *greatest* common measure. The remark which follows the rule would be proved in nearly a similar manner.

This problem and problem III. are given in this place, because, though they respect whole numbers, they are chiefly useful in the second and fourth problems in this article.—It is easy to see, that the greatest common measure of three or more numbers, would be found by applying the preceding rule to two of the numbers; then to the result and another of the numbers, &c.

Exercises. Find the greatest common measures of the following numbers:

1. 285 and 465 .... *Answ.* 15	4. 2145 and 3471 .... *Answ.* 39
2. 532 ..... 1274 .... *Answ.* 14	5. 1879 ..... 2425 .... *Answ.* 1
3. 888 ..... 2775 .... *Answ.* 111	6. 693 ..... 1815 .... *Answ.* 33

A fraction is said to be in its LOWEST TERMS, or in its SIMPLEST FORM, when it is expressed in the least whole numbers possible.

Problem II. *To reduce a fraction to its lowest terms.*

RULE I. (1.) Divide the terms of the given fraction by any number that will measure both: the quotients will be the numerator and denominator of an equivalent fraction in lower terms. (2.) Let this fraction, if possible, be reduced in like manner; and proceed thus, till a fraction is obtained for whose terms no common measure can be found.

The application of this rule will often be facilitated by the following directions and remarks:—1. If the terms of the fraction end in ciphers, cut an equal number from both. 2. If they end each in 5, or one in 5, and the other in a cipher, divide them both by 5; or double them, and reject a cipher from each of the results. 3. If 2 measure the last figure of each term, it will measure the terms themselves. In like manner, if 4 measure the number expressed by the two last figures, or 8 that expressed by the three last. 4 in the one case, and 8 in the other,

will measure both terms. 4. If 3 or 9 measure the sum of the digits of each term, 3 in the one case, and 9 in the other, will measure both terms.—*See page* 36.

Exam. 2. Reduce $\frac{108}{162}$ to its lowest terms.

Here, we divide the given terms by 2; those of the result by 9; and those of that result by 3; and we see that $\frac{108}{162}$ is equal to $\frac{54}{81}$, $\frac{6}{9}$, or $\frac{2}{3}$. The last of these has evidently no common measure greater than unity; it is therefore the expression required, being the simplest form of the given fraction. The same result would have been obtained rather more quickly by dividing by 6 and 9.

$$\begin{array}{r} 9)\quad 3) \\ 2)\frac{108}{162}=\frac{54}{81}=\frac{6}{9}=\frac{2}{3}. \end{array}$$

Exam. 3. Reduce $\frac{21000}{37800}$ to its simplest form.

Here, two ciphers are cut from the end of each term, which is equivalent to the division of each by 100. The quotients are then divided by 6, and the results by 7, and the fraction is reduced to the form $\frac{5}{9}$, which is its simplest form.

$$\begin{array}{r} 7) \\ 6)\frac{210|00}{378|00}=\frac{35}{63}=\frac{5}{9}. \end{array}$$

The reason of this rule is evident from the first of the principles established in page 88.

Exercises. Reduce the following fractions to their simplest forms:

Ex.	*Answ.*	*Ex.*	*Answ.*	*Ex.*	*Answ.*
7. $\frac{63}{105}$	$\frac{3}{5}$	10. $\frac{240}{312}$	$\frac{10}{13}$	13. $\frac{176}{1000}$	$\frac{22}{125}$
8. $\frac{55}{121}$	$\frac{5}{11}$	11. $\frac{126}{162}$	$\frac{7}{9}$	14. $\frac{960000}{1152000}$ ...	$\frac{5}{6}$
9. $\frac{120}{320}$	$\frac{3}{8}$	12. $\frac{57}{63}$	$\frac{19}{21}$	15. $\frac{144000}{1920000}$ ...	$\frac{3}{40}$

This method of reducing fractions to their lowest terms is very convenient and easy in practice, when the terms of the proposed fractions are not very large, or when the divisors are readily discovered. It labours under the disadvantage, however, of not determining, in many cases, whether the fraction is in its lowest terms or not; and besides its being a process which depends on trial, and which therefore is not strictly mathematical, the measures are generally discovered with difficulty, unless they are some of the smaller numbers. Thus, if the fraction $\frac{399}{627}$ were proposed, we should readily discover that it may be reduced, by division by 3, to the form $\frac{133}{209}$, which we would perhaps conceive to be its simplest form, not knowing that it is still farther reducible by 19, and that the simplest form is $\frac{7}{11}$. The following method, though often tedious, is perfect in principle, always reducing the fraction to its simplest form, and not depending on trial.

Rule. II. (1.) Find, by problem I, the greatest common measure of the numerator and denominator. (2.) Divide them both by this, and the quotients will be the numerator and denominator of the required fraction.

It is often of advantage to carry the reduction as far, by the first rule, as can readily be done, and then to apply the second rule to the result.

Exam. 4. Reduce the fraction $\frac{679}{1746}$ to its lowest terms.

In this example, by dividing the denominator by the numerator, the numerator by the remainder, &c. we find the greatest common measure to be 97: and dividing both terms of the given fraction by this, we obtain $\frac{7}{18}$, which is the equivalent fraction in its lowest terms.

Exam. 5. Reduce $\frac{8280}{11385}$ to its lowest terms.

Here, because the terms end in 5 and 0, 5 is a measure, and reduces the fraction to $\frac{1656}{2277}$. This again is reducible by 9, because 9 measures the sum of the digits of each term; and the result is $\frac{184}{253}$, which, by the second rule, is reduced to $\frac{8}{11}$.

Exam. 6. Reduce $\frac{468}{1266}$ to its most simple expression.

We see here immediately, that 6 is a common measure; for 3 measures the sum of the digits of each term, and 2 must be a measure, since the unit figures are even. After division by 6, therefore, the fraction becomes $\frac{78}{211}$. In applying the second rule to this, 55 at the third division is to be divided by 23, which is a prime number, and is not a measure of 55. $\frac{78}{211}$, therefore, we conclude to be the simplest form of the fraction; and thus we are saved the trouble of three or four divisions.

It may be proper for the pupil to recollect, that

At the conclusion of every operation, if there be a fraction, it should be reduced to its simplest form.

Reduce the following fractions to their lowest terms;

Ex.	*Answ.*	*Ex.*	*Answ.*	*Ex.*	*Answ.*
16. $\frac{272}{425}$	$\frac{16}{25}$	19. $\frac{6465}{7335}$	$\frac{431}{489}$	22. $\frac{10395}{16819}$	$\frac{945}{1529}$
17. $\frac{594}{2142}$	$\frac{33}{119}$	20. $\frac{1236}{1632}$	$\frac{103}{138}$	23. $\frac{5415}{30105}$	$\frac{361}{2007}$
18. $\frac{5170}{8754}$	$\frac{235}{397}$	21. $\frac{285714}{999999}$	$\frac{2}{7}$	24. $\frac{15863}{21489}$	$\frac{547}{741}$

Problem III. *To find the least common multiple of two or more given numbers.*

Rule I. (1.) Arrange the given numbers in succession, and find by inspection a number which will measure as many of them as possible. (2.) By this number divide all the given numbers which it measures, and write the quotients and the undivided numbers in a new line. (3.) Proceed, if possible, in the same manner with the numbers in this line; and thus continue the process till no number greater than a unit will measure any two or more of the numbers last found. (4.) Then multiply all the numbers in the last line, and all the divisors employed in the operation continually together, and the result will be the common multiple required.

The work is often much shortened by rejecting, in any

line, any number that is contained without remainder in any other number in the same line.

If no two of the given numbers have any common measure greater than unity, their continual product will be the least common multiple.

RULE II. (1.) Find by problem I, the greatest common measure of two of the given numbers. (2.) By this, divide one of those two numbers, and multiply the quotient by the other. (3.) Perform a similar operation on the product and another of the given numbers. (4.) Continue the process thus with each successive quotient, till all the numbers have been used, and the final result will be the least common multiple required.

Exam. 7. Required the least common multiple of 24, 10, 9, 32, 6, 45, and 25.

By Rule I.

2)	24	10	(9)	32	(6)	45	25
3)	12	(5)		16		45	25
5)	(4)			16		15	25
				16		3	5

$2 \times 3 \times 5 \times 16 \times 3 \times 5 = 7200$, *Ans.*

Here, the given numbers being placed in the same line, as we see by inspection, that 9 is contained exactly in 45, and 6 in 24, these are neglected. Then using 2 as a divisor, we obtain the quotients 12, 5, and 16, and we place in the line with them the undivided numbers, 45 and 25: and since the quotient 5 is contained exactly in either of these, it is rejected. We then divide by 3, and obtain the quotients 4 and 15, which with the undivided numbers, 16 and 25, are placed in a new line. 4 is then rejected as it is a measure of 16: and dividing by 5, we place in the next line the quotients 3 and 5, and the undivided number 16. Then, as no two of these have any common measure greater than unity, the three divisors and the three numbers in the last line are multiplied continually together, and the product, 7200, is the common multiple required.

By Rule II. Rejecting 9 and 6 as before, we divide 24 by 2, the greatest common measure of it and 10, and we multiply the quotient 12 by 10: the product, 120, is the least common multiple of 24 and 10. Then, the greatest common measure of 120 and 32 being 8, we have $32 \div 8 = 4$, and $120 \times 4 = 480$, the least common multiple of 24, 10, and 32. In the next place, the greatest common measure of 480 and 45 is 15; and, since $45 \div 15 = 3$, we have $480 \times 3 = 1440$, the least common multiple of 24, 10, 32, and 45. Lastly, since the greatest common measure of 1440 and 25 is 5, and since $25 \div 5 = 5$, we have $1440 \times 5 = 7200$, the least common multiple of 24, 10, 32, 45, and 25, and consequently of 9 and 6, of which 45 and 24 are multiples.

With respect to *the reason of these rules*, it is difficult to give a strict, and, at the same time, an easily comprehended proof; and

for most learners the following illustration may perhaps be preferable:

In the operation by the first rule, $2 \times 12 \times 5 \times 16 \times 45 \times 25$, the product of the first divisor and the numbers in the second line, is evidently a multiple of each of the given numbers, 24 being contained in it $5 \times 16 \times 45 \times 25$ times; 10, the second number, $12 \times 16 \times 45 \times 25$ times, &c. Again, $2 \times 3 \times 4 \times 16 \times 15 \times 25$, the product of the two first divisors and the numbers in the third line, is also a multiple of each of the given numbers, 24 being contained in it $16 \times 15 \times 25$ times; 10, the second number, $3 \times 4 \times 16 \times 3 \times 25$ times, (since $2 \times 15 = 30$, and $30 = 10 \times 3$;) 32, the fourth number, $3 \times 4 \times 15 \times 25$ times, &c.: and the illustration may be extended in a similar manner to the rest of the operation. That the given number 6 may be rejected in the operation, will readily appear from considering, that 6 is contained exactly 4 times in 24, and will therefore be contained without remainder in any multiple of 24, and 4 times as often as 24. In like manner it would appear, that 9 may be rejected, as 45 is a multiple of it. That 5 and 4 may be rejected in the succeeding lines will be evident from this, that they would disappear were the lines that contain them divided respectively by 5 and 4.

The proof of the second rule depends on the principle, that *if the product of any two numbers be divided by any factor which is common to both, the quotient will be a common multiple of the two numbers.* Thus, if 48, the product of 6 and 8, be divided by 2, a factor of both, the quotient 24, will be a multiple of both, since it may be regarded either as 8 multiplied by the quotient of 6 by the factor 2, or as 6 multiplied by the quotient of 8 by the same factor. Now this being so, it is obvious that the greater the common measure is, the less will be the multiple; and consequently, the greatest common measure will produce the least common multiple. When the common multiple of the two first numbers is found, it is evident, that any number which is a common multiple of it and of the third number, will be a multiple of the first, second, and third numbers: and thus the reason of every part of the second rule is manifest.

It may be remarked with respect to the practical application of these rules, that the second always gives the *least* common multiple. The multiple found by the first is not always the *least* possible; but it will be such, if care be always taken to use such a divisor as will measure more of the given numbers, than any other divisor would. This rule therefore, being very easy in practice, is generally preferred to the other. It may be farther remarked, that by practice the pupil will gradually become able to discover, in many instances, the common multiples by inspection, especially when the given numbers are not large.

Exercises. Required the least common multiples of the following numbers:

Exercises.	*Answ.*	*Exercises.*	*Answ.*
25. 6 10 15 18	90	28. 63 12 84 7	252
26 7 11 13 3	3003	29. 54 81 63 14	1134
27. 8 12 20 24 25	600	30. 2 3 4 5 6 7 8 9 ...	2520

Problem IV. *Any number of fractions having different denominators being given; to find equivalent fractions having a common denominator.*

Rule I. (1.) Find, by problem III. a common multiple of all the denominators: this will be the common denominator. (2.) Then, divide the common multiple by the first of the given denominators, and multiply the quotient by the first of the given numerators: the product will be the first of the required numerators. The other numerators are found in a similar manner.

Rule II. (1.) Multiply each numerator by all the denominators, except its own, and the product will be the numerator of the equivalent fraction. (2.) Multiply all the denominators continually together for the common denominator.

Exam. 7. Reduce $\frac{5}{8}$, $\frac{7}{12}$, $\frac{11}{18}$, and $\frac{9}{20}$, to fractions having a common denominator.

By Rule I. Here, by problem III. the least common multiple is found to be 360; and the rest of the work will stand as in the margin. The correctness of the results would be proved by reducing them to their simplest forms, as the given fractions would thus be obtained. Thus, $\frac{225}{360}$ would be reduced to $\frac{5}{8}$; $\frac{210}{360}$ to $\frac{7}{12}$, &c.

8)360	12)360	18)360	20)360
45	30	20	18
5	7	11	9
225	210	220	162

$\frac{225}{360}$ $\frac{210}{360}$ $\frac{220}{360}$ $\frac{162}{360}$, *Answ.*

By Rule II. $5 \times 12 \times 18 \times 20 = 21600$, the first numerator; $7 \times 8 \times 18 \times 20 = 20160$, the second numerator; $11 \times 8 \times 12 \times 20 = 21120$, the third numerator; $9 \times 8 \times 12 \times 18 = 15552$, the fourth numerator; and $8 \times 12 \times 18 \times 20 = 34560$, the common denominator. Hence, the equivalent fractions found by this rule, are $\frac{21600}{34560}$, $\frac{20160}{34560}$, $\frac{21120}{34560}$, and $\frac{15552}{34560}$, which being reduced to their lowest terms, would in like manner, be shown to be equivalent to the given fractions.

In this example the results found by the second rule are expressed in numbers inconveniently large; and the same is very generally the case. Hence, the other rule should always be preferred, except when the

given denominators are *prime to each other*; that is, when they have no common measure greater than a unit.

With respect *to the reason of these rules*, it is evident, that the operation by the second is nothing else than the multiplication of both the terms of each fraction by all the denominators, except its own; and in this way the rule might have been expressed. Thus, the first of the given fractions is $\frac{5}{8}$, which by the operation is transformed into $\frac{5\times12\times18\times20}{8\times12\times18\times20}$, or $\frac{21600}{34560}$; and this by the first of the properties proved in page 88, is equal to the proposed fraction $\frac{5}{8}$.

In the first rule, the division of the common multiple by the given denominator is evidently nothing else than the finding of the number by which if both the numerator and the denominator of the given fraction be multiplied, the denominator of the result will be the same as the common multiple; and, by the principle before referred to, the result will be equal to the given fraction. Thus, in the preceding example, when 360 is divided by 8, the quotient is 45; by which if both terms of the fraction $\frac{5}{8}$ be multiplied, there results $\frac{225}{360}$, for the equivalent fraction required.

Exercises. Reduce the following sets of fractions to other fractions having common denominators:

	Exercises.	*Answers.*
31.	$\frac{5}{12}, \frac{9}{16}, \frac{11}{24}, \frac{17}{33}, \frac{3}{8}$	$\frac{220}{528}, \frac{297}{528}, \frac{242}{528}, \frac{272}{528}, \frac{198}{528}$
32.	$\frac{11}{27}, \frac{19}{24}, \frac{5}{6}, \frac{7}{15}, \frac{3}{5}, \frac{1}{9}$	$\frac{440}{1080}, \frac{855}{1080}, \frac{900}{1080}, \frac{504}{1080}, \frac{648}{1080}, \frac{120}{1080}$
33.	$\frac{17}{30}, \frac{19}{60}, \frac{11}{60}, \frac{43}{75}, \frac{4}{9}, \frac{5}{36}$	$\frac{765}{3600}, \frac{1140}{3600}, \frac{660}{3600}, \frac{2544}{3600}, \frac{1600}{3600}, \frac{500}{3600}$
34.	$\frac{11}{35}, \frac{19}{45}, \frac{25}{42}, \frac{1}{100}, \frac{79}{81}$	$\frac{17820}{58700}, \frac{23940}{58700}, \frac{38750}{58700}, \frac{567}{58700}, \frac{55300}{58700}$
35.	$\frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \frac{1}{11}$	$\frac{693}{3465}, \frac{495}{3465}, \frac{385}{3465}, \frac{315}{3465}$
36.	$\frac{11}{12}, \frac{17}{18}, \frac{29}{30}, \frac{47}{48}, \frac{7}{16}$	$\frac{660}{720}, \frac{680}{720}, \frac{696}{720}, \frac{705}{720}, \frac{315}{720}$
37.	$\frac{17}{100}, \frac{17}{1000}, \frac{17}{10000}$	$\frac{1700}{10000}, \frac{170}{10000}, \frac{17}{10000}$
38.	$\frac{41}{60}, \frac{13}{90}, \frac{13}{25}, \frac{1}{105}, \frac{4}{9}$	$\frac{4305}{6300}, \frac{910}{6300}, \frac{3276}{6300}, \frac{60}{6300}, \frac{2800}{6300}$
39.	$\frac{4}{7}, \frac{1}{10}, \frac{13}{15}, \frac{5}{16}, \frac{9}{28}, \frac{2}{35}$	$\frac{960}{1680}, \frac{168}{1680}, \frac{1456}{1680}, \frac{525}{1680}, \frac{540}{1680}, \frac{96}{1680}$
40.	$\frac{1}{3}, \frac{2}{9}, \frac{4}{27}, \frac{8}{81}, \frac{16}{243}, \frac{32}{729}$	$\frac{243}{729}, \frac{162}{729}, \frac{108}{729}, \frac{72}{729}, \frac{48}{729}, \frac{32}{729}$

Problem V. *To reduce an improper fraction to a whole or mixed number.*

Rule. Divide the numerator by the denominator; the quotient will be the whole number required: and if there be any remainder, write it over the given denominator for the fractional part of the required result.

The reason of this rule and of that for the next problem, is evident from the nature of fractions.

Exam. 8. Reduce $\frac{56}{7}$ and $\frac{200}{9}$ to whole or mixed numbers.

$$\begin{array}{r} 56 \\ \hline 7)56 \\ \hline 8 \end{array} \quad \text{Answ.} \qquad \begin{array}{r} 200 \\ \hline 9)200 \\ \hline 22\frac{2}{9} \end{array} \quad \text{Answ.}$$

Exercises. Reduce the following fractions to whole or mixed numbers:

Ex.	*Answ.*	*Ex.*	*Answ.*	*Ex.*	*Answ*
41. $\frac{18}{18}$	1	43. $\frac{2588}{375}$	$20\frac{12}{125}$	45. $\frac{600}{14}$	$42\frac{6}{14}$
42. $\frac{6437}{296}$	$21\frac{171}{296}$	44. $\frac{10000}{111}$	$90\frac{10}{111}$	46. $\frac{720}{24}$	30

Problem VI. *To reduce a mixed number to an improper fraction.*

RULE. Multiply the whole number by the denominator of the fractional part, and to the product add the numerator; the sum will be the required numerator, below which write the given denominator.

A whole number may be expressed in a fractional form by writing a unit below it; or by multiplying it by any whole number, and writing that number below the product as denominator. Thus, $9=\frac{9}{1}=\frac{36}{4}$, &c.

Exam. 9. Reduce $5\frac{11}{12}$ to an improper fraction.

$$\begin{array}{l} 5\frac{11}{12} \\ \underline{12} \\ \frac{71}{12}, \textit{ Answ.} \end{array}$$

Exercises. Reduce the following numbers to improper fractions:

Ex.	*Answ.*	*Ex.*	*Answ.*	*Ex.*	*Answ.*
47. $15\frac{1}{4}$	$\frac{61}{4}$	49. $46\frac{5}{8}$	$\frac{373}{8}$	51. $1\frac{999}{1000}$	$\frac{1999}{1000}$
48. $3\frac{17}{55}$	$\frac{182}{55}$	50. $21\frac{117}{583}$	$\frac{12360}{583}$	52. $19\frac{6}{7}$	$\frac{138}{7}$

Problem VII. *To express any given quantity as a fraction of another given quantity of the same kind, considered as an integer.*

RULE. Make the integer the denominator of the required fraction, and the other given quantity its numerator, both being reduced to the same denomination, if they be not of the same already.

Exam. 10. Reduce 13/9 to the fraction of £1.

In 13/9 there are 165 pence, and in £1, 240 pence; the fraction therefore is £$\frac{165}{240}$, which, by reduction to its lowest terms, becomes £$\frac{11}{16}$. The reason of this is evident, since 1 penny is $\frac{1}{240}$ of a pound, and in 13/9 there are 165 pence, or $\frac{165}{240}$ of a pound.

Exam. 11. Express a pound troy weight as a fraction of a pound avoirdupois.

A pound avoirdupois weight contains 7000 grains, and a pound troy weight 5760 grains.—(*See page* 39.) Hence, the fraction is $\frac{5760}{7000}$, or in its lowest terms, $\frac{144}{175}$.

This rule enables us to find the ratio of two quantities. Thus, the ratio of 13/9 to £1 is that of $\frac{11}{16}$ to 1, or 11 to 16; and a pound avoirdupois is to a pound troy, as 1 to $\frac{144}{175}$, or as 175 to 144.—(*See page* 89, *No.* 4.)

Exercises.			*Answers.*
53. Reduce 12/6 to the fraction of £1			$\frac{2}{3}$
54. £32 10		£100	$\frac{13}{40}$
55. 2/8½		£1 2 9	$\frac{5}{42}$
56. 1 qr. 20 lbs.		1 cwt. long weight	$\frac{5}{14}$
57. 2 hours		23 h. 56 m. 4 s.	$\frac{1800}{21541}$
58. 2/1		6/8	$\frac{5}{16}$

59. The height of Ben Nevis, the highest mountain in Britain, is 4350 feet, and that of mount Ararat 9500 feet: express the former as a fraction of the latter in its simplest form. *Answ.* $\frac{87}{190}$.

60. Express 96 pages as a fraction of a book containing 432 pages. *Answ.* $\frac{2}{9}$.

61. If one farm contain 37 a. 2 r. 14 p. and another 168 a. 0 r. 28 p. what is the simplest form of the fraction expressing their comparative magnitudes? *Answ.* $\frac{97}{437}$.

Problem VIII. *To find the value of a fraction in the denominations contained in the integer.*

RULE. Consider the numerator as expressing the integer taken as often as there are units in the numerator, and divide it by the denominator.

This problem is, in strictness, a particular case of the next. On account of its frequent use, however, it is perhaps better in a separate form.

Exam. 12. Required the value of £$\frac{4}{7}$.

Here the integer is £1; and the numerator, being regarded as 4 times that integer, or £4, is divided by the denominator: the quotient 11/5$\frac{1}{7}$ is the value of the fraction. The reason of the rule is manifest from the nature of fractions.

	£	s.	d.
7)	4	0	0
	£0	11	5$\frac{1}{7}$

Ex. 13. Required the value of $\frac{3}{8}$ of £5 13 9.

Here the integer is £5 13 9; then, the numerator expressing three times this quantity, we multiply by 3, and divide by the denominator 8; the quotient, £2 2 7$\frac{7}{8}$, is the required value.

	£	s.	d.
	5	13	9
			3
8)	17	1	3
	£2	2	7$\frac{7}{8}$

Exam. 14. The height of M'Gillicuddy's Reeks, the highest mountain in Ireland, is 3610 feet: if a person have ascended through $\frac{3}{8}$ of this space, to what height has he ascended?

Here, the height of the mountain is the integer; and the numerator expressing three times this quantity, we multiply by 3 and divide by 8, and we find for the answer 1353$\frac{3}{4}$ feet.

3610 feet.
3
8)10830
1353$\frac{3}{4}$ feet.

Exercises.	*Answers.*	*Exercises.*	*Answers.*
62. £$\frac{5}{8}$	$=11/1\frac{1}{8}$	66. $\frac{6}{13}$ foot.....	$=5\frac{7}{13}$ inches.
63. £$\frac{3}{16}$.....	$=3/9$	67. $\frac{5}{24}$ Irish mile.	$=1$ f. 26 p. 4 y. 2 f.
64. £$\frac{341}{1000}$.....	$=6/9\frac{21}{25}$	68. $\frac{3}{8}$ of £1 2 9..	$=13/7\frac{4}{5}$
65. $\frac{17}{20}$ shilling.	$=10\frac{1}{5}$d.	69. $\frac{6}{11}$ of £148 5.	$=$£80 17 $3\frac{3}{11}$

70. $\frac{5}{8}$ of 175 tons.....=145 tons, 16 cwt. 2 qrs. $18\frac{2}{3}$ lbs.

71. The earth revolves once on its axis in 23 hours, 56 minutes, 4 seconds; in what time does it perform $\frac{4}{5}$ of a rotation? *Answ.* 10 hours, 38 minutes, $15\frac{1}{5}$ seconds.

72. By the articles of the Union of Great Britain and Ireland, which took place in 1801, during the first twenty years Britain was to contribute $\frac{15}{17}$, and Ireland $\frac{2}{17}$, of the amount of the public expenditure. How much did each country contribute in making a million sterling? *Answ.* £882,352 18 $9\frac{15}{17}$, and £117,647 1 $2\frac{2}{17}$.

Problem IX. *To reduce a given fraction to another of a lower denomination.*

RULE. As in common Reduction, reduce the given numerator, considered as an integer, to the denomination to which the fraction is to be reduced, and write below it the given denominator.

Exam. 15. Reduce £$\frac{3}{1600}$ to the fraction of a penny.

Here, by reducing £3 to pence, and writing 1600 below it, we have for the required fraction $\frac{720}{1600}$d. which by reduction to its lowest terms, becomes $\frac{9}{20}$d. The reason of this is manifest, since by the nature of fractions, the given fraction expresses a sixteen-hundredth part of £3, and it must therefore express a sixteen-hundredth part of the pence in £3.

Exercises.	*Answers.*
73. Reduce £$\frac{1}{35}$ to the fraction of a shilling	$\frac{4}{7}$ shil.
74. ..... $\frac{3}{20}$ cwt. to the denomination of pounds	$16\frac{4}{5}$ lbs.
75. What part of a second is $\frac{1}{1000000}$ day?	$\frac{54}{625}$.

The converse of this problem, or the reduction of a fraction to a higher denomination, is seldom of use. It is performed by operating on the *denominator* in the way mentioned in the rule. Thus, if it be required to reduce $\frac{9}{20}$d. to the fraction of a pound, let the denominator be multiplied by 20 and 12, and the numerator be retained: the result, £$\frac{9}{4800}$, or, in its simplest form, £$\frac{3}{1600}$, is the result required.*

* The method of reducing compound to simple fractions will be given in Multiplication of Fractions, and that of reducing complex fractions to simple ones, in Division of Fractions. These are the natural and proper positions of these problems; but should any teacher conceive their introduction necessary here, with a view to prepare fractional quantities for Addition, Subtraction, &c. it will be easy for the learner to turn over for them to Multiplication and Division.

ADDITION OF FRACTIONS.

RULE. (1.) Reduce the given fractions to others, having a common denominator, if they be not such already. (2.) When they are in this state, add all the numerators together, and below their sum write the common denominator: the result will be the sum of the fractions; which, if it be an improper fraction, must be reduced to a whole or mixed number. (3.) If some of the given quantities be mixed numbers, the fractions are to be added by the preceding part of the rule; and the whole numbers, with any integral part that may be obtained by the adding of the fractions, are to be added by Simple Addition.

Exam. 1. Add together $\frac{14}{15}$, $\frac{11}{15}$, and $\frac{8}{15}$.

Here, the sum is $\frac{33}{15}$, which becomes, by reduction to a mixed number, $2\frac{3}{15}$, or, by the reduction of the fractional part to its lowest terms, $2\frac{1}{5}$, the sum required. The reason of the process is so obvious as scarcely to require illustration. The first of the given fractions expresses fourteen-fifteenths of the integer; the next, eleven of these parts: consequently, the two taken together must express twenty-five of these parts. In like manner the addition of any others may be illustrated.

$$\left.\begin{array}{r} 14 \\ 11 \\ 8 \end{array}\right\} 15$$
$$\begin{array}{r} \hline 33 \\ \hline \end{array}$$
$$15)33(2\tfrac{3}{15}, \text{ or } 2\tfrac{1}{5},$$
Answ.

Exam. 2. Add together $3\frac{31}{40}$, $\frac{29}{36}$, and $6\frac{23}{25}$.

In this exercise, by reducing the given fractions to equivalent ones having a common denominator, we find the first to be $\frac{1395}{1800}$, the second $\frac{1450}{1800}$, &c. Then, by adding the numerators, we find the sum of the fractions to be $\frac{4501}{1800}$, or $2\frac{901}{1800}$, by reduction to a mixed number.

$$\left.\begin{array}{rl} 3\frac{31}{40} \ldots\ldots & 1395 \\ \frac{29}{36} \ldots\ldots & 1450 \\ 6\frac{23}{25} \ldots\ldots & 1656 \end{array}\right\} 1800$$
$$\begin{array}{rl} \hline 11\frac{901}{1800} & 4501 \\ \textit{Answ.} & \hline \end{array}$$
$$1800)4501(2\tfrac{901}{1800}$$
$$\begin{array}{r} \hline 901 \end{array}$$

We then write the fractional part beneath the given fractions; and carrying 2 to the whole numbers, we find the required sum to be $11\frac{901}{1800}$.

	Exercises.	*Answers.*
1.	$\frac{8}{15}+\frac{9}{20}$	$=\frac{59}{60}$
2.	$\frac{7}{8}+\frac{1}{12}+\frac{17}{18}+\frac{23}{24}+\frac{20}{27}$	$=3\frac{65}{108}$
3.	$\frac{7}{8}+\frac{7}{12}+\frac{13}{16}+\frac{11}{18}+\frac{19}{24}$	$=3\frac{97}{144}$
4.	$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}$	$=\frac{127}{128}$
5.	$\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+\frac{31}{32}+\frac{63}{64}+\frac{127}{128}$	$=6\frac{1}{128}$

	Exercises.	*Answers.*
6.	$\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$	$3\frac{127}{140}$
7.	$90\frac{3}{4}+67\frac{2}{3}+4\frac{17}{20}+15\frac{5}{12}+34\frac{14}{67}$	$212\frac{1469}{1740}$
8.	$5\frac{9}{10}+73\frac{3}{8}+37\frac{1}{10}+\frac{19}{70}+1\frac{31}{48}+2\frac{61}{90}$	$120\frac{4889}{5040}$
9.	$\frac{3}{8}+3\frac{14}{15}+10\frac{2}{5}+\frac{9}{22}$	$15\frac{31}{264}$
10.	$7\frac{29}{36}+33\frac{5}{12}+\frac{17}{21}+4\frac{7}{16}+10\frac{22}{27}+\frac{1}{63}$	$57\frac{907}{3024}$
11.	$3\frac{5}{8}+12\frac{1}{6}+13\frac{83}{84}$	$29\frac{131}{168}$
12.	$2\frac{3}{5}+4\frac{7}{8}+5\frac{3}{10}$	$12\frac{31}{40}$
13.	$\frac{1}{3}+\frac{4}{7}+\frac{21}{32}$	$1\frac{377}{672}$
14.	$\frac{5}{8}+\frac{5}{14}+\frac{1}{6}$	$1\frac{25}{168}$
15.	$\frac{2}{5}+\frac{23}{36}+\frac{3}{5}$	$1\frac{23}{36}$
16.	$\frac{1}{4}+\frac{65}{189}$	$\frac{449}{756}$
17.	$35\frac{9}{10}+21\frac{11}{19}+44\frac{3}{5}+8\frac{99}{101}$	$111\frac{227}{3838}$

SUBTRACTION OF FRACTIONS.

RULE. (1.) Set the less quantity below the greater, and prepare them both as in Addition of Fractions. (2.) Then, if possible, subtract the lower numerator from the upper; below the remainder write the common denominator, and if there be whole numbers, find their difference as in Simple Subtraction. (3.) But if the lower numerator exceed the upper, subtract it from the common denominator; to the remainder add the upper numerator; write the common denominator beneath the sum; and carry one to the whole number in the lower line.

The reason of this rule is evident from the explanations already given of Simple Subtraction, and of Addition of Fractions.

Exam. 1.	*Exam.* 2.		*Exam.* 3.	
From $87\frac{8}{11}$	$73\frac{5}{6}$ 25	30	$10\frac{2}{25}$ 14	175
take $42\frac{5}{11}$	$17\frac{4}{15}$ 8		$3\frac{22}{35}$ 110	
Rem. $45\frac{8}{11}$	$56\frac{17}{30}$ 17		$6\frac{79}{175}$ 79	

	Exercises.	*Answ.*		*Excrcises.*	*Answ.*
1.	$\frac{37}{42}-\frac{13}{126}$	$\frac{7}{9}$	7.	$\frac{16}{21}-\frac{8}{15}$	$\frac{8}{35}$
2.	$3\frac{5}{14}-1\frac{13}{21}$	$1\frac{31}{42}$	8.	$\frac{9}{55}-\frac{1}{15}$	$\frac{16}{165}$
3.	$12\frac{3}{4}-10\frac{13}{16}$	$1\frac{15}{16}$	9.	$\frac{19}{96}\backsim\frac{43}{216}$*	$\frac{1}{864}$
4.	$\frac{1}{7}-\frac{1}{11}$	$\frac{4}{77}$	10.	$\frac{11}{75}\backsim\frac{7}{45}$	$\frac{2}{225}$
5.	$12\frac{23}{28}-9\frac{27}{35}$	$3\frac{1}{20}$	11.	$5\frac{23}{32}-2\frac{2}{7}$	$3\frac{97}{224}$
6.	$4\frac{1}{24}-3\frac{1}{16}$	$\frac{47}{48}$	12.	$11\frac{2}{3}+8\frac{7}{9}-9\frac{19}{22}$	$10\frac{115}{198}$

* This sign, $\backsim$, is used to denote the difference of two quantities, without indicating which of them is the greater. In the exercises in which it occurs, therefore, the fractions are to be reduced to others having a common denominator, and the less taken from the greater.

MULTIPLICATION OF FRACTIONS.

RULE: (1.) If any of the quantities be whole or mixed numbers, reduce them to improper fractions. (2.) Find the product of the numerators for the numerator of the required result, and the product of the denominators for its denominator.

As a contraction, if a numerator and any denominator be the same, reject them; or divide a numerator and any denominator by any number which will measure them, and instead of them employ the results.

Compound fractions are reduced to simple ones in the same manner.

Exam. 1. Multiply $\frac{7}{8}$ by $\frac{12}{19}$. $\frac{7}{8}\times\frac{12}{19}=\frac{84}{152}=\frac{21}{38}$, *Answ.*

Here, by taking the products of the numerators, and of the denominators, we find for the required result $\frac{84}{152}$, or, in its simplest form, $\frac{21}{38}$.

Exam. 2. Multiply $\frac{4}{7}$ by $\frac{7}{11}$. $\frac{4}{7}\times\frac{7}{11}=\frac{28}{77}=\frac{4}{11}$.

In this example, as one of the numerators is 7, and also one of the denominators the same, by neglecting this numerator and denominator, we have $\frac{4}{11}$ for the required result, the same as before.

$$\frac{4}{(7)}\times\frac{(7)}{11}=\frac{4}{11}$$

The *reason of this rule* will appear evident from the principles established in page 88. Thus, since $\frac{12}{19}$ is the same as one-nineteenth of 12, it is evident, that the product of $\frac{7}{8}$ by $\frac{12}{19}$ is $\frac{1}{19}$ of its product by 12. Hence it appears, that to find the required product, we are to multiply $\frac{7}{8}$ by 12 and to divide the result by 19; and by the principles referred to, this will be effected by multiplying the numerator by 12, and the denominator by 19: that is, the numerator of the final result is the product of the given numerators, and its denominator the product of the given denominators. The contraction evidently produces the same result that would be obtained if the terms of the product found by the rule, were divided by the number rejected, or by the common measure employed in the contraction.

These principles will assist in removing the difficulty which learners often feel in accounting for the product of a quantity by a fraction being less than the quantity itself. From what we have seen above, it appears, that in every case, in what is called Multiplication of Fractions, there is not only a *multiplication* but also a *division*. Hence, in the first example, since we multiply $\frac{7}{8}$ by 12, and divide the product by 19, the result must obviously be less than the multiplicand $\frac{7}{8}$. From this also it is evident, that when one factor is less than a unit, the product is less than the other; and consequently, when both factors are less than units, the product is less than either.

Exam. 3. Multiply $17\frac{5}{6}$ by $12\frac{3}{4}$.

These quantities, by reduction to improper fractions, become $\frac{107}{6}$ and $\frac{51}{4}$. Then, $\frac{107}{6} \times \frac{51}{4} = \frac{5457}{24} = \frac{1819}{8}$, by reduction to its lowest terms; and this being reduced to a mixed number, becomes $227\frac{3}{8}$, the product required.

The product may also be found, as in the margin, without reducing the factors to improper fractions. In this mode, 12 times $\frac{5}{6} = \frac{60}{6}$, or 10, which is carried to the product of 17 and 12. We have then to find the products of 17 and $\frac{3}{4}$, and of $\frac{5}{6}$ and $\frac{3}{4}$. The former is $12\frac{3}{4}$, and the latter $\frac{5}{8}$. The sum of all the three partial products is $227\frac{3}{8}$, the same as the answer found already.*

$$\begin{array}{lll} 17\frac{5}{6} & & \\ 12\frac{3}{4} & & \\ \hline 214 & & \\ 12\frac{3}{4} & \ldots\ldots\ldots 6 & \left.\right\} 8 \\ \frac{5}{8} & \ldots\ldots\ldots 5 & \\ \hline 227\frac{3}{8} & \frac{11}{8}, \text{ or } 1\frac{3}{8}. & \end{array}$$

Exam. 4. Reduce the compound fraction $\frac{3}{4}$ of $\frac{8}{9}$ of $3\frac{1}{3}$ to a simple fraction.

Here the mixed number $3\frac{1}{3}$ is reduced to $\frac{10}{3}$. Then, by taking the product of the numerators, 3, 8, and 10; and also that of the denominators 4, 9, and 3, we obtain $\frac{240}{108}$, or in its lowest terms $\frac{20}{9}$, the simple fraction required, which might be reduced to the mixed number $2\frac{2}{9}$. In this example, the work might be contracted by rejecting the numerator and the denominator 3, and by dividing the denominator 4 and the numerator 8 by 4, and the same result would be obtained.

Exercises.	*Answ.*	*Exercises.*	*Answ.*
1. $\frac{5}{12} \times \frac{13}{17}$	$\frac{65}{204}$	13. $13\frac{9}{10} \times 1\frac{11}{14}$	$24\frac{23}{28}$
2. $\frac{12}{13} \times \frac{7}{8}$	$\frac{21}{26}$	14. $2\frac{9}{13} \times \frac{8}{9}$	$2\frac{46}{117}$
3. $\frac{11}{15} \times \frac{29}{33}$	$\frac{29}{45}$	15. $\frac{5}{344} \times 10\frac{1}{299}$	$\frac{149855}{102856}$
4. $\frac{9}{14} \times \frac{3}{17}$	$\frac{27}{238}$	16. $\frac{25}{29} \times \frac{25}{29}$	$\frac{625}{841}$
5. $\frac{36}{55} \times \frac{23}{28}$	$\frac{72}{115}$	17. $12\frac{7}{27} \times 12\frac{7}{27}$	$150\frac{211}{729}$
6. $\frac{23}{27} \times \frac{45}{82}$	$\frac{115}{246}$	18. $\frac{4}{5}$ of $\frac{8}{9}$	$\frac{32}{45}$
7. $\frac{15}{79} \times \frac{19}{20}$	$\frac{57}{316}$	19. $\frac{5}{12}$ of $\frac{9}{20}$	$\frac{3}{16}$
8. $19\frac{7}{19} \times 1\frac{5}{77}$	$20\frac{916}{1463}$	20. $\frac{5}{7}$ of $9\frac{2}{3}$	$6\frac{19}{21}$
9. $34\frac{25}{36} \times 4\frac{36}{49}$	$164\frac{118}{441}$	21. $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$	$\frac{3}{7}$
10. $21\frac{5}{6} \times 21\frac{5}{7} \times 21\frac{5}{8}$	$10252\frac{13}{42}$	22. $\frac{7}{8}$ of $\frac{5}{6}$ of 7	$5\frac{5}{48}$
11. $\frac{99}{100} \times \frac{47}{100}$	$\frac{4653}{10000}$	23. $\frac{5}{9} \times \frac{7}{10} + \frac{3}{5} \times \frac{7}{12}$	$\frac{28}{45}$
12. $\frac{3}{17} \times 1\frac{9}{11} \times \frac{4}{9}$	$\frac{80}{561}$	24. $\frac{7}{11} \times \frac{28}{33} \times \frac{46}{49}$	$\frac{4}{7}$

DIVISION OF FRACTIONS.

Rule (1.) Reduce mixed or whole numbers to improper fractions, and compound fractions to simple ones, if any

* When the pupil has learned the method of aliquot parts, he will see, that it may often be employed with ease and advantage in the multiplication of fractional quantities.

such be given. (2.) Then invert the divisor,* or rather conceive it to be inverted, and proceed as in Multiplication.

A complex fraction is reduced to a simple one by dividing the numerator by the denominator, according to the preceding rule.

Exam. 1. Divide $\frac{5}{12}$ by $\frac{7}{12}$. $\frac{5}{12} \div \frac{7}{12} = \frac{60}{336} = \frac{10}{21}$, *Answ.*

Here, the divisor being conceived to be inverted, and the fractions being multiplied, we have for the answer $\frac{5}{12} \times \frac{12}{7} = \frac{60}{336} = \frac{10}{21}$.

The *reason of this operation* will be understood from considering, that $\frac{7}{12}$ is only one-twelfth of 7; and consequently, the quotient obtained by dividing by $\frac{7}{12}$ must be twelve times the quotient obtained by dividing by 7. We must therefore divide by 7, and multiply the result by 12, which is the same that is done by the inversion of the divisor.

From this it appears, that in Division of Fractions, there is in reality, both a division and a multiplication. It will also readily appear, that if the divisor be a proper fraction, the quotient will be greater than the dividend.

Exam. 2. Divide $5\frac{4}{15}$ by $2\frac{1}{8}$.

Here, $5\frac{4}{15} = \frac{79}{15}$, and $2\frac{1}{8} = \frac{17}{8}$: we have therefore for the required result, $\frac{79}{15} \times \frac{8}{17} = \frac{79}{8} \times \frac{8}{17} = \frac{632}{255} = 2\frac{82}{255}$.

By the application of the principles established in page 88, a fractional number may be more easily divided by a whole number, than it is by the general rule. Thus,

Exam. 3. Divide $75\frac{7}{10}$ by 9.

Here, 9 is contained 8 times in 75, with the remainder 3. Then $3\frac{7}{10}$, by reduction to an improper fraction, becomes $\frac{37}{10}$, which by division by 9, becomes $\frac{37}{90}$, the remaining part of the quotient.†

$$9)75\tfrac{7}{10}$$
$$8\tfrac{37}{90}$$

Exam. 4. Required the value of the complex fraction $\frac{5\frac{1}{8}}{6\frac{2}{3}}$.

Here, $5\frac{1}{8} = \frac{41}{8}$, and $6\frac{2}{3} = \frac{20}{3}$; and the simple fraction required is $\frac{41}{8} \div \frac{20}{3}$, or $\frac{123}{160}$.

It may be useful on some occasions to know, that if the given fractions be reduced to equivalent ones having a common denominator, the quotient of the resulting numerators will be the required quotient. Thus, the fractions in the first example are equivalent to $\frac{10}{36}$ and $\frac{21}{36}$:

* That is, take its numerator as denominator, and its denominator as numerator. It may be remarked, that the fraction resulting from the inversion of another fraction, is called its *reciprocal*; since *any two numbers whose product is a unit, are called the reciprocals of each other.* Thus, $\frac{12}{7}$ and $\frac{7}{12}$, $\frac{1}{8}$ and $\frac{8}{1}$ or 8, are the reciprocals of each other, as $\frac{12}{7} \times \frac{7}{12} = 1$, and $\frac{1}{8} \times 8 = 1$.

† This mode of dividing furnishes an explanation of the rule in page 27, for finding the true remainder in dividing by the factors of a given divisor.

dividing therefore the numerator 10, by the numerator 21, we have $\frac{10}{21}$, the same quotient as before; and thus may all the exercises in this rule be performed.

Exercises.	*Answ.*	*Exercises.*	*Answ.*
1. $4\frac{3}{4} \div 5\frac{1}{8}$	$=\frac{38}{41}$	12. $4\frac{1}{2} \div 15$	$=\frac{3}{10}$
2. $19\frac{1}{20} \div 2\frac{1}{15}$	$=9\frac{27}{124}$	13. $\frac{2}{3} \div \frac{3}{11}$	$=2\frac{4}{9}$
3. $\frac{13}{60} \div \frac{23}{60}$	$=\frac{13}{23}$	14. $\frac{3}{11} \div \frac{2}{3}$	$=\frac{9}{22}$
4. $16\frac{100}{151} \div \frac{48}{49}$	$=17\frac{17}{1812}$	15. $\frac{3}{5}$ of $\frac{8}{9} \div \frac{6}{7}$ of $\frac{3}{4}$ ..	$=\frac{112}{135}$
5. $\frac{7}{9} \div \frac{15}{16}$	$=\frac{112}{135}$	16. $\frac{13}{14} \div \frac{13}{15}$	$=1\frac{1}{14}$
6. $\frac{3}{19} \div \frac{1}{54}$	$=8\frac{10}{19}$	17. $\frac{7}{9} \div \frac{1}{8}$	$=6\frac{2}{9}$
7. $2\frac{16}{17} \div 2\frac{17}{18}$	$=\frac{900}{901}$	18. $3\frac{5}{9} \div 2\frac{5}{8}$	$=1\frac{67}{189}$
8. $19 \div \frac{199}{450}$	$=42\frac{192}{199}$	19. $\frac{7}{22} \div \frac{113}{355}$	$=\frac{2485}{2486}$
9. $1 \div \frac{8}{700}$	$=233\frac{1}{3}$	20. $\frac{7}{8}$ of $\frac{6}{7} \div \frac{4}{5}$ of $\frac{8}{9}$ ..	$=1\frac{7}{128}$
10. $2\frac{1}{2} \div \frac{1}{1728}$	$=4320$	21. $\frac{2}{3} \div \frac{5}{7} - \frac{5}{8} \div \frac{10}{11}$	$=\frac{59}{240}$
11. $7\frac{7}{9} \div 8$	$=\frac{35}{36}$	22. $1\frac{3}{4} \div 2\frac{1}{2} + 5\frac{1}{2} \div 3\frac{1}{8}$.	$=2\frac{23}{50}$

THE RULE OF PROPORTION IN FRACTIONS.

In fractions, in the rule of Proportion, whether Simple or Compound, the terms are arranged in the same manner as if they were whole numbers; but they are reduced, multiplied, and divided by the rules for the reduction, multiplication, and division of fractions.

When only three terms are given, it is evident from the rules for the multiplication and division of fractions, that if the first term, after the necessary reduction, be inverted, the continual product of the result and the other two terms, will be the fourth proportional required.

The work may also be performed by reducing the first and second terms to fractions having a common denominator. The common denominator may then be rejected and only the numerators employed.

Exam. 1. If $3\frac{4}{7}$ cwt. of sugar cost £$17\frac{4}{15}$, how much may be bought for £$4\frac{1}{7}$?

Here, as £$17\frac{4}{15}$: £$4\frac{1}{7}$: : $3\frac{4}{7}$ cwt.; or, by reduction to improper fractions, as £$\frac{259}{15}$: £$\frac{29}{7}$: $\frac{25}{7}$ cwt.; $\frac{15 \times 29 \times 25}{259 \times 7 \times 7}$ cwt. $=\frac{10875}{12691}$ cwt. the fourth proportional required, the value of which by problem VIII. page 98, is found to be 3 qrs. $11\frac{1765}{1813}$ lbs.

Exam. 2. If £$\frac{8}{9}$ purchase $1\frac{5}{6}$ lbs. of indigo, how much may be bought for $\frac{16}{19}$ shilling?

As £$\frac{8}{9}$: $\frac{16}{19}$s. : : $1\frac{5}{6}$ lbs.; or, by reducing the first term to the denomination of shillings, and the third to an improper fraction,

as $\frac{160}{6}$s. : $\frac{16}{19}$s. : : $\frac{11}{6}$ lb.: $\frac{9\times16\times11}{160\times19\times6}$ lbs.$=\frac{3\times11}{10\times19\times2}$ lbs.$=$ $\frac{33}{380}$ lbs.; or, by multiplying by 16, $=\frac{37}{95}$ oz.

Exam. 3. If $\frac{5}{6}$ lb. of tea cost 6/8, what cost $\frac{7}{3}$ lb.?

As $\frac{5}{6}$ lb. : $\frac{7}{3}$ lb. : : 6/8 : $\frac{5}{6}\times\frac{7}{3}\times$6/8, or $\frac{14}{15}\times$6/8, or 6/2$\frac{2}{3}$. The work may also be as follows, since 6/8$=$£$\frac{80}{240}$, or £$\frac{1}{3}$: as $\frac{5}{6}$ lb. : $\frac{7}{3}$ lb. : : £$\frac{1}{3}$: £$\frac{6\times7}{5\times9\times3}$$=$£$\frac{14}{45}$; or by problem VIII. page 98, $=$ 6/2$\frac{2}{3}$.

Since $\frac{5}{6}=\frac{15}{18}$, and $\frac{7}{3}=\frac{14}{18}$, the answer might also be obtained very easily by the following analogy:—as 15 : 14 : : 6/8 : 6/2$\frac{2}{3}$.

Exercises. 1. If $\frac{7}{12}$ cwt. cost £$\frac{4}{11}$, what cost 1$\frac{7}{9}$ cwt.? *Answ.* £1 2 1$\frac{5}{7}$.

2. How much wine may be bought for £17 8 4,* if 5$\frac{5}{11}$ gallons cost £7 7 6? *Answ.* 12$\frac{53}{55}$ gallons.

3. How much sugar may be bought for £81 14 6, if 1$\frac{5}{8}$ cwt. cost £6$\frac{4}{7}$? *Answ.* 19 cwt. 3 qrs. 16$\frac{348}{575}$ lbs.

4. If $\frac{7}{8}$ lb. cost $\frac{13}{14}$s. what cost 2$\frac{11}{12}$ cwt.? *Answ.* £17 6 8.

5. If the yearly rent of 45 a. 2 r. 20 p. be £44$\frac{7}{16}$, what is the rent of 1$\frac{5}{8}$ acre? *Answ.* £1 11 7$\frac{53}{55}$.

6. If 3$\frac{5}{8}$ shillings pay for 1$\frac{1}{12}$ yard, how much may be bought for £6$\frac{17}{18}$? *Answ.* 42 yds. 1 qr. 1$\frac{1}{12}$ nail.

7. What cost 49$\frac{3}{11}$ yards of broadcloth, if 7$\frac{5}{8}$ yards cost £7 18 4? *Answ.* £51 3 1$\frac{333}{671}$.

DECIMAL FRACTIONS.†

A DECIMAL FRACTION, or a DECIMAL, is a fraction whose denominator is 10, or some number produced by the continued multiplication of 10 as factor, such as 100, 1000, &c.‡

Hence all the rules for the management of fractions in general, are applicable to decimal fractions.

In the notation of decimals, the denominator is usually omitted; and, to indicate its value, a point is placed to the left of as many figures of the numerator, as there are ci-

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* This, by problem VII. page 97, becomes £17$\frac{5}{12}$: and £7 7 6 is equivalent to £7$\frac{3}{8}$.

† The following article on *decimal fractions* is merely an adaptation of the general principles of fractional quantities, already delivered, to a particular and important class of fractions. It therefore contains no principles that can be properly called new; but rather short and easy modes of applying those already given. Every operation on decimal fractions, indeed, may be performed by reducing them to common fractions, as will appear hereafter, and then applying to the results, the rules in the preceding articles.

‡ It will be seen hereafter, that *the number which results from the continued multiplication of any number as factor, is called a* power *of that factor.* Thus, 100 is the second power of 10; 1000, its third power, &c.

phers in the denominator. Should there not be a sufficient number of figures in the numerator, as many ciphers are prefixed as supply the deficiency.

Thus, $\frac{7}{10}$, $\frac{8}{100}$, $\frac{3}{100000}$, $\frac{475}{100}$, are decimals; and $\frac{7}{10}$ is written ·7; $\frac{9}{100}$, ·09; $\frac{3}{100000}$, ·00003; $\frac{475}{100}$, 4·75, &c. Hence, conversely,

The denominator of a decimal, thus expressed, is the number denoted by a unit with as many ciphers annexed, as there are figures in the given number.

Thus, ·37 is $\frac{37}{100}$; ·004, $\frac{4}{1000}$; ·00083, $\frac{83}{100000}$, &c.

From this notation it is evident, that the figure immediately following the decimal point, denotes tenths; the next figure, hundredths; the third, thousandths, &c.

Thus, since 476 is equivalent to 400+70+6, the fraction ·476, or $\frac{476}{1000}$ is equivalent to $\frac{400}{1000}+\frac{70}{1000}+\frac{6}{1000}$, or to $\frac{4}{10}+\frac{7}{100}+\frac{6}{1000}$, by dividing the terms of the first of these fractions by 100, and those of the second by 10. In a similar manner it would appear, that ·709 is equivalent to seven tenths, no hundredths, and nine thousandths.

Hence, the values of figures in decimals, as well as in whole numbers, are increased in a tenfold ratio by removing them one place towards the left hand, and diminished in the same ratio by removing them one place to the right.

Thus, in the decimal ·004, by removing the point one place to the right, and consequently the figures of the decimal one place to the left, we have ·04, which denotes $\frac{4}{100}$, or $\frac{40}{1000}$, and is ten times the given fraction, $\frac{4}{1000}$; but by introducing a cipher after the point, which removes the figures a place to the right, we have ·0004 or $\frac{4}{10000}$, which is evidently only a tenth part of the given fraction $\frac{4}{1000}$, or $\frac{40}{10000}$. From these principles it will readily appear, that

A decimal is multiplied by 10, if the separating point be removed one place towards the right hand; by 100, if two places; by 1000, if three places, &c.: and, conversely, a decimal is divided by 10, if the separating point be removed one place towards the left hand; by 100, if two places; by 1000, if three places, &c.; vacant places, when such occur, being supplied in both cases by ciphers.

Thus, ·7248×10=7·248, or $7\frac{248}{1000}$; 6·347×100=634·7; 6·3×1000=6300. Also, 78·33÷10=7·833; ·736÷100=·00736; 7·3÷100=·073, &c. It appears also from the same principles, that

The value of a decimal is not changed by annexing a cipher to the end of it, nor by taking one away, as in each case the significant figures retain the same positions in relation to the separating point.

Thus, ·50=·5=·500, each being equivalent to one half.

From this view of the nature of decimal fractions, it appears, that there is in every respect the closest resemblance between them and whole numbers; and hence all operations on decimals are performed in ex-

actly the same manner as those on whole numbers—due attention being paid to the position of the separating point. This last circumstance, indeed, demands the utmost care, as *the point* is the characteristic of the decimal; and from what precedes, it is evident how much depends on its proper position.

Exercises in Notation and Numeration of Decimal Fractions.

Write the following fractions according to the notation of decimal fractions:

1. Three hundred and six ten-thousandths. 2. Three hundred-thousandths. 3. One ten-millionth. 4. One ten-thousandth. 5. One tenth. 6. Fifty-seven hundred-thousandths. 7. Two hundred ten-millionths. 8. Five hundred and nine hundredths.

Express the following decimal fractions in words:

1.	·58	3.	·007	5.	31·73	7.	·00004	9.	·034567
2.	·106	4.	·0007	6.	3·173	8.	·00041	10.	·00000011

REDUCTION OF DECIMAL FRACTIONS.

Problem I. *To reduce a common fraction to a decimal.*

RULE. (1.) Let the numerator, with a cipher annexed, be divided by the denominator, and to the significant figure or cipher placed in the quotient, prefix a point. (2.) Then, if there be a remainder, annex to it more ciphers, and continue the division till nothing remain, or till the result consist of as many figures as may be thought necessary.

A given quantity may be reduced to the decimal of another given quantity of the same kind, considered as an integer, by first reducing it to a common fraction by problem VII. page 97, and then reducing the fraction thus found to a decimal.

Exam. 1. Reduce $\frac{3}{512}$ to a decimal.

Here, 3 by the addition of one cipher, becomes 30, in which 512 is not contained; and therefore a cipher is put in the quotient, and a point prefixed. By the annexing of another cipher, 30 becomes 300, in which the divisor not being contained, another cipher is put in the quotient. After this the division proceeds in the usual way, a cipher being added each time, and the decimal is found to be ·005859375, or $\frac{5859375}{1000000000}$, which by reduction to its lowest terms, would become $\frac{3}{512}$, the given fraction; and thus the work is proved to be correct.

```
512)3000(·005859375
    ----
    4400
    ----
     3040
     ----
      4800
      ----
       1920
       ----
        3840
        ----
         2560
         ----
          ...
```

With respect to the *reason of the rule*, the given fraction, by annexing to each of its terms nine ciphers, (the number that was annexed in the preceding operation,) becomes $\frac{3000000000}{512000000000}$; and this, by dividing each term by 512, becomes $\frac{5859375}{1000000000}$, which in the notation of decimals, is ·005859375, the same as before. Hence it is obvious, that the preceding reduction is in reality the multiplication of each term of the given fraction by 1000000000, and the division of each of the results by 512; and it is evident from the principles established in page 88, that the result must be equivalent to the given fraction.

This operation might also be illustrated in other modes. Thus, according to the fourth principle in page 89, we should have 512 : 3 : : 1000000000 : 5859375, the numerator of the required fraction; which in relation to the denominator 1000000000 would be expressed by ·005859375, according to the notation of decimals. Another illustration might also be derived from the latter part of the rule for problem IV. page 95.

Exam. 2. Reduce $\frac{5}{12}$ to a decimal.

In this example, each remainder after the two first is 8, and hence the same figure must be repeated perpetually.

```
12)500000
   -------
   ·41666, &c.
```

Exam. 3. Reduce $\frac{6}{11}$ to a decimal.

Here, after two figures have been obtained in the quotient, the same series of remainders, and consequently the same series of figures in the quotient, must recur; and therefore, were the work pursued, the first two figures of the decimal would recur without end.

```
11)60000
   -------
   ·5454, &c.
```

Exam. 4. Reduce 13/8 to the decimal of a pound sterling.

Here, 13/8=164 pence=£$\frac{164}{240}$= £$\frac{41}{60}$; and £$\frac{41}{60}$, reduced to a decimal, in the manner already shown, becomes £·68333, &c. The same result may be obtained by writing, as in the margin, 13 shillings below 8 pence; then dividing by 12 and by 20, we find 13/8 equivalent to 13·6666, &c. shillings, or to £·68333, &c. as before.

```
12)80000
   -------
20)13·6666, &c.
   -------
   £·6833, &c.
```

The first three figures of the decimal of a pound may be found very readily, thus: Take half the number of shillings for the first figure; and for the other figures increase the farthings in the given pence and farthings by 1, if they amount to 24 or more, and by 50, if there were a remainder of 1 shilling. Thus, if 13/8 be given, the half of 13 is 6, and 1 remains; and in 8 pence there are 32 farthings, which exceeding 24, is made 33, and this, because 1 shilling remained, is increased by 50, and becomes 83: hence the three first figures are ·683.

A decimal which cannot be exactly expressed, but which may be continued to an unlimited number of figures, is

called an INTERMINATE DECIMAL, to distinguish it from others, which in respect to it, are called TERMINATE.

When a decimal is expressed either by the continual repetition of the same figure, or of the number expressed by two or more figures, it is called a PERIODICAL or a CIRCULATING DECIMAL; and the figure, or number, so repeated is called the PERIOD.

Thus the decimals in the second and third examples are periodical, the period in the one consisting of one figure, and that in the other of two. When only one figure is repeated, the application of the terms *period* and *periodical*, though convenient, and perhaps best on the whole, is scarcely correct, as each of the terms suggests the idea of more figures than one. On this account, some writers term such decimals *repeaters*, or *repeating decimals*.

A periodical decimal is said to be MIXED, if it consist of one or more figures prefixed to a periodical part; others are called PURE.

For the sake of brevity, in writing decimals of this kind, it will often be sufficient to write the period but once, and to denote its continuation by putting a trait, or stroke, over the first figure of the period, and another over the last, or one over the repeating figure, if there be but one figure in the period.

Thus, ·47333, &c. may be expressed by ·473′, and ·5637637, &c. by ·56′37′.

The following considerations will tend to explain still farther the nature of these fractions.—By the preceding rule we find the following results: $\frac{1}{9}=\cdot1111$, &c.; $\frac{1}{99}=\cdot010101$, &c.; $\frac{1}{999}=\cdot001001$, &c.; $\frac{1}{9999}=\cdot00010001$, &c. These decimals are all periodical; and if any of them be multiplied by a whole number, the result will also be periodical. Thus, if we multiply the third by 128, we find $\frac{128}{999}=\cdot128128$, &c.; if by 998, we get $\frac{998}{999}=\cdot998998$, &c. Here we see, that the value of the periodical fraction is the common fraction, whose denominator is the number expressed by as many 9's as there are figures in the period, and whose numerator is the period itself: and the same may be shown in every case. Hence, we have the following rule:—*To find the value of a pure periodical decimal, take the period for numerator, and as many 9's as there are figures in the period, for denominator.* Thus, ·8181, &c.$=\frac{81}{99}=\frac{9}{11}$; $2'97'=\frac{297}{999}=\frac{11}{37}$; $\cdot3'=\frac{3}{9}=\frac{1}{3}$; ·999, &c.$=\frac{9}{9}=1$, &c.

If the decimal be mixed, its value may be easily found by the same principle. Thus, if it were required to find the common fraction which would produce the decimal ·12436436, &c. by multiplying by 100 we should find $12\cdot4'36'$, or $12\cdot\frac{436}{999}$. Dividing this by 100, we obtain, for the value of the proposed fraction, $\frac{12}{100}+\frac{436}{99900}$, or by reduction to a common denominator, $\frac{12\times999}{99900}+\frac{436}{99900}=\frac{12424}{99900}=\frac{3106}{24975}$.

From this we may readily derive a rule which will be easier in prac-

tice. For in the last result, since 12 is multiplied into 999, or into 1000—1, the product is 12000—12; to which if the other numerator be added, we have for the required numerator 12436—12, and the denominator remains the same as before. From a due consideration of this result, we may derive the following rule: *To find the value of a mixed periodical decimal, from the number expressed by the finite part with the period annexed, subtract the finite part for the numerator; and, for the denominator, to as many 9's as there are figures in the period, annex as many ciphers as there are figures in the finite part.* Thus, to find the value of ·83′, from 83 take 8, and there will remain 75, and the required fraction is $\frac{75}{90}$, or $\frac{5}{6}$. Again, if ·5416′ be given, we have 5416—541, or 4875 for numerator, and the fraction is $\frac{4875}{9000}$, or $\frac{13}{24}$. In like manner, to find the value of ·263′01′, we have for numerator 26301—26=26275, and for denominator 99900. Hence the required fraction is $\frac{26275}{99900}$, or $\frac{1051}{3996}$.—The value of such fractions may also be found by the summation of series, as will appear hereafter.

By the method thus shown, interminate decimals may be reduced to common fractions, and subjected to the rules for managing such quantities. Unless when complete precision is required however, this is not necessary: and indeed in all useful cases their values, instead of being found with entire accuracy, are to be *approximated*, by carrying the decimals out to as many figures as may be necessary in any particular case. In thus approximating the value of a decimal, *if it be carried to two places, the error will be less than a hundredth part of the integer; if to three places, less than a thousandth; if to four places, less than a ten-thousandth*, &c. Thus, if the decimal ·27′, be carried only to ·27, the error will be less than $\frac{1}{100}$, since the true value is more than $\frac{27}{100}$, and less than $\frac{28}{100}$; if to the three figures ·272, the error will be less than $\frac{1}{1000}$, since the true value is greater than $\frac{272}{1000}$ and less than $\frac{273}{1000}$, &c.

It may be remarked here, that when a vulgar fraction is in its lowest terms, and its denominator contains no simple factors except 2 or 5, (the two prime factors of 10,) the equivalent decimal is finite, but in every other case it is interminate. The cause of this is, that neither 10, 100, nor any similar number, is divisible by any of the nine digits except 2 and 5. It may also be observed, that the number of circulating figures must always be less than the units in the denominator. This is obvious from the consideration, that the number of remainders different from each other, which can arise in any operation in division, must be less than the units in the divisor. Thus, in dividing by 7, it is evident, that the only possible remainders are 1, 2, 3, 4, 5, and 6; and since, in reducing to decimals, a cipher is annexed to each remainder, there cannot be more than six dividends, and consequently, six figures in the quotient, all different.

It may be added, on this part of the subject, that *any prime number, except 2 and 5, is contained, without remainder, in the number expressed in the common notation, by as many 9's as there are units, wanting one, in the prime number itself.** Thus, 3 is a measure of 99; 7 of 999,999; and 13 of 999,999,999,999. It is easy to show from this property, that every prime number, except 2, 3, and 5, is a measure of the number

* For a demonstration of the general proposition of which this is a particular case, see Legendre's *Théorie des Nombres*, No. 129.

expressed in the common notation, by as many 1's as there are units, wanting one, in the prime number itself. Thus, 7 is a measure of 111,111; and 13 of 111,111,111,111, &c. It may also be remarked, that while the prime number is always a measure of the number expressed by the number of 9's or 1's above specified, it is sometimes contained exactly in the number expressed by one-half, one-third, or some other submultiple, or exact part, of the number of those figures. Thus, while 11 is a measure of 9,999,999,999, it is also contained exactly in 99, the number expressed by two (one-fifth of 10,) 9's; and 13 is a measure of 999,999, or 111,111.

Exercises. Reduce the following fractional quantities to decimals:

Exercises.	*Answers.*	*Exercises.*	*Answers.*
1. $\frac{7}{16}$	·4375	9. $\frac{4}{99}$	·0′4′
2. $\frac{3}{32}$	·09375	10. $\frac{37}{79}$	·4′683544303797
3. $\frac{5}{1024}$	·0048828125	11. $\frac{4}{909}$	·0′044′
4. $\frac{11}{40}$	·275	12. $\frac{4}{9009}$	·0′00444′
5. $\frac{1}{13}$	·0′76923′	13. $\frac{1}{41}$	·0′2439′
6. $\frac{7}{12}$	·583′	14. $\frac{1}{9}$	·1′
7. $\frac{3}{256}$	·01′04895′	15. $\frac{1}{81}$	·0′12345679′
8. $\frac{6}{10001}$	·0′0059994′	16. $\frac{1}{44}$	·022′7′

	Exercises.		*Answers.*
17.	10/9 to the decimal of	£1	·5375
18.	0/10½	£1	·04375
19.	17/7	£1	·87916′
20.	3 r. 11 p.	an acre	·81875
21.	2 qrs. 8 lbs.	cwt	·5′71428′
22.	37 perches,	mile	·115625
23.	3 hours, 30 minutes,	day	·14583′
24.	15 minutes, 30 seconds,	hour	·2583′
25.	16/8	£1 2 9	·7′32600′
26.	3 c. 1 q. 7 lbs.	ton	·165625
27.	6½d.	shilling	·5416′

Ex. 28. If the diameter of a circle be 1, the circumference is $3\frac{1}{7}$ nearly, or $3\frac{16}{113}$ more nearly. Express each of these decimally.* *Answ.* 3·1′42857′, and 3·1415929, &c.

29. If the circumference of a circle be 1, the diameter is $\frac{7}{22}$ nearly, or $\frac{113}{355}$ more nearly. Express each of these decimally. *Answ.* ·31′8′, and ·318309859, &c.

30. An English acre is $\frac{121}{196}$ of an Irish acre: required the equivalent decimal. *Answ.* ·61734693877551, &c.

31. Reduce a quarter of wheat containing 456 lbs. to the decimal of a hundred weight. *Answ.* 4·07′14285′.

32. The old Irish liquid gallon contained 217·6 cubic inches, and the old English wine gallon 231. Reduce the former to the decimal of the latter. *Answ.* ·94′19913′.

* The circumference true to twenty decimal places, is 3·14159265[illegible]3238[illegible].

Problem II. *To find the value of a given decimal in the parts contained in the integer.*

RULE. Multiply the given decimal by the numbers which, were it an integer, would reduce it to the lower denominations contained in it, and, after each multiplication, point off for decimals as many figures towards the right hand as there were figures in the given decimal. The figures remaining on the left will express the required value.

Exam. 5. Required the value of ·3945 of a day in hours, &c.

Here, the given decimal is multiplied by 24, the number of hours in a day, and four figures being cut off towards the right, it appears that ·3945 day is equal to $9\frac{4680}{10000}$ or $9\frac{468}{1000}$ hours. The decimal ·468 (the cipher being omitted, see page 107,) is then multiplied by 60, and three figures being cut off, there results 28·080 or 28·08 minutes. By continuing the process, the value of the given decimal is found to be 9 h. 28 min. 4 sec. 48 thirds. With respect to the reason of this process, it is only necessary to observe, that it is exactly the same as finding the value of $\frac{3945}{10000}$ day by problem VIII. page 98, the pointing off of the decimals serving the purpose of dividing by the denominator.

·3945 day
24
15780
7890
9·4680 hours
60
28·080 minutes
60
4·80 seconds
60
48·0 thirds.

Exam. 6. Required the value of £·5937.

Here, by multiplying by 20, we find, that the given decimal is equivalent to $11\frac{874}{1000}$ shillings. Then, by multiplying by 12, we find ·874s. to be equivalent to 10·488 pence. In like manner, ·488d. is shown to be equivalent to $1\frac{952}{1000}$ farthing, or a halfpenny, very nearly; and consequently, the required value is 11/10½, nearly.

£·5937
20
11·8740 shillings
12
10·488 pence
4
1·952 farthings.

Answ. 11/10½, nearly.

The following rule is easy and useful in practice:—*To find the value of the decimal of a pound sterling to the nearest farthing; (1.) Take a fifth of the number expressed by the two first figures of the decimal, for the shillings of the result: (2.) Diminish the number expressed by the remainder with the third figure of the decimal annexed, by a twenty-fifth of itself, and what remains will be the farthings in the rest of the required value.* Thus, in finding the value of £·5937, the fifth part of 59 is 11, the shillings of the answer; the remainder is 4, which being prefixed to 4, (the third figure increased by a unit, because the next figure 7, is greater than five,) we have

44, the twenty-fifth part of which is more nearly 2 than 1; we therefore reject 2, and have remaining 42 farthings, or $10\frac{1}{2}$; and hence, the answer is $11/10\frac{1}{2}$, nearly, the same as before. With respect to the reason of this process, it is evident that the given decimal is more nearly equal to ·5940 than ·5930; we use therefore the former, or its equal 594. Now this is equivalent to ·55+·044, the value of the former part of which would be found by the general rule for this problem, by multiplying by 20 and dividing by 100, which is the same as dividing by 5, since 20 is one-fifth of 100. Then, for the value of £·044 or $£\frac{44}{1000}$, by diminishing the denominator by one twenty-fifth part of itself, we have 960; and by diminishing the numerator by a similar part of itself, we have 42 nearly: hence $£\frac{44}{1000}$ is nearly equal to $£\frac{42}{960}$, or 42 farthings, since $£\frac{1}{960}$=1 farthing.—Practice will soon enable the learner to estimate with sufficient correctness, the effect of the fourth figure of the decimal, in respect to the quantity to be rejected. In this example, instead of using 44 farthings, we might use 43·7, and it would be easy to see, that the twenty-fifth of this would be about 1·7; and rejecting this, we should have 42, as before.

Exam. 7. Required the value of ·805′ of a yard in long measure.

This, and similar exercises, may be wrought either by converting the proposed decimal into a common fraction, in the way shown in page 111, or more easily by employing an approximative process, as in the margin. Here, we carry 1 to the product of 3 and 5, because, had the decimal been continued farther, it is evident 1 must have been carried from the preceding product.

·8055′ of a yard.
3
———
2·4166 feet.
12
———
4·9999 inches.

Answ. 2 feet, 5 inches.

For a similar reason, 7 is carried to the product of 12 and 6. The result is found to be 2 feet, 4·9′ inches, which is equivalent to 2 feet, 5 inches.—*(See page 110.)* Another 5 was added to the given decimal, that the result might be more distinct and certain.

Exercises. Required the values of the following decimals:—

	Exercises.	*Answers.*
33.	·0675 of a cwt.	$7\frac{14}{25}$ lbs.
34.	·4625 of a ton	9 cwt. 1 qr.
35.	£·0484	$11\frac{72}{125}$d.
36.	·8845 of an acre	3 r. $21\frac{3}{25}$ p.
37.	·00213 of a day	3 minutes, $4\frac{4}{125}$ sec.
38.	£·7′	$15/6\frac{1}{4}$, nearly.
39.	·2′85714′ of a cwt.	1 qr. 4 lbs.
40.	·113′6′ of a mile	36 perches, 2 yards.
41.	·615 of a shilling	$7\frac{18}{50}$d.
42.	£·483′5′	9/8 nearly.
43.	·2385 of a degree	14′ 18″ 36‴
44.	·63′12′ of a cwt. *long weight*	2 qrs. $15\frac{4}{7}$ lbs. nearly

	Exercises.	*Answers.*
45.	£·06′	1/4
46.	·47916′ lbs. troy	5 oz. 15 dwts.
47.	£·42′8′	8/6¾, nearly.
48.	·4375 of a shilling	5¼d.
49.	·09375 of an acre	15 perches.
50.	·4′ foot, *long measure*	5⅓ inches.

ADDITION OF DECIMAL FRACTIONS.

RULE. (1.) Arrange the given numbers so that the separating points may all be in the same column. (2.) Find the sum as in Simple Addition. (3.) Point off as many places of decimals, as there are in the given number which contains most.

If any of the given numbers contain periodical decimals, let these be carried out to as many places as there are in the longest of the finite decimals; or, if greater accuracy be required, let them be carried as far as may be judged necessary.

In the application of decimals to practical purposes, it is generally known, from the nature of the case under consideration, to how many places it is necessary, that the result should be true. *When a result is thus required to be true to an assigned number of places of decimals, it is proper to carry the decimals which consist of more places, to at least one place beyond the assigned number, and to reject the last figure.* In this case, it is proper to observe, that *when a decimal is not carried out to its full length, the last figure of the part retained, should be increased by a unit, if the succeeding figure be 5, or greater than 5.*

Exam. 1. Add together 81·4632, 9·75, and 47·388.

Here, the numbers are arranged as in the margin, and added as in common Addition. The reason of the arrangement and operation is manifest, those figures being added together which are of the same local value.

```
 81·4632
  9·75
 47·388
 ———————
138·6012, sum.
```

Exam. 2. Add together 3·7′3′, ·873, 51·7′, 108·2, and 73·463128, so that the result may have four places of decimals true.

In this example, the first, third, and fifth numbers are carried to five places, each, and the last figure of the third is made 8, because the next figure would be 7. In like manner the fifth figure of the last line is made 3, because the succeeding figure is 8. The reason of this

```
 3·73737
  ·873
 51·77778
 108·2
 73·46313
 ————————
238·0513, sum.
```

is evident, since 30 is nearer 28 than 20 is, and 30, by the rejection of the last figure, becomes 3. In the addition, the sum of the last column is 18, from which 2 is carried, because 18 is nearer 20 than 10.—The correct sum, found by carrying [illegible] decimals farther, is 238·0512795′1′, which by retaining only [illegible] figures of the decimal, and increasing the last of them by a unit [illegible] because it is followed by 79, &c. becomes 238·0513, the same as [illegible].

Exercises. 1. Add together 1·83, 5·674, ·3125, 18·3, 100, 38·62, 4·3957, and ·5. *Answ.* 169·6322.

2. Required the sum of 93·617843, 7·836, 12·25, ·71375, 4·391, 7·839, 3·7674285, and ·8693. *Answ.* 131·2843215.

3. Required the sum, true to six places, of 51·25, 3·4′, ·63′7′, 7·885′, 7·875, 7·8′75′, and 11·1′. *Answ.* 90·0793′60724′.

4. Required the sum of ·7354, ·7354′, ·735′4′, ·7′354′, ·07354, and ·0735′4′. *Answ.* 3·088857′991′.

5. Add together ·3, ·3′, ·45, ·4′5′, ·3′51′, ·6468, ·6468′, ·646′8′, and ·64′68′. *Answ.* 4·47663′45618′.

6. Add together ·8′, ·8′7′, and ·8′76′. *Answ.* 2·6′44553′.

7, 8, and 9. Required the sum, true to five places, of the numbers given in exercises 5th, 6th, and 13th of Addition of Fractions, the several fractions being previously reduced to decimals. *Answ.* 6·0078125, 3·907′14285′, and 1·561011′90476′.

SUBTRACTION OF DECIMAL FRACTIONS.

RULE. (1.) Set the less number so that each figure in it may stand below a figure of the same local value in the greater. (2.) Then find the difference as in Simple Subtraction, and place the separating point as in Addition of Decimals.

Exam. 1. From 3·5′4′ take 1·34265.

Here, the greater number is extended, and the remainder is found to be 2·202804′5′.

```
3·5454[illegible]5
1·34265
---------
2·202804′5′, diff.
```

Exam. 2. Required the difference of 8·6 and 2·7′.

Here, the less number is carried to four places, that the true answer may be discovered with greater certainty. In the subtraction, ciphers are conceived to be annexed to the greater number, and 1 is carried to the repeating figure first used, because this must have been done, had the less number been carried one place farther. The answer is found to be 5·82′.

```
From  8·6
Take  2·7777
-----------
Rem.  5·8222
```

Exercises.	*Answers.*	*Exercises.*	*Answers.*
1. 3·468—1·2591.....	=2·2089	7. 5·83—4·1′7′....	=1·658′2′
2. 6·45—1·34′5′.....	=5·104′5′	8. 17·4′—·4′8′....	=16·9′5′
3. 56·4297—29·68534	=26·74436	9. 3·3′42′—1·7′5′.	=1·5′84766′
4. 34·528—10·63475.	=23·89325	10. $17\frac{3}{8}$—$7\frac{3}{4}$*.....	=9·94642857
5. ·682—·09647.....	=·58553	11. $7\frac{5}{12}$—$4\frac{7}{15}$.....	=2·9617521
6. 13·6′—4·345.....	=9·3216′	12. $15\frac{1}{17}$—$13\frac{6}{25}$...	=1·8188235

MULTIPLICATION OF DECIMAL FRACTIONS.

RULE I. Multiply the factors as in Simple Multiplication, and point off in the product as many places of decimals as there are in both factors, supplying the deficiency, when any occurs, by prefixing ciphers.

Exam. 1. Multiply ·582 by 66·3.

Here, because there are three places of decimals in the one factor, and one in the other, there must be four places of decimals in the product.

The reason of the rule will be understood from considering, that when the denominators are supplied, the first factor becomes $\frac{582}{1000}$, and the second $66\frac{3}{10}$, or $\frac{663}{10}$, which, by the rule for the multiplication of fractions, page 102, give for product $\frac{582 \times 663}{10000}$; whence it appears, that the product of 582 and 663 must be divided by 10000, which is effected by cutting off four figures. It is evident, that the divisor must contain as many ciphers as there are in both denominators; that is, as there are decimal figures in both factors.

```
  ·582
  66·3
  ----
  1746
 3492
3492
-------
38·5866, product.
```

Exam. 2. Multiply ·13 by ·7.

Here a cipher must be prefixed to the product 91, as there are two places of decimals in the one factor, and one in the other.

```
 ·13
  ·7
 ----
 ·091
```

Exercises.	*Answers.*	*Exercises.*	*Answers.*
1. ·78 × ·42	=·3276	4. ·144 × ·144	=·02073
2. 7·8 × 4·2.....	=32·76	5. 1·05 × 1·05 × 1·05	=1·15762
3. 7·49 × 63·1.....	=472·619	6. 36·48 × ·475	=17·328

Exercises.	*Answers.*
7. ·1 × ·1 × ·1 × ·1 × ·1	=·00001
8. 13·825 × 5·128	=70·8946
9. ·08 × ·036	=·00288

* In this exercise and the two next, the given fractions are to be reduced to decimals, and the difference taken according to the rule

Exercises.	*Answers.*
10. ·31 × ·32	=·0992
11. 3·18 × 41·7	=132·606
12. 62·38 × 7	=436·66

When the number of decimal figures is great, or the factors numerous, the decimal figures resulting from the application of the preceding rule, are, in many cases, unnecessarily and inconveniently numerous. The following approximative rule will be found extremely useful in such cases.

RULE II. (1.) Count off, after the separating point in the multiplicand, (annexing ciphers, if requisite,) as many figures of decimals as it is necessary to have in the product. (2.) Below the last of these, write the unit figure of the multiplier, and write its other figures in reversed order. (3.) Then multiply by each figure of the multiplier thus inverted, neglecting all the figures of the multiplicand to the right of that figure, except to find what is to be carried; and let all the partial products be so arranged, that their right hand figures may stand in the same column. (4.) Lastly, from the sum of these partial products, cut the assigned number of decimal places.

In carrying from the rejected figures, we should always take what is *nearest* the truth, whether it be too great or too small.

Exam. 3. Multiply 7·24651 by 81·4632, so that there may be only three places of decimals in the product.

Here 1, the unit figure of the multiplier, is written below 6, the third decimal figure of the multiplicand; 8, the figure which precedes 1, is written after it; 4, the figure which follows it, is set before it, &c. We then say, 8 times 5=40, and 1, carried for 8 times 1,=41: 1 is then set down, and 4 carried, and the rest of the multiplication by 8 proceeds in the usual way. Then, in multiplying 7·246 by 1, we add 1 to the product for 51, because 51 is nearer 100 than 0, and therefore it is nearer the truth to carry 1 than 0. In multiplying 7·24 by 4, three is carried for the product that would have resulted for the rejected figures: for going two places back, we have 4 times 5=20; 4 times 6=24, and 2=26, which being nearer 30 than 20, we carry 3. For a similar reason, in multiplying 7·2 by 6, we carry 3 from the rejected figures; and thus we proceed in similar cases. In finding what is to be carried for the rejected figures, it is generally sufficient to go one figure back; but in doubtful cases it is better to go farther.

```
  7·24651
 2364·18
 -------
 579721—
   7247—
   2899—
    435—
     22—
      1+
 -------
590·325, product.
```

The reason of the preceding operation will be seen from the adjoined work at full length, in which a vertical line is drawn, cutting off the part rejected in the abbreviated process. The result in this way is 590·323, or rather 590·324, on account of the following figures, and is less by a thousandth part of a unit, than the result before obtained. The reason of this difference is, that all the partial products in the contracted mode, except the last, happen to be rather too great. If, as in the preceding example, the results which are too great be marked by the sign —, and those that are too small by +, it may enable us in some degree to judge of the accuracy of the result, as we may suppose it to be nearly correct, if the number of the signs of each kind be nearly the same, since the excesses and the defects will then probably balance each other.

```
          7·24651
          81·4632
          ———————
        1|449302
       21|73953
      434|7906
     2898|604
     7246|51
   579720|8
   ———————|———————
  590·323|893432
```

Exam. 4. Multiply ·681472 by ·01286, so that the decimal in the product may contain five figures.

In this example, since the multiplier contains no integer, a cipher is placed below the fifth figure of the multiplicand; and then, the multiplier being written in reversed order, the work proceeds as in the last example.

```
 ·681472
68210·0
———————
   681+
   136+
    55—
     4+
———————
·00876, product.
```

Exam. 5. Multiply 7·94′ by 3·69, so that there may be four places of decimals in the product.

Here the multiplicand is carried out to four places, and by a process similar to those which precede, the answer is found to be 29·315, which is quite correct.

```
 7·9444′
   96·3
———————
 238333
  47666
   7150
———————
29·3150, product.
```

	Exercises.	*Answers.*
13.	1·123674 × 1·123674 to 6 places	=1·262643
14.	7·2′85714′ × 36·74405 to 6 places	=267·706650
15.	24·6′3′ × ·2347 to 6 places	=5·78215′4′
16.	·863541 × ·10983 to 5 places	=·09484
17.	·1347866 × ·288793 to 7 places	=·0389254
18.	·26736 × ·28758 to 4 places	=·0769
19.	2·656419 × 1·7′23′ to 6 places	=4·578932
20.	1·6′5′ × 1·4′8′ to 5 places	=2·45975
21.	·053437 × ·047126 to 6 places	=·002521

22, 23, 24, 25. Required the product, true to six places of decimals, of the numbers given in exercises 4, 6, 14, and 17 in

Multiplication of Fractions; the several fractions being previously reduced to decimals. *Answ.* ·113445, ·467480, 2·393162, and 150·289438.

DIVISION OF DECIMAL FRACTIONS.

RULE. (1.) If the divisor and dividend do not contain the same number of places of decimals, supply the deficiency by annexing ciphers, or in a periodical decimal, the next figures of the period. (2.) Then rejecting the separating points, divide as in whole numbers, and the quotient will be a whole number. (3.) If there be a remainder after all the figures of the dividend have been employed, ciphers or periodical figures may be annexed, till there be no remainder, or till as many figures be found as may be judged necessary. The part of the quotient thus obtained will be a decimal.

If after the rejection of the separating points, the divisor be greater than the dividend, the quotient will contain no whole number, and the work will proceed according to the rule for problem I. in Reduction of Decimals.

When the divisor is large, the work will be shortened, if, instead of annexing a cipher or periodical figure to each remainder, a figure be cut off from the divisor. In this case, each product is to be increased by *carrying* from the product of the figure last cut off, and of the figure last placed in the quotient.

It may facilitate the use of this contraction, if after the rejection of the separating points, so many figures be annexed to the divisor and dividend, or taken from them, that the divisor may contain one, or for greater accuracy, two figures more than are required to be in the quotient. Other directions might be given; but the following examples and illustrations will perhaps be preferable.

Exam. 1. Divide 1346·5 by 43·68.

Here, by annexing a cipher to the dividend, and rejecting the points, we have for divisor 4368, and for dividend 134650. Hence, dividing in the common way, we find 30 for the integral part, and annexing ciphers to the remainders, and continuing the operation, we get ·826465, &c.; and the entire answer is 30·826465, &c. The work is left for the learner to perform.

With respect to the *reason* of the operation, the value of 1346·5 is not changed by the annexing of a cipher; and the removal of the points merely multiplies each of the given numbers by 100.—

(See page 107.*)* It is evident therefore, that the value of the quotient will not be affected, since, while the dividend is multiplied by 100, the divisor is increased in the same ratio. We might also consider the dividend as the numerator, and the divisor as the denominator of a fraction; and then the reason of the process would depend on the first of the principles established in page 88. The reason of removing the points, is to make the dividend and divisor whole numbers, and thus to render the operation as much as possible the same as in Simple Division.

Exam. 2. Divide ·1342 by 67·1.

Here, by annexing three ciphers to the divisor and rejecting the points, we get for divisor 671000, and for dividend 1342. Then the dividend being the greater, the quotient will contain no integral part; and the annexing of a cipher to the dividend, gives one cipher for the quotient: the annexing of a second gives another cipher; but the addition of a third gives 2. Hence the quotient is ·002.

When the number of places of decimals in the divisor is not greater than in the dividend, the number of figures of decimals in the quotient is to be equal to the difference between the number of places in the divisor and dividend, as is evident from Multiplication of Decimals; and in this way the number of decimal figures in the quotient is often very easily determined.

Exam. 3. Divide 2·3748 by 1·4736, so that the quotient may contain three places of decimals.

In this example, the numbers being prepared according to the rule, and the first figure of the quotient being found, instead of adding a cipher to the remainder 9012, we omit the last figure of the divisor, to denote which, a point is placed below it. Then 6 being put in the quotient, we multiply 6, the figure cut off, by it, and without setting any thing down, we carry 4, because the product 36 is nearer 40 than 30. After that, 3 is cut off in like manner, and then 7. The quotient is found to be 1·611, or more nearly 1·612, because the remainder 8 is rather more than the half of 14. The annexed operation at full length, will explain the reason of the contracted process, the vertical line cutting off the rejected part.

```
14736)23748(1·611      14736)23748(1·611
   ··· 14736                14736
       ------                ------
        9012                 9012|0
        8842                 8841|6
        ----                 ----|--
         170                  170|40
         147                  147|36
         ---                  ---|---
          23                   23|040
          15                   14|736
          --                   --|---
           8                    8|304
```

Exam. 4. Divide 73·64 by ·43232, so that the quotient may have four places of decimals.

Here it is easy to see, that the quotient will contain three places of whole numbers.—(This would be seen by dividing 736 by 4.) Hence the quotient must contain seven figures. Extending therefore the divisor to eight places, and the dividend to the same number of places of decimals, the process will stand as in the margin. In the work, instead of bringing down the two last ciphers of the dividend, two figures may be cut in succession from the divisor, and the rest of the operation will proceed as before.

```
43232323)7364000000(170·3355
         43232323
         --------
          30407677
          30262626
          --------
           145051
           129697
           ------
            15354
            12970
            -----
             2384
             2162
             ----
              222
              216
              ---
                6
```

The pupil should work the following exercises by both methods, particularly by the abbreviated process:

Exercises.	*Answers.*	*Exercises.*	*Answers.*
1. 47·58 ÷ 26·175 =	1·81776504	7. 1 ÷ 10·473654.. =	·09547766
2. ·3412 ÷ 8·4736 =	·040266239	8. 7·5 ÷ 37·38..... =	·20064205
3. 468·7′ ÷ 3·365.. =	139·309889	9. 5·09′ ÷ 6·2′..... =	·8′1′
4. ·5′8′ ÷ 77·482... =	·00756122	10. ·625 ÷ ·4′28571′ =	1·4583′
5. 75·347 ÷ ·3829 =	196·779838	11. ·0′9′ ÷ ·2′30769′ =	·3′9′
6. 6·5′ ÷ 7·06249.. =	·928222	12. 2·1′62′ ÷ 3·125.. =	·69′18′

Exercises.	*Answers.*
13. ·079085 ÷ ·83497.....	= ·094716
14. ·6′1′ ÷ 13·543516.....	= ·04549495
15. 23·6 ÷ ·037538.....	= 628·6962545
16. 7·126491 ÷ ·531.....	= 13·420887005
17. ·879454 ÷ ·897.....	= ·98043924
18. 52·7′3′ ÷ 52·734567.....	= 1·000053224
19. 2·3′70′ ÷ 4·9′23076′.....	= ·4′81′

After the full illustration of the multiplication and division of decimals, which has been given in the preceding pages, it appears unnecessary to give their application in the Rule of Proportion; as in thus applying them the pupil can encounter no difficulty, the terms being arranged in the manner already explained, and the product of the second and third terms, in like manner, divided by the first. Should it be thought necessary however, the exercises in page 106 may be wrought in this manner, by reducing the common fractions which they contain, to decimal fractions.

PRACTICE.

An ALIQUOT PART of a number is such a part as, when taken a certain number of times, will exactly make that number. Thus, 5 is an aliquot part of 20, 3 of 12, &c.

What is generally called PRACTICE is only an abridged method of performing operations in the Rule of Proportion by the use of aliquot parts; and is generally employed in calculating the prices of commodities.

TABLES OF ALIQUOT PARTS.

MONEY.

10/0 ..... = £$\frac{1}{2}$	2/0 .... = £$\frac{1}{10}$	4d .... = $\frac{1}{3}$ of 1/0
6/8 ..... = £$\frac{1}{3}$	1/8 .... = £$\frac{1}{12}$	3d .... = $\frac{1}{4}$ of 1/0
5/0 ..... = £$\frac{1}{4}$	1/4 .... = £$\frac{1}{15}$	2d .... = $\frac{1}{6}$ of 1/0
4/0 ..... = £$\frac{1}{5}$	1/3 .... = £$\frac{1}{18}$	1$\frac{1}{3}$d ... = $\frac{1}{8}$ of 1/0
3/4 ..... = £$\frac{1}{6}$	1/0 .... = £$\frac{1}{20}$	1d .... = $\frac{1}{12}$ of 1/0
2/6 ..... = £$\frac{1}{8}$	6d ..... = $\frac{1}{2}$ of 1/0	$\frac{3}{4}$d ... = $\frac{1}{16}$ of 1/0

COMMON WEIGHT.

2 qrs..... = $\frac{1}{2}$ cwt.	14 lbs..... = $\frac{1}{8}$ cwt.	4 lbs.. = $\frac{1}{28}$ cwt.
1 qr..... = $\frac{1}{4}$ cwt.	8 lbs..... = $\frac{1}{14}$ cwt.	14 lbs.. = $\frac{1}{2}$ of a qr.
16 lbs..... = $\frac{1}{7}$ cwt.	7 lbs..... = $\frac{1}{16}$ cwt.	7 lbs.. = $\frac{1}{4}$ of a qr.

LONG WEIGHT.

2 qrs..... = $\frac{1}{2}$ of a cwt.	15 lbs..... = $\frac{1}{8}$ of a cwt.
1 qr..... = $\frac{1}{4}$ of a cwt.	12 lbs..... = $\frac{1}{10}$ of a cwt.
20 lbs..... = $\frac{1}{8}$ of a cwt.	10 lbs..... = $\frac{1}{12}$ of a cwt.

LAND MEASURE.

2 roods..... = $\frac{1}{2}$ acre	20 perches = $\frac{1}{8}$ acre	10 perches = $\frac{1}{4}$ rood
1 rood ..... = $\frac{1}{4}$ acre	16 perches = $\frac{1}{10}$ acre	8 perches = $\frac{1}{5}$ rood

These tables may be constructed by problem VII. page 97; or rather by dividing £1, 1 acre, &c. by 2, 3, 4, &c. and selecting such of the quotients as are free from fractions. The following continuation of them may often be found useful. By its means the pupil may frequently be assisted in discovering what aliquot parts may be most advantageously employed in many of the operations which he may be required to perform. The more obvious and less useful parts are omitted.

MONEY.

2/6..... = { $\frac{1}{4}$ of 10/0; $\frac{1}{2}$ of 5/0 }	1/4... = $\frac{1}{3}$ of 4/0	10d.... = { $\frac{1}{12}$ of 10/0; $\frac{1}{6}$ of 6/8; $\frac{1}{4}$ of 5/0; $\frac{1}{3}$ of 3/4 }
1/8..... = { $\frac{1}{4}$ of 10/0; $\frac{1}{2}$ of 5/0 }	1/3... = { $\frac{1}{8}$ of 10/0; $\frac{1}{4}$ of 5/0; $\frac{1}{2}$ of 2/6 }	

10d. ...= { $\frac{1}{3}$ of 2/6 ; $\frac{1}{2}$ of 1/8 }

8d. ..= { $\frac{1}{6}$ of 4/0 ; $\frac{1}{5}$ of 3/4 ; $\frac{1}{3}$ of 2/0 }

7½d....= { $\frac{1}{8}$ of 5/0 ; $\frac{1}{4}$ of 2/6 ; $\frac{1}{2}$ of 1/3 }

5d....= { $\frac{1}{12}$ of 5/0 ; $\frac{1}{8}$ of 3/4 ; $\frac{1}{6}$ of 2/6 ; $\frac{1}{4}$ of 1/8 }

4d....= { $\frac{1}{12}$ of 4/0 ; $\frac{1}{10}$ of 3/4 ; $\frac{1}{6}$ of 2/0 }

3d....= { $\frac{1}{10}$ of 2/6 ; $\frac{1}{8}$ of 2/0 }

2½d....= { $\frac{1}{12}$ of 2/6 ; $\frac{1}{8}$ of 1/8 ; $\frac{1}{6}$ of 1/3 ; $\frac{1}{4}$ of 10d. }

1½d. ...= { $\frac{1}{10}$ of 1/3 ; $\frac{1}{8}$ of 1/0 ; $\frac{1}{4}$ of 6d. }

¾d....= { $\frac{1}{8}$ of 6d. ; $\frac{1}{4}$ of 3d. }

COMMON WEIGHT.

14 lbs..... = $\frac{1}{4}$ of 2 qrs.

8 lbs..... = { $\frac{1}{7}$ of 2 qrs. ; $\frac{1}{2}$ of 16 lbs. }

7 lbs..... = { $\frac{1}{8}$ of 2 qrs. ; $\frac{1}{2}$ of 14 lbs. }

4 lbs. ..... = { $\frac{1}{7}$ of 1 qr. ; $\frac{1}{4}$ of 16 lbs. }

2 lbs. ..... = { $\frac{1}{8}$ of 16 lbs. ; $\frac{1}{4}$ of 8 lbs. }

In the calculation of prices, the quantity of the commodity may be of *one denomination*, or of *more than one*: and accordingly, the subject divides itself into two branches, with several varieties, as will appear from the following rules and illustrations.

RULE I. In finding the price of a commodity, *when the price of each article*, as well as the quantity, *is of one denomination*, the product of the given price and of the number of articles, will be the price required.

Exam. 1. Required the price of 289 cwt. of beef at £2 ℔ cwt.

Here the price of 289 cwt. at £1 ℔ cwt. is evidently £289; and the price of the same at £2 ℔ cwt. must obviously be twice that amount.

289 cwt. at £2 ℔ cwt.

£289...=price of 289 cwt. at £1 ℔ cwt.
2

£578...=.....£2 .....

Exercises.	*Answ.*	*Exercises.*	*Answ.*
1. 311 at £3	£933	3. 197 at £4	£788
2. 1286 at £5	£6430	4. 309 at £1	£309

RULE II. *When the price is an aliquot part of a higher denomination*, take a like part of the number of articles, and the result will be the price in the higher denomination.

Exam. 2. What cost 532 lbs. of tea at 6/8 ℔ lb.?

532 lbs. at 6/8 ℔ lb.

£532.....=price of 532 lbs. at £1 each

6/8=⅓ of £1...£177 6 8=.....6/8.....

In this example, since 532 articles, at £1 each, would cost £532, it is evident, that at 6/8, the same number of articles would cost one-third of that amount, 6/8 being one-third of £1. We therefore divide £532 by 3, and the quotient, £177 6 8, is the required price.

Exam. 3. Required the price of 537 yards of ribbon at 3d. ℔ yard.

537 yards at 3d. ℔ yard.

537 s.....=price of 537 at 1/0 ℔ yard.

3d.=¼ of 1/0.....134/3.....=..... at 3d. .....

£6 14 3, *Answ.*

Here, the price of 537 yards at 1 shilling would be 537 shillings; and the price of the same at 3d. would evidently be one fourth of that, or 134/3, which by reduction becomes £6 14 3.

Exercises.	*Answ.*	*Exercises.*	*Answ.*
5. 389 at 10/0.....	£194 10 0	7. 739 at 3/4	£123 3 4
6. 538 at 5/.....	£134 10 0	8. 645 at 2/6	£80 12 6

Rule III. *When the price of each article is not an aliquot of a higher denomination,* it is to be divided into such parts, that the price of the whole quantity at each of these prices, may be found by the first or second rule; and the sum of the prices thus obtained will be the whole price required.

Exam. 4. Required the price of 479 cwt. of sugar at £4 9 6 ℔ cwt.

479 cwt. at £4 9 6 ℔ cwt.

£479..... =price of 479 cwt. at £1 ℔ cwt.
4

	£1916.....	=.....	£4 .....
5/0=¼ of £1	119 15	=.....	5/ .....
4/0=⅕ of £1	95 16	=.....	4/ .....
6d.=$\frac{1}{10}$ of 5/0	11 19 6	=.....	6d. .....
Answer,	£2143 10 6	=.....	£4 9 6 .....

In this example, at £1 ℔ cwt. 479 cwt. would cost £479, and hence, to find the price at £4 ℔ cwt. we multiply by 4. Again, since 5/=one-fourth of a pound, and 4/=one-fifth of a pound, the prices at these rates will be found by rule II. by taking the fourth and the fifth of £479. It still remains to find the price at 6d.: now, since 6d. is one-tenth of 5/, the price at 6d. will evidently be one-tenth of the price at 5/, and therefore we take one-

tenth of £119 15 the price at 5/, and the result is £11 19 6, the price of 479 cwt. at 6d. ℔ cwt. Finally, the price of the whole quantity at £4 ℔ cwt. being £1916; at 5/, £119 15; at 4/, £95 16; and at 6d. £11 19 6; the price at £4 9 6 will be £2143 10 6, the sum of these.

Exam. 5. Required the price of 647 yards of linen at 3/9 ℔ yd

647 yards at 3/9 ℔ yard.

£647.....=price of 647 yards at £1 ℔ yard.

2/6=$\frac{1}{8}$ of £1	80 17 6=.....	2/6 .....
1/3=$\frac{1}{2}$ of 2/6	40 8 9=.....	1/3 .....
Answer, £121 6 3=.....		3/9 .....

In this example, the price at £1 ℔ yard is obviously £647, and as 2/6 is one-eighth of a pound, the price at 2/6 will be one-eighth of £647, or £80 17 6. Now, 2/6 with 1/3 make up the given price 3/9; and 1/3 being one-half of 2/6, the price at 1/3 will be one-half of the price at 2/6, and hence we take the half of £80 17 6. Then the price at 2/6 being £80 17 6, and that at 1/3 being £40 8 9, the price at 2/6 and 1/3, or at 3/9, will be £121 6 3, the sum of these.—This exercise might be wrought with nearly equal facility by finding the prices at 3/4 and 5d. and taking their sum.

Exam. 6. Required the price of 247 cwt. of flour at £1 5 ℔ c.

247 cwt. at £1 5 ℔ cwt.

£247.....=price at £1 ℔ cwt.

5/0=$\frac{1}{4}$ of £1...	61 15 = ...	5/ .....
Answer, £308 15 = ...		£1 5 .....

This and the preceding examples have been wrought at full length for showing the reason of the operations, and thus the pupil should be accustomed to work similar questions, till he thoroughly understand the reason of the process. Afterwards, however, the work may be properly contracted as in the margin, which is the mode commonly employed.

247 cwt. at £1 5 ℔ cwt.
5/=£$\frac{1}{4}$... 61 15

£308 15, *Answer*.

Exam. 7. Required the price of 195 lbs. of raisins at 1/3 ℔ lb.

195 lbs. at 1/3 ℔ lb.

1/3=$\frac{1}{16}$ of £1... { 4)£195.....=price at £1 ℔ lb.
4)48 15

Answer, £12 3 9=.....1/3 .....

Or thus, 195 lbs. at 1/3 per lb.

195s....= price at 1/0 per lb.
3d.=¼ of 1/0...48 9 = ... 0/3 .....

2|0)24|3 9 = ... 1/3 .....

£12 3 9, *Answer*, as before.

In the first of the preceding operations, since 1/3 is one-sixteenth of a pound, we divide £195, the price at a pound sterling per pound, by 4, and the result again by 4. In the second method, the price at one shilling being 195 shillings, and the price at 3d. one-fourth of that amount, or 48/9, the sum of both is 243/9, the price required, which by reduction becomes £12 3 9, the same as before. This operation may also stand as in the margin.

195 lbs. at 1/3 per lb.
3d.=¼ of 1/0...48 9

2|0)24|3 9

£12 3 9, *Answer*.

Example 8. What cost 1257 yards of ribbon at 6¾d. per yard?

1257 yards at 6¾d. per yard.

6d.=½ of 1/0	628	6	= price at	6d. per yard.
¾d.=⅛ of 6d.	78	6¾	=	¾d. .....
	2|0)70|7	0¾	=	6¾d. .....

£35 7 0¾, *Answer*.

Exam. 9. Required the price of 347 cwt. of coffee at £7 11 6¾ per cwt.

347 cwt. at £7 11 6¾ per cwt.
7

	£2429	.....	=	price at	£7	0	0	per cwt.
10/=½ of £1	173	10	0	=	0	10	0	.....
1/3=⅛ of 10/	21	13	9	=	0	1	3	.....
0/3=⅕ of 1/3	4	6	9	=	0	0	3	.....
¾d.=¼ of 0/3	1	1	8¼	=	0	0	0¾	.....
				=				
	£2629	12	2¼	=	£7	11	6¾	.....

Or thus, 347 cwt. at £7 11 6¾ per cwt.
7

	£2429			= price at	£7	0	0	per cwt.
10/ = ½ of £1	173	10	0	=	0	10	0	
1/ = $\frac{1}{10}$ of 10/	17	7	0	=	0	1	0	
0/6 = ½ of 1/0	8	13	6	=	0	0	6	
¾d. = ⅛ of 6d.	1	1	8¼	=	0	0	0¾	
Answ.	£2629	12	2¼	=	£7	11	6¾	

Exam. 10. If a tradesman have 3/9½ per day, how much is his yearly salary, the days of labour in the year being 313?

313 days at 3/9½ per day.

3/4 = ⅙ of £1	£52	3	4	= amount at	3/4	per day.
5d. = ⅛ of 3/4	6	10	5	=	0/5	
½d. = $\frac{1}{10}$ of 5d.	0	13	0½	=	0/0½	
Answ.	£59	6	9½	=	3/9½	

This exercise might have been wrought by distributing 3/9½ into the parts, 2/6, 1/3, and ½d.; but those employed above are much preferable, as, in the other case, ½d. being one-thirtieth of 1/3, we must have divided by a number inconveniently large. Unless, indeed, in particular circumstances, we should, if possible, avoid taking parts that would require us to divide by any number greater than 12. The seventh example affords an instance in which no inconvenience or difficulty arises from employing a larger divisor; and every person's experience will point out others.

In taking aliquot parts, it sometimes shortens the work to take the same part twice, as the result of the first operation may be copied without working for it again. Thus, 18/6 may be distributed into 10/, 4/, 4/, and 6d. Sometimes also the price at a small rate may be found, and from it the price of a greater may be obtained by multiplication. Thus, 16/4 may be divided into 2/, 14/, and 4d. the price at 14/ being 7 times the price at 2/. In like manner, for 17/1 we may take 1/8, 15/, and 5d. Other remarks on this subject will be found at the end of this article.

		Exercises.				*Answers.*		
		£	*s.*	*d.*		£	*s.*	*d.*
9.	1625 at	0	2	8½		220	1	0½
10.	1429	1	2	9		1625	9	9
11.	1973	0	6	10		674	2	2
12.	749	0	5	8		212	4	4
13.	1689	0	4	10½		411	13	10½
14.	2476	0	18	6		2290	6	0
15.	5926	0	11	8		3456	16	8
16.	313	0	8	8		135	12	8
17.	5934	1	5	10		7664	15	0
18.	3576	0	11	4½		2033	17	0
19.	958	1	18	8		1852	2	8

	Exercises.	£	s.	d.		Answers. £	s.	d.
20.	1898 at	0	6	10½	.....	652	8	9
21.	1594	0	13	6	.....	1075	19	0
22.	695	0	14	10	.....	515	9	2
23.	2386	1	6	6	.....	3161	9	0
24.	725	1	7	8	.....	1002	18	4
25.	589	1	11	6	.....	927	13	6
26.	286	0	12	1	.....	172	15	10
27.	7649	0	0	5¾	.....	183	5	1¾
28.	5728	0	0	7½	.....	173	0	8
29.	6491	0	0	10¾	.....	290	14	10¼
30.	991	0	1	3½	.....	64	0	0½
31.	436	4	17	8	.....	2129	2	8
32.	3725	0	5	8¼	.....	1059	5	11¼
33.	1677	0	5	1	.....	426	4	9
34.	7913	2	16	10½	.....	22502	11	10½
35.	4265	1	14	1½	.....	7277	3	1½
36.	249	5	13	9	.....	1416	3	9
37.	576	0	18	11½	.....	546	0	0
38.	6485	2	10	2½	.....	16280	1	0½

RULE IV. *The price of any number of articles at 2 shillings each*, is found by doubling the last figure of the number for shillings, and taking the number expressed by the preceding figures as pounds.

Exam. 11. What cost 647 yards of muslin, at 2 shillings ℔ yard?

The reason of this rule is, that 2 shillings are one-tenth of a pound, and the work is no more than a contracted division by ten. Thus, in the annexed example, by dividing by 10, we should have £64, with the fraction $\frac{7}{10}$, or, by doubling both the terms, $\frac{14}{20}$, or 14 shillings.

647 yards at 2/
———
£64 14 0, *Answ.*

RULE V. *If the rate be an even number of shillings*, multiply by half that number, and in multiplying, double the last figure of the product for shillings; the rest will be pounds.

Exam. 12. Required the price of 273 lbs. of indigo, at 8 shillings ℔ lb.

This rule and operation are evidently nothing more than an extension of the last. In the annexed example, the price is $\frac{8}{20}$ or $\frac{4}{10}$ of a pound; and hence we multiply by 4, and the doubling of the last figure of the product for shillings, is equivalent to a division by 10.

273 lbs. at 8/ ℔ lb.
4
———
£109 4 0, *Answ.*

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RULE VI. *When the number of shillings is odd,* we may find, by the last rule, the amount at one shilling less than the given rate, and for that shilling take $\frac{1}{20}$ of the price at one pound.

Exam. 13. What cost 787 lbs. of nutmegs, at 17 shillings pound?

In this example, the price is first found at 16/ ℔ pound, by multiplying by 8, and doubling the last figure of the product for shillings; then, for the remaining shilling, a twentieth part of £787, the price at £1 per lb. is taken; and the sum of both results is £668 19, the price at 17 shillings.

787 lbs. at 17/ ℔ lb.
8
£629 12=price at 16/ ℔ lb.
1/=£$\frac{1}{20}$ 39 7=..... 1/ .....
£668 19=.....17/ .....

Exam. 14. Required the amount of 233 cwt. of pearl ashes, at £3 9 per cwt.

In this example, we first find the price at £3 by multiplying by 3; then the price at 8/ by multiplying by 4, and doubling the last figure of the product for shillings; and lastly, the price at 1/, by taking a twentieth part of £233, the price at £1 per cwt.—It is evident, that we might have taken one eighth of the price at eight shillings, for the price at one shilling.

233 cwt. at £3 9 per cwt.
3 4
699 =price at £3 per lb.
93 4=..... 8/ .....
1/=£$\frac{1}{20}$...11 13=..... 1/ .....
Answer, £803 17=.....£3 9 .....

Exercises.	*Answers.* £	s.	d.	*Exercises.*	*Answers.* £	s.	d.
39. 397 at 2/	39	14	0	46. 675 at 13/	438	15	0
40. 418 3/	62	14	0	47. 512 14/	358	8	0
41. 763 6/	228	18	0	48. 319 16/	255	4	0
42. 378 7/	132	6	0	49. 868 17/	737	16	0
43. 841 8/	336	8	0	50. 470 18/	423	0	0
44. 937 9/	421	13	0	51. 983 £3 4...	3145	12	0
45. 949 12/	569	8	0	52. 724 £5 13..	4090	12	0

RULE VII. The price may often be determined very easily, *by finding the amount at a rate higher than the given rate, and deducting from the amount the price at the difference between the given and the assumed rates.*

This method is generally of little use, unless the difference between the given and the higher rate be an aliquot part of the higher. This difference may be called the COMPLEMENT of the given rate.

Exam. 15. What cost 189 tons of coals at 17/6 per ton?

189 tons at 17/6 per ton.

£189..... =price at £1 per ton.
2/6=£$\frac{1}{8}$ 23 12 6= .....2/6 .....

Answ. £165 7 6=..... 17/6 .....

Or simply thus:
189 at 17/6
2/6=£$\frac{1}{8}$...23 12 6

£165 7 6

In this example, since 17/6 is less than a pound by 2/6, which is an aliquot part of a pound, the required price is found by taking from £189, the price at £1 per ton, £23 12 6, the price at 2/6 per ton.

Exam. 16. What cost 257 feet of plank, at 10d. per foot?

Here, from 257 shillings, the price at 1/ per foot, we take 42/10, the price at 2d. per foot: the remainder is 214/2, or by reduction £10 14 2, the price at 10d. per foot.

257 feet at 10d. ℔ foot.
2d.=$\frac{1}{6}$ of 1/... 42 10

20)214 2

£10 14 2, *Answer.*

Exam. 17. What cost 514 gallons of seal oil at 3/11 ℔ gallon?

514 gallons, at 3/11 ℔ gallon.
4

2056/.....=price at 4/ ℔ gallon.
1d.=$\frac{1}{12}$ of 1/... 42 10...=..... 1d. .....

2|0)201|3 2...=..... 3/11.....

£100 13 2, *Answer.*

Exam. 18. Required the price of 193 cwt. of Turkey figs, at £3 18 ℔ cwt.

In this example, the price at 2/ ℔ cwt. (found by doubling the last figure, &c.) is taken from the price at £4.

193 cwt. at £3 18 ℔ cwt.
4

£772.....=price at £4 0 ℔ cwt.
19 6...=..... 0 2 .....

£752 14...=..... 3 18 .....

	Exercises.	£	s.	d.		*Answers.* £	s.	d.
53.	358 at	0	13	4	.....	238	13	4
54.	599	0	0	10½	.....	26	4	1½
55.	967	0	0	8	.....	32	4	8
56.	275	2	15	0	.....	756	5	0

	Exercises.	£	s.	d.		*Answers.* £	s.	d.
57.	361 at	3	12		.....	1299	12	0
58.	889	0	0	9	.....	33	6	9
59.	483	5	16	8	.....	2817	10	0
60.	189	0	4	7	.....	43	6	3
61.	997	0	0	11	.....	45	13	11
62.	753	0	1	9	.....	65	17	9
63.	649	0	1	5½	.....	47	6	5½
64.	721	0	19	0	.....	684	19	0

We now proceed to problems of the second class; and in resolving them, we may employ either of the two following general rules.

RULE VIII. *When the quantity is not expressed by a whole number of one denomination,* find the price of the integral quantity according to the method already illustrated, and then find the price of the fractional parts, or lower denominations, from the given rate, by means of aliquot parts, or otherwise; the sum of all will be the whole price required. Or,

RULE IX. *When the quantity is not expressed by a whole number of one denomination,* find the price of the entire given quantity at £1† for each unit of the integral part, valuing the subordinate parts at the same rate; and then the work will proceed in the manner already explained, without the necessity of farther work for the subordinate parts.

Exam. 19. Required the price of 79¾ yards of broadcloth, at £1 2 11 $\frac{3}{4}$ yard.

By Rule VIII.

79¾ yards, at £1 2 11.

2/6=£⅛	£9	17	6	=	price at 2/6
5d.=⅛ of 2/6	1	12	11	=	..... 5d.
½ of £1 2 11	0	11	5½	=	..... of ½ yard
½ of 11/5½	0	5	8¾	=	..... ¼ .....
	£91	7	7½,		*Answer.*

* £3 12 is the difference between £4 and 8 shillings, and 8/=one-tenth of £4.—Exercise 62 will be wrought by finding the price at two shillings, and then proceeding in the usual manner; and the answers of exercises 60 and 63 may be derived from the prices at 5/0 and 1/8.

† This rule is restricted to *pounds*, which is almost the only useful or necessary application of the principle. The principle however, is universal in its nature, and might be extended to any other rate in a similar manner.

With respect to the eighth and ninth rules, it may be observed, that the ninth (of which rules X. XI. XII. and XIII. are particular applications,) is very elegant, on ac

In this mode of resolving this exercise, the price of 79 yards is first found, (or rather the parts of which it is made up are found;) and then for $\frac{1}{2}$ yard the half of £1 2 11 is taken, and for $\frac{1}{4}$ yard the half of that is taken: then the sum of all these parts is £91 7 $7\frac{1}{4}$, the result required.

By Rule IX.

79¾ yards at £1 2 11.

	£79	15	0	=price at £1 ℔ yard.
2/6=£⅛	9	19	$4\frac{1}{2}$	=..... 2/6.....
5d.=⅛ of 2/6	1	13	$2\frac{3}{4}$	=..... 5d.....
	£91	7	$7\frac{1}{4}$	=.....£1 2 11.....

In this method we first find the price of 79¾ yards at £1 ℔ yard. This is obviously £79 15; for the price of 79 yards is £79, and the price of a quarter of a yard being evidently 5/0, the price of ¾ of a yard is 15/. Then, the price at £1 ℔ yard being £79 15, the price at 2/6 ℔ yard will be one-eighth of this, or £9 19 $4\frac{1}{2}$; and the price at 5d. one-sixth of the price at 2/6, or £1 13 $2\frac{3}{4}$. The sum of these prices is £91 7 $7\frac{1}{4}$, the whole price, as before.

Exam. 20. What cost 69⅝ yards of cambric, at 13/10 ℔ yard?

By Rule VIII.

69⅝ at 13/10

10/=£½	£34	10	0
3/4=⅓ of 10/	11	10	0
6d.=$\frac{1}{20}$ of 10/	1	14	6
⅝ yd.=½ yd.	0	6	11
⅛ yd.=¼ of ⅝ yd.	0	1	$8\frac{3}{4}$
Answer,	£48	3	$1\frac{3}{4}$.

Here, after finding, in the way already illustrated, the parts whose sum will be the price of 69 yards at 13/10 ℔ yard, we take for ⅝ of a yard the half of 13/10, the price of one yard, and for the remaining eighth we take ¼ of 6/11, and the result is 1/$8\frac{3}{4}$, the price of ⅛ of a yard. The sum of all these partial prices is £48 3 $1\frac{3}{4}$, the whole price required.

By Rule IX.

69⅝ yards at 13/10 ℔ yard.

	£69	12	6	=price at £1 ℔ yard.
10/=£½.....	34	16	3	=..... 10/ .....
3/4=⅓ of 10/...	11	12	1	=..... 3/4 .....
6d.=$\frac{1}{20}$ of 10/	1	14	$9\frac{3}{4}$	=..... 0/6 .....
	£48	3	$1\frac{3}{4}$	=.....13/10.....

count of the principle on which it depends; and in many cases it affords the neatest and most concise mode of resolving problems. In some cases, however, it gives origin

In this method, since, at £1 ℔ yard, one-eighth of a yard would cost 2/6, five-eighths would cost 5 times 2/6, or 12/6; and consequently, at that rate, 69⅝ yards would cost £69 12 6. The rest of the work proceeds in the usual manner.—With respect to both modes, it may be observed, that on account of the large divisor, 20, the work would have perhaps been as short, had we taken 10/, 3/4, 5d. and 1d. though it would appear to be a line longer.

	Exercises.	£	s.	d.		*Answers.* £	s.	d.
65.	328¾ at	0	6	6	.....	106	16	10½
66.	675½	2	14	4	.....	1835	2	2
67.	7538¾	0	2	4	.....	879	10	5
68.	176c. 3q.	0	18	8	.....	164	19	4
69.	164⅞	2	5	6	.....	375	1	9¾
70.	239a. 3r.	0	10	10	.....	129	17	3½
71.	172 7/10	3	15	10	.....	654	16	5
72.	257 11/12	2	9	4	.....	636	3	10½

RULE X. *In calculating the prices of hundreds, quarters, and pounds, common weight,* multiply the pounds by 2¼, and consider the product as pence; and multiply the quarters by 5, and consider the product as shillings: the rest of the operation will be performed by rule IX.

The reason of this rule is evident, since at £1 ℔ cwt. each quarter would evidently cost 5/, and each pound a twenty-eighth of this, or 2¼ pence.

Exam. 21. Required the price of 198 c. 2 qrs. 21 lbs. of cast steel, at £4 8 8 ℔ cwt.

By Rule VIII.

	c.	q.	lbs.	
	198	2	21	at £4 8 8 per cwt.
	4			
	£792			..... =price of 198 cwt. at £4 ℔ cwt.
8/=1/10 of £4	79	4	0	=..... 8/ .....
8d.=1/12 of 8/	6	12	0	=..... 8d. .....
2 qr.=½ of 1 c.	2	4	4	=price of 2 qrs. at £4 8 8 ℔ cwt.
14lbs.=½ of 2 qr.	0	11	1	=..... 14 lbs.....
7lbs.=½ of 14lb.	0	5	6½	=..... 7 lbs.....
	£880	16	11½	=price required.

fractions, which, when perfect accuracy is required, render the operation tedious in a considerable degree. In such cases the eighth rule is sometimes not so objectionable.—For mercantile purposes, however, as it is unnecessary to work for the precise fractions, the ninth rule, when managed as will be shown in the succeeding rules abovementioned, will perhaps be found superior in all the principal and more difficult calculations. Let the teacher and calculator judge, however, and employ whichever they may consider preferable.

By Rule X.

c.	q.	lbs.	
198	2	21	at £4 8 8 ℔ cwt.
	5	2¼	

£198 13 9 = price of 198 c. 2 q. 21 lbs. at £1 0 0 ℔ cwt.
4

	79¼	15	0	=	4	0	0
1/10	79	9	6	=	0	8	0
1/12	6	12	5½	=	0	0	8
An.	880	16	11½	=	£4	8	8

The first mode of working this exercise is sufficiently explained in the operation itself. In the second mode, we say, one-seventh of 21 is 3, and twice 21=42, and 3=45 pence, or 3/9, which is the value of the pounds at £1 ℔ cwt. We then set down 9d. and carry 3 to 5 times 2, or 10 shillings, the price of 2 quarters; and we thus find the price of 2 q. 21 lbs. at £1 ℔ cwt. to be 13/9, to which £198, the price of 198 cwt. at the same rate, is prefixed. The remaining part of the operation proceeds in the usual manner.

	Exercises.							*Answers.*		
	cwt.	*q.*	*lbs.*		£	*s.*	*d.*	£	*s.*	*d.*
73.	75	3	21	at	4	14	6	358	16	1
74.	285	3	7		3	17	8	1109	18	1¼
75.	117	1	7		0	17	4	101	13	5
76.	84	3	14		12	11	8	1068	0	2½
77.	134	1	21		0	18	4	123	4	8¼
78.	836	2	21		2	8	4	2021	19	10¾
79.	812	3	7		6	12	8	5391	13	1½
80.	176	1	14		2	16	10	501	3	11¾

Exam. 22. What cost 319 c. 3 q. 16 lbs. of glue, at £2 12 6 ℔ cwt.?

By Rule VIII.

c.	q.	lbs.	
319	3	16	at £2 12 6 per cwt.
2			

	£638	0	0	= price of 319 cwt. at £2 per cwt.		
10/=£½.....	159	10	0	=	10/	
2/6=¼ of 10/	39	17	6	=	2/6	
2 q.=½ of 1 c.	1	6	3	= price of 2 qrs. at £2 12 6 per cwt.		
1 q.=½ of 2 q.	0	13	1½	=	1 qr.	
16lb.=¼ of 1 c.	0	7	6	=	16 lbs.	

Answer, £839 14 4½, the price required.

By Rule IX.

	c.	q.	lbs.	
	319	3	16	at £2 12 6 per cwt.
		5	$2\frac{1}{7}$	
	£319	17	$10\frac{3}{7}$	=price at £1 per cwt.
			2	
	£639	15	$8\frac{4}{7}$	=.....£2.....
10/=£$\frac{1}{2}$	159	18	$11\frac{1}{7}$	=..... 10/.....
2/6=$\frac{1}{4}$ of 10/	39	19	$8\frac{11}{14}$	=..... 2/6.....
Answer,	£839	14	$4\frac{1}{2}$	=.....£2 12 6.....

The latter mode of performing this example gives origin to fractions, which render the operation more tedious. This may be obviated by conducting the operation on the same principle, but converting the fractions into decimals, which it will be found to be sufficient to carry out to two places each. Thus, the work may be as follows:

	c.	q.	lbs.	
	319	3	16	at £2 12 6.
		5	$2\frac{1}{7}$	
	£319	17	10·29	
			2	
	639	15	8·58	
10/=£$\frac{1}{2}$	159	18	11·14	
2/6=$\frac{1}{4}$ of 10/	39	19	8·78	

Answer, £839 14 4·50, or £839 14 $4\frac{1}{2}$, as before.

Here, in taking $\frac{1}{4}$ of 16, we have 2 to carry, and 2 remaining; then, conceiving a cipher annexed to the remainder, and dividing 20 by 7, we set down the quotient 2, and conceiving a cipher annexed to the remainder 6, we have 7 contained most nearly 9 times in 60. We then proceed in the same manner as before, and find for the price at £1 per cwt. £319 17 10·29 nearly. After this the work proceeds exactly as before, except that in each line the pence and the decimal are multiplied and divided as if they were a single whole number, the point alone being preserved.—Thus, in finding the price at 2/6, after having found £39 19, we have 2/11, or 35d. remaining; we then divide 35·14 by 4, as if it were all one number, and find for the quotient 878, or, the point being placed before the two last figures, 8·78. The final result is £839 14 4·50, or £839 14 $4\frac{1}{2}$, the same as by the other processes.

In working by this method, it should be recollected, that ·25*d.* is a *farthing*; ·50*d.* a *halfpenny*; and ·75*d.* three *farthings*: and in valuing the

decimal found in the answer, the pupil should consider to which of these it is nearest, and value it accordingly.

It may be observed, that in consequence of the method of managing the decimals in all operations of this kind being uniformly the same, *any pupil may practise this method, whether he have studied decimal fractions or not.*

The following exercise is wrought several ways for the purpose of showing their comparative advantages and disadvantages, and of affording to the pupil an additional example of the mode of performing calculations of this kind.

Exam. 23. What cost 212 c. 3 q. 19 lbs. of barilla, at £1 13 2 ℔ cwt.?

By Rule VIII.

	c.	q.	lbs.
	212	3	19 at &c.
10/=£½.....	106	0	0
2/=£$\frac{1}{10}$.....	21	4	0
1/=½ of 2/....	10	12	0
2d.=⅙ of 1/....	1	15	4
2q.=½ of 1c....	0	16	7
1q.=½ of 2q....	0	8	3½
16lb.=¼ of 1c....	0	4	8$\frac{4}{7}$
2lb.=⅛ of 16lb.	0	0	7$\frac{3}{28}$
1lb.=½ of 2lb.	0	0	3$\frac{11}{56}$
	£353	1	10$\frac{1}{56}$.

Or thus:

	c.	q.	lbs.
	212	3	19 at &c.
10/=£½.....	106	0	0
2/=£$\frac{1}{10}$.....	21	4	0
1/=½ of 2/....	10	12	0
2d.=⅙ of 1/....	1	15	4
2q.=½ of 1c....	0	16	7
1q.=½ of 2q....	0	8	3·50
16lb.=¼ of 1c....	0	4	8·86
2lb.=⅛ of 16lb.	0	0	7·11
1lb.=½ of 2lb.	0	0	3·55

Answer, £353 1 10·02, or
£353 1 10, nearly.

By Rule X.

	c.	q.	lbs.
	212	3	19 at &c.
		5	2$\frac{1}{7}$
	£212	18	4$\frac{5}{7}$
10/=£½....	106	9	2$\frac{5}{14}$
2/=£$\frac{1}{10}$...	21	5	10$\frac{1}{14}$
1/=½ of 2/	10	12	11$\frac{1}{28}$
2d.=⅙ of 1/	1	15	5$\frac{47}{56}$
Answer,	£353	1	10$\frac{1}{56}$

Or thus:

	c.	q.	lbs.
	212	3	19 at &c.
		5	2$\frac{1}{7}$
	£212	18	4·71
10/=£½....	106	9	2·35
2/=£$\frac{1}{10}$...	21	5	10·07
1/=½ of 2/	10	12	11·03
2d.=⅙ of 1/	1	15	5·84
Answer,	£353	1	10·00

Exercises.

	c.	q.	lbs.	£	s.	d.		Answers. £	s.	d.
81.	75	1	16 at	1	9	9	.....	112	2	11½
82.	538	2	17	0	10	4	.....	278	6	0¾
83.	346	1	4	1	12	7	.....	564	3	1¾
84.	786	2	8	0	18	9	.....	737	8	2½
85.	647	2	11	6	10	8	.....	4230	19	6
86.	238	0	3	3	19	7½	.....	947	12	10½
87.	181	3	13	2	13	4	.....	484	19	6¼

	Exercises.								*Answers.*		
	c.	q.	lbs.	£	s.	d.			£	s.	d.
88.	251	2	1	at 1	17	1	.....		466	6	9½
89.	103	0	27	5	14	10	.....		592	15	6¼
90.	418	2	17	2	0	8	.....		851	5	2
91.	179	3	25	3	10	3	.....		632	3	1½
92.	246	3	24	3	5	4	.....		806	15	0
93.	319	1	9	0	18	7½	.....		297	7	6¼
94.	90	2	10	5	2	4½	.....		463	14	1

RULE XI. *In computing prices in long weight*, multiply the pounds by 2 for pence, and the quarters by 5 for shillings; and perform the rest of the work by rule IX.

The *reason of this rule* is evident, since at £1 $\frac{20}{20}$ cwt. long weight, each quarter would cost 5/, and each pound one-thirtieth of this, or 2d.

Exam. 24. Required the price of 216 c. 2 q. 17 lbs. *(long weight)* of beef, at £1 17 3 $\frac{20}{20}$ cwt.

By Rule VIII.

	c.	q.	lbs.	
	218	2	17	at &c.
10/=£½.....	109	0	0	
5/=½ of 10/..	54	10	0	
2/=£$\frac{1}{10}$.....	21	16	0	
3d.=⅛ of 2/...	2	14	4	
2q.=⅛ cwt.....	0	18	7½	
12lb.=⅕ of 2q...	0	3	8$\frac{7}{10}$	
5lb.=$\frac{1}{12}$ of 2q...	0	1	6$\frac{1}{2}$	
Answer,	£407	4	4$\frac{13}{40}$	

By Rule XI.

	c.	q.	lbs.	
	218	2	17	at &c.
		5	2	
	£218	12	10	
10/=£½.....	109	6	5	
5/=½ of 10/...	54	13	2·50	
2/=$\frac{1}{10}$ of £1.	21	17	3·40	
3d.=⅛ of 2/...	2	14	7·92	
	£407	4	4·82, or	
	407	4	4$\frac{13}{25}$, nearly	

	Exercises in long weight.							*Answers.*		
	c.	q.	lbs.	£	s.	d.		£	s.	d.
95.	281	3	3	at 1	13	7½	.....	473	14	8¼
96.	86	3	17	1	14	1½	.....	148	5	2¼
97.	598	3	13	2	12	6	.....	1572	0	0¾
98.	238	2	19	1	16	6	.....	435	11	0¼
99.	216	0	2	1	13	10	.....	365	8	6¾
100.	411	1	28	0	12	3½	.....	252	17	9¾
101.	384	0	17	1	0	8	.....	396	18	11¼
102.	277	2	23	3	8	3	.....	947	12	5⅛

RULE XII. *In computing the values of acres, roods, and perches*, multiply the perches by 1½ for pence, and the roods by 5 for shillings, and the rest of the work will proceed according to rule IX.

James Keating Book

The *reason of this rule* is also evident, since at £1 ℔ acre, each rood would be worth 5/, and each perch worth one-fortieth of this, or 1½d.

Exam. 25. What is the yearly rent of 136 a. 3 r. 29 p. of land, at £1 4 3 ℔ acre?

The work by rule VIII. in this exercise and some others, is left for the pupil to perform, if it be thought necessary. The exact answer to this question is £166 0 6$\frac{159}{160}$, as would be found by employing common fractions instead of decimals.

	A.	R.	P.	
	136	3	29	at £1 4 3.
		5	1½	
	£136	18	7·50	
4/=£⅕.....	27	7	8·70	
3d.=$\frac{1}{16}$ of 4/	1	14	2·79	
Answer,	£166	0	6·99, or	
	166	0	7, nearly.	

Exercises. *Answers.*

	a.	r.	p.		£	s.	d.		£	s.	d.
103.	561	2	20	at	1	17	6	.....	1053	0	11¼
104.	45	2	35		0	16	6	.....	37	14	4¼
105.	77	1	30		0	12	0	.....	46	9	3
106.	586	1	31		1	7	3½	.....	800	5	0¼
107.	674	2	29		0	11	4½	.....	383	14	6
108.	311	2	26		1	2	9	.....	354	10	3¾
109.	1268	3	17		1	6	6	.....	1681	4	8¼
110.	139	3	39		2	7	10	.....	334	16	4½

RULE XIII. *In calculating the price of tons, hundreds, and quarters, at a given rate per ton,* take the tons and hundreds as pounds and shillings, and multiply the quarters by 3 for pence: and *in calculations in troy weight, at a given rate per ounce,* take the ounces as pounds, the penny weights as shillings, and half the grains as pence.

The reason of this rule depends on the principle, that at £1 ℔ ton, every hundred would cost a shilling, and every quarter 3d.; and that at £1 ℔ ounce, every penny weight would cost a shilling, and every grain a halfpenny.

Exam. 26. Required the price of 98 tons, 14 c. 3 q. of bark, at £9 14 ℔ ton.

	t.	c.	q.	
	98	14	3	at £9 14 ℔ ton.
			3	
	£98	14	9	=price at £1 ℔ ton.
			9	
	888	12	9	
10/=£½	49	7	4½	
4/=£⅕	19	14	11¼,	nearly.
	£957	15	0¾,	nearly, *Answer.*

Thorn

Ambrose

Exam. 27. Required the price of a set of silver articles weighing 216 oz. 14 dwts. 18 grains, at 9/2 ₩ oz.

	oz.	dwt.	grs.	
	216	14	18	at 9/2 ₩ oz.
			½	
£216	14	9		price at £1 ₩ oz.
6/8=£⅓	72	4	11	
2/6=£⅛	27	1	10½	
£99	6	9⅛,		*Answer.*

	Exercises.						*Answers.*		
111.	175 t.	18 c.	1 q.	at £38	13	0	6799	0	4½
112.	219 t.	16 c.	3 q.	11	7	6	2500	13	0¼
113.	98 oz.	7dwt.	15 gr.	0	10	4	48	4	11¼
114.	263 oz.	16dwt.	9 gr.	0	11	3	148	7	11½

The preceding are the most useful applications of the principle on which the ninth and the succeeding rules are founded. Various others might be given, which however are omitted, as they are of less importance, and as the pupil who is well acquainted with those already explained, will find it easy to apply the same principles in other cases.

RULE XIV. In many calculations, instead of multiplying the quantity by the price, it is better to *multiply the price by the quantity.* This is often the case, when Compound Multiplication can conveniently be employed.

Exam. 28. Required the price of 12c. 3q. 8 lbs. of hops, at £23 18 6 ₩ cwt.

	c.	q.	lbs.		£.	s.	d.		
	12	¾	8	at	23	18	6	₩ cwt.	
							12		
				£287	2	0		=price of 12 cwt.	
2 qrs.=½ c.				11	19	3		=	2 qrs.
1 qr.=½ of 2 q.				5	19	7½		=	1 qr.
8 lbs.=¼ of 2 q.				1	14	2¼		=	8 lbs.
Answer,				£306	15	0 8/14		=	12c. 3q. 8 lbs.

	Exercises.	£	s.	d.		*Answers.* £	s.	d.
115.	8 c. 2 qr. 12 lbs. at	1	15	9		15	7	8½
116.	8 a. 3 r. 19 p.	0	18	10		8	7	0¼
117.	9 tons, 13 cwt.	5	19	8		57	14	9½
118.	12 cwt. 1 q. 10 lbs. *(long weight)*...at	2	10	1½		30	18	2½
119.	11 a. 1 r. 23 p.	1	3	7½		13	9	2¼
120.	1 yd. 3 q. 2 nails	1	9	3		2	14	10½

Reading

121. The quantity of rum imported into Great Britain from Jamaica in 1811, was 4,604,771 gallons. Required the amount of the prime cost at 3/9½, and of the importation duty at 10/10 ℔ gallon. Required also the drawback, (or sum refunded on exportation,) at 7/7 ℔ gallon, on one-third of the same quantity. *Answ.* £872987 16 8½; £2494250 19 2; and £581991 17 9½.

122. The quantity of wool exported from Ireland in 1806, was 4337 c. 3 q. 8 lbs.: what was the value at £3 19 7½ ℔ cwt.? *Answ.* £17269 19 0¼.

123. What is the importation duty on 359 gallons of port wine, at 7/7 ℔ gallon? *Answ.* £136 2 5.

124. What is the duty on 710 gallons of Madeira wine, at 7/8¼ ℔ gallon? *Answ.* £273 12 11.

125. Required the amount of the duty on 124 gallons of French wine, at 11/4½ ℔ gallon. *Answ.* £70 10 6.

126. What is the duty on a cwt. of opium, at 8/9 ℔ lb.? *Answ* £49.

127. Required the duty on 47 c. 0 q. 19 lbs. of mother-of-pearl shells, at £4 13 4 ℔ cwt.; and on 197 lbs. of tortoise shell, at 3/11½ ℔ lb. *Answ.* £220 2 6, and £38 19 9½.

128. What is the duty on 517 lbs. of East India coffee, at £5 2 8 ℔ cwt.? *Answ.* £23 13 11.

129. Required the duty on 179 c. 2. q. 12 lbs. of Muscatel raisins, at £2 3 6½ ℔ cwt. *Answ.* £391 0 4¾.

130. In 1800, the quantity of foreign calicoes and muslins printed in England and Wales, was 1,577,536 yards; of British calicoes and muslins, 28,692,790 yards; and of linens and stuffs, 3,232,073 yards. Required the amount of the duty paid on each quantity, the first at 7d. and the second and third at 3½ ℔ yard. *Answ.* £46011 9 4; £418436 10 5; and £47134 7 11½.

131. The quantity of tea imported by the East India Company, in 1791, was 16,299,854 lbs. the average prime cost of which in India was 1/6¾ per pound. Required the entire prime cost. *Answ.* £1273426 1 10½.

132. The quantity of wheat consumed in England and Wales, in 1809, is computed to have been 7,876,100 quarters. Required the value at £4 15 7 per quarter, the average price of wheat during that year: required also the value, at the same rate, of 46,598 quarters, the part of that quantity which was imported from Ireland. *Answ.* £37,641,194 11 8, and £222,699 12 2.

133.

Belfast, 13*th Oct.* 1824.

Mr. Henry Wilson,

Bought of Samuel Jones.

725 yards fine white linen, at 4/1½ .....£
87 yards cambric, at 12/10 .....
417 yards muslin, at 3/9½ .....

£284 8

Lie the

134*

Joseph O'Reilly, Esq.

1823. To David White & Co. Dr

Sept. 24th, To fine scale sugar, 4 c. 3 q. 22 lbs.
at £4 17 4 ℔ cwt. ..... £

Dec. 1st, To tea, for 1 chest, containing 83 lbs.
at 7/4 ℔ lb. .....

£54 10 1½

135. Dublin, 8th December, 1824. Mr. William Joyce buys from Patrick M'Neale, 138 gallons port wine, at 16/7½ ℔ gallon; 130 gallons sherry, at 16/4; 110 gallons Madeira, at 26/9; and 120 gallons Teneriffe, at 14/10. Required the whole amount. *Answ.* £457 0 1.

136. Mr. Edw. Stone buys of Hugh Sinclaire of Cork, Jan. 3d, 1824, 156 tierces prime beef, at £5 9 8; 313 barrels ditto, at £3 5 8; Feb. 8th, 93 barrels prime pork, at £3 8 3; Feb. 26th, 64 barrels inferior ditto, at £3 3 6. Required the entire amount. *Answ.* £2403 12 11.

137. Belfast, Jan. 2d, 1830. Mr. Alexander Jefferson buys from William Fitzpatrick, 218 yards linen at 3/2 per yard; 173 yards muslin, at 1/4½ per yard; 2 pieces printed calico, containing 56½ yards, at 1/2 per yard; and one piece ditto, containing 27¾ yards, at 10d. per yard: and he pays in part £32 12 6. How much remains due? *Answ.* £18 4 9.

138. Mr. Robert Bellingham buys from Patrick Cunningham of Limerick, Feb. 5th, 1830, 3 cwt. of fine scale sugar, at 8/10 per stone; 2 cwt. coarse ditto, at 7/4 per stone; a chest of tea containing 86 lbs. at 4/6 per lb; 38 gallons whiskey, at 6/10 per gallon; and 10 gallons rum, at 13/6 per gallon. Required the amount. *Answ.* £55 11 0.

The method of aliquot parts in its application in finding prices, having been fully developed and illustrated in the preceding pages, it may be proper to conclude this article with some miscellaneous matter that could not with propriety have been intermixed with the general principles already exemplified.

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* In this exercise and the preceding, which are called *Bills of Parcels*, the prices of the several articles are to be found and set in the blank spaces towards the right hand: the sum of these partial amounts is the entire amount required. The form of the first of these exercises is that which is usually employed, when all the articles are bought at the same time: but when the times are different, the form is generally that of the next exercise. The remaining exercises of this kind are left for the pupil to write out, in proper form, for his own improvement.

Operations in the Rule of Proportion may often be abbreviated by the method of aliquot parts, whether the first term is a unit or not. Thus, if it were proposed to find how much flour might be bought for £6 5 8, if 7 c. 2 q. 16 lbs. cost £11; the terms being arranged in the usual way, we may multiply the third term by £6, and take parts of it for 5/8. By this means, the product of the second and third terms is found to be 48 c. 0 q. $2\frac{8}{15}$ lbs.; and this being divided by 11, by Compound Division, the quotient is 4 c. 1 q. 13 lbs. nearly, the quantity required.

	£	£	s.	d.	c.	q.	lbs.
As	11 :	6	5	8 : :	7	2	16
							6
					45	3	12
4/= £$\frac{1}{5}$					1	2	$3\frac{1}{5}$
1/8= £$\frac{1}{12}$					0	2	$15\frac{1}{2}$
				11)	48	0	$2\frac{8}{15}$
				Answer,	4	1	13, nearly.

Thus, also, if it be required to find the price of 365 bottles of wine, at £2 13 6 per dozen (question 53, page 79), the work will stand as in the margin.—Experience will enable the student to judge when this method may be employed with advantage, and when the common method is preferable.

	b.	b.	£	s.	d.
As	12 :	365 : :	2	13	6
		2			
		730			
10/= £$\frac{1}{2}$		182	10	0	
2/6= $\frac{1}{4}$ of 10/		45	12	6	
1/0= $\frac{1}{10}$ of 10/		18	5	0	
	12)	976	7	6	
		£81	7	$3\frac{1}{2}$, *Answ.*	

As an application of this method to quantities of another kind, let it be required to find the sun's mean apparent motion in 10 days, 7 hours, 20 minutes, the mean space which he apparently describes each day in the ecliptic, being 59′ 8″·3. Here the daily space being multiplied by 10 by Compound Multiplication, there results 9° 51′ 23″, for the space described in 10 days. Then for 6 hours, a fourth of the daily space is taken; for one hour, a sixth of the result; and for 20 minutes, a third of this last result. The sum of all these partial results is 10° 9′ 27″·2 nearly, the mean space required.

		59′	8″·3	
			10	
h. m.	9°	51	23	
6 0= $\frac{1}{4}$ day		14	47·1	
1 0= $\frac{1}{6}$ of 6 h.		2	27·8	
20= $\frac{1}{3}$ of 1 h.		0	49·3	
	10	9	27 2	

The following is another example of the application of this principle. When the radius of a circle, that is, half its diameter, is 1, the half of the circumference is 3·14159265; hence, let it

required to find the length of a part of the circumference containing 38° 44′ 33″.

Here, half the circumference being 180°, the reason of the process is obvious; and the answer is true, except the last figure, which should be 3, the difference being occasioned by the rejection of remainders in the divisions.

				3·14159265
30°	0′	0″	$= \frac{1}{6}$ of 180°	·52359877
6	0	0	$= \frac{1}{5}$ of 30°	·10471975
2	0	0	$= \frac{1}{3}$ of 6°	·03490658
	40	0	$= \frac{1}{3}$ of 2°	·01163553
	4	0	$= \frac{1}{10}$ of 40′	·00116355
		30	$= \frac{1}{8}$ of 4′	·00014544
		3	$= \frac{1}{10}$ of 30″	·00001454
			Answ.	·67618416

Various abbreviated modes of finding prices in particular cases, are given in works on Mercantile Arithmetic. These may be useful for the pupil who has had considerable experience in arithmetical calculations, and who is well acquainted with the more common and general rules for such purposes; but for the less experienced pupil they are quite unfit, as from their variety and want of connexion they must tend only to perplex and puzzle him, and to make him conceive the subject to be more difficult than it really is. It has been thought better, therefore, that no abbreviations except such as are of the most general and obvious nature, and such as are likely to be most frequently useful, should be introduced in the preceding part of this article, but that such others as might appear to be worthy of notice, should be inserted at the end of the article, that the attention of the pupil might be directed to them or not, as the teacher should reckon best.

To find the price of any number of articles at 2d. each; since 2d.$=£\frac{1}{120}$, we divide by 120: then since 2d.$=\frac{1}{6}$s. we take one-sixth of the remainder, regarded as shillings, for the remaining part of the answer. This can all be done in a single line, the division by 120 being performed by conceiving the last figure to be cut off, and then dividing by 12. In like manner the price at 3d. 4d. or 6d. may be found.

4532 at 2d.

2d.$=£\frac{1}{120}=\frac{1}{6}$ of 1s. ... £37 15 4

To find the price at 17/4; find the price at 16/ by the method given in page 129, and for the price at 1/4 take a twelfth of the result: and to find the price at 14/8, from the price at 16/ take a twelfth of itself. In like manner, to find the price at 15/2, to the price at 14/ add a twelfth of itself; and to find the price at 12/10, from the price at 14/ take a twelfth of itself: and lastly, to find the price at 19/6, to the price at 18/ add a twelfth of itself, and to find the price at 16/6, from the price at 18/ take a twelfth of itself.

3729 at 17/4
8

2983 4 for 16/
248 12 ..... 1/4

£3231 16

The price at 5/7½ may be found by adding to the price at 5/ an eighth of itself; the price at 3/1½ by adding to the price at 2/6 a fourth of itself; and the price at 2/9¾ by adding to the price at 2/6 an eighth of itself. The prices at 4/4½, 1/10⅛, and 2/2¼, may be found by the same rules, except that we are to *subtract*, instead of adding.

To find the price at £6 15, to the price at £6, add an eighth of itself; and to find the price at 6/9, to the price at 6/, add one-eighth of itself. In like manner, to find the price at £3 7 6, to the price at £3, add an eighth of itself; and to find the price at £2 12 6, take an eighth of itself from the price at £3.

The price of 24 articles may often be found very readily by taking each penny in the price as 2/; the price of 48, by taking each halfpenny as 2/; and the price of 96, by taking each farthing as 2/. Hence, the price of 25 yards at 3/6, is readily found to be £4 7 6; for in 3/6 there are 42 pence; and by doubling the last figure of this, we have £4 4 for the price of 24 yards; to which 3/6, the price of one yard, being added, the sum £4 7 6 is the required price.

Because 112 farthings=2/4, to find the price of 112 articles, reduce the price of one to farthings, and doubling the last figure for shillings, take the rest as pounds, and to the result add a sixth of itself. Thus, since in 9¾d. there are 39 farthings, to find the price of 112 at 9¾d. each, by doubling the last figure of 39, we have £3 18, to which a sixth of itself being added, we have £4 11, the price required.

To find the price of 120, reckon every penny in the given rate 10/. Thus, 120 at 4d. each, amount to 40/, or £2; and at 5½d. to 55/, or £2 15.—The same results would be obtained by taking the farthings in the given rate as pounds, and dividing by 8.

Almost numberless other abbreviations might be added. Should any person choose, however, to make himself master of the preceding, he will find it easy to discover many others.

TARE AND TRET.

THE whole weight of any commodity, together with the weight of the box, barrel, &c. which contains it, is called its GROSS WEIGHT.

The weight of the box, barrel, &c. which contains any commodity, is called its TARE.

The SUTTLE WEIGHT is what remains after the tare is deducted.

TRET is an allowance of 4 lbs. for each 104 lbs. of the suttle, in goods that are liable to waste.

Ambrose

CLOFF is an allowance of 2 lbs. in every 3 cwt. of what remains after the other deductions are made. This allowance is for the turning of the beam in weighing the goods.

The NEAT OR NET WEIGHT is what remains after all deductions have been made.

The deductions made in different places vary; the following examples, however, will illustrate the mode in which they are to be made, whatever they may be.

Exam. 1. Required the neat weight of 39 c. 3 q. 21 lbs. tare 18 lbs. ℔ cwt. tret and cloff as usual.

In the several parts of this and the following operations, for the purpose of avoiding fractions, for the last figure of each quotient, that which is nearest the truth has been taken. If greater accuracy be required, one or two places of decimals may be wrought for.—The reason of the process is sufficiently manifest.

	c.	q.	lbs.
	39	3	21
16 lbs. = ¼ of 1 c.	5	2	23
2 lbs. = ⅛ of 16 lbs.	0	2	24
Tare,	6	1	19
Suttle,	33	2	2
4 lbs. = $\frac{1}{28}$ of 104 lbs....	1	1	4
	32	0	26
2 lbs. = $\frac{1}{16}$ of 3 c....	0	0	22
Neat weight,	32	0	4

Exam. 2. Required the neat weight of 59 casks of butter, weighing gross, 40 c. 0 q. 4 lbs. tare 12 lbs. ℔ cask.

	59 casks.			
8 lbs. = $\frac{1}{14}$ of 1 c.	4	0	24	
4 lbs. = ½ of 8 lbs.	2	0	12	
Tare,	6	1	8	subt.
Gross,	40	0	4	
Neat weight,	33	2	24	

Exam. 3. Required the neat weight of 24 c. 1 q. 16 lbs. tare 2½ ℔ cent.

Here, 2½ being a fortieth of 100, the tare is a fortieth of the gross weight.

	c.	q.	lbs.
	24	1	16
2½ = $\frac{1}{40}$ of 100....	0	2	12
Neat weight,	23	3	4

Exercises. Find the neat weights of the following quantities:—

1. 166 c. 0 q. 8 lbs. tare 4 ℔ cent. *Answ.* 159 c. 1 q. 20 lbs.
2. 164 c. 1 q. 12 lbs. tare 5 ℔ cent. tret as usual. *Answ.* 150 c. 0 q. 15 lbs.
3. 104 cwt. tare 12 lbs. ℔ cwt. tret as usual. *Answ.* 89 c. 1 q. 4 lbs.
4. 125 c. 2 q. 17 lbs. tare 15 lbs. ℔ cwt. *Answ.* 108 c. 3 q. 8 lbs.
5. 75 c. 1 q. 26 lbs. tare 13 lbs. ℔ cwt. tret as usual. *Answ.* 64 c. 0 q. 17½ lbs. nearly.

6. 94 c. 3 q. 4 lbs. tare 12 lbs. ℔ cwt. tret and cloff as usual. *Answ.* 80 c. 3 q. 16 lbs.

7. 133 c. 0 q. 10 lbs. tare 17 lbs. ℔ cwt. tret as usual *Answ.* 108c. 2 q. 5 lbs.

8. 91 c. 1 q. tare 5 ℔ cent. *Answ.* 86 c. 2 q. 21 lbs.

9. 88 c. 2 q. tare 7½ ℔ cent. *Answ.* 81 c. 3 q. 13 lbs.

10. 24 c. 2 q. 7 lbs. tare 6 ℔ cent. *Answ.* 23 c. 0 q. 10 lbs.

11. 19 c. 1 q. tare 4 ℔ cent. *Answ.* 18 c. 1 q. 26 lbs.

12. 22 bags weighing gross 45 c. 1 q. 19 lbs. tare 5 lbs. ℔ bag, tret as usual. *Answ.* 42 c. 2 q. 26 lbs.

13. 39 barrels, weighing gross 83 c. 2 q. tare 18 lbs. ℔ barrel, tret as usual. *Answ.* 74 c. 1 q. 1 lb.

14. 96 tierces, weighing gross 213 c. tare 23 lbs. ℔ tierce, cloff as usual. *Answ.* 192 c. 0 q. 15 lbs.

15. 141 c. 0 q. 25 lbs. tare 6½ ℔ cent. tret and cloff as usual. *Answ.* 126 c. 0 q. 23½ lbs. nearly.

INTEREST.

THE sum to be paid by a person for the use of money which he owes, is called the INTEREST of that money.

The money due is called the PRINCIPAL.

The sum of the principal and interest is called the AMOUNT.

The RATE is the money allowed for the use of one hundred pounds for any given time, but usually for a year.

When interest is charged on the *original principal only*, it is termed SIMPLE INTEREST.*

When interest is charged, not only on the original principal, but also *on the interest as it becomes due*, it is called COMPOUND INTEREST.

It is scarcely necessary to remark, that *per cent.* means *per hundred*, and *per annum*, *per year*.

SIMPLE INTEREST.*

RULE I.—*To find the interest of a given sum for a year, at a given rate per cent. per annum;* multiply the principal by the rate, and divide the product by 100. Or,

As £100 are to the rate ℔ cent. ℔ annum, so is the principal to its interest for one year.

* In Interest five quantities are concerned, the *principal*, the *rate*, the *time*, the *interest*, and the *amount*; and any three of these, except the principal, the interest, and the amount, being given, the rest can be found. Hence, calculations in Interest

Exam. 1. Required the interest of £576 5 8½ for 1 year, at 6 ₱ cent. ₱ annum.*

£576	5	8½	
		6	
£34|	57	14	3
	20		
11|	54		
	12		
6|	51		
	4		
	2,04		

Or thus:—(See note, last page.)

£576	5	8½	
		6	
100)	3457	14	3

Answer, £34 11 6½

Here, by the first, or exact method, the answer is £34 11 6½, with $\frac{4}{100}$ or $\frac{1}{25}$ of a farthing.

The reason of the operation is quite evident, as it is nothing more than this: as the principal £100 is to its interest £6, so is

---

mit of several problems. The most useful however, and consequently that which claims the greatest degree of attention, is that in which the principal, the time, and the rate, are given, to find the interest or amount. This problem may be resolved in all cases by means of the first or second rule: the third and fourth, present modified, and in many cases, shorter methods of effecting the same. The following rule may also be found useful:—

To divide money by 100, for the pounds of the quotient, take the pounds of the dividend, except the two last figures, which are to be divided by 5 for shillings: from the remainder with half the shillings of the dividend annexed, reject a twenty-fifth part, and regard what remains as farthings. If there be pence, or an odd shilling, their effect in modifying the quotient may be estimated as nearly as possible. Thus, let it be required to divide £8947 13 8 by 100. Here, by cutting off two figures, we have £89; and one-fifth of 47 is 9, the shillings required, and the remainder is 2. This remainder with 7, the half of 14 shillings, annexed, becomes 27, from which 1, nearly its twenty-fifth part, being rejected, we have 26 farthings, or 6½d. We use 14 shillings in this example, because 13/8 is nearly 14/.

	£	s.	d.
100)	8947	13	8
	£89	9	6½

The reason of the rule will be understood from the paragraph commencing near the bottom of page 113.

As another example, let it be required to divide £2658 16 10 by 100. Here, after cutting off two figures, we have £26, and the fifth of the remainder is 11 for shillings, and the remainder 5. This remainder being prefixed to 8½, the half of 17/. to which 16/10 is nearly equal, we have 38½, the twenty-fifth part of which is obviously about 1½. Then 38½ being diminished by this quantity, the remainder is 37 farthings, or 9¼d. which completes the quotient.

£2658 16 10

£26 11 9¼

* The rate of interest has varied much at different periods, and in different countries, but it has been generally observed to diminish as commerce extends. In Italy, about the beginning of the the thirteenth century, it varied between 20 and 30 per cent.; and in the Netherlands it was fixed by Charles V. in 1560, at 12 per cent. By an act of the 37th year of Henry VIII, interest in England was not to exceed 10 per cent. By 21st James I. it was reduced to 8 per cent. Soon after the Restoration, it was reduced still lower, to 6 per cent.; and in the 12th of Anne, to 5 per cent. the present rate. The rate of interest in Ireland is at present 6 per cent. per annum.

the principal, £576 5 8½, to its interest; and it is evident, that as often as the one principal contains its interest, so often will the other contain its interest: that is, by the nature of proportion, the interest will be proportional to the principal.—*See the commencement of Simple Proportion.*

Exam. 2. Required the interest and the amount of £619 9 6 for 1 year, at 5½ ℔ cent. ℔ annum.

	£619	9	6	at 5½ ℔ cent.
			5½	
	3097	7	6	for 5 ℔ cent.
	309	14	9	for ½ .....
100)	3407	2	3	for 5½ .....
Add	34	1	5	=interest.
	619	9	6	=principal.
	£653	10	11	=amount.

The division at full length is as follows:—

£34|07	2	3
20		
1|42		
12		
5|07		

In the work of this example the principal is multiplied by 5 for 5 per cent. and, for ½ per cent. half the principal is added to the product. At the conclusion of the contracted division, it gives the result more nearly true to reject *one* than nothing from 21, though less than 25, and more especially as there are three pence in the dividend.—In the examples that follow, the division at full length will be omitted; it may be proper however for the pupil occasionally to work exercises both ways.

Exam. 3. What is the interest of £1374 1 9 for 1 year, at 5⅝ per cent. per annum?

	£1374	1	9	at 5⅝ ℔ cent.
			5⅝	
	6870	8	9	
⅝=⅛ of 5...	858	16	1	
100)	7729	4	10	
	£77	5	10,	*Answer.*

Since 5=$\frac{1}{20}$ of 100, this example might be wrought by taking a twentieth of the principal, and increasing the result by an eighth of itself.

Exercises. Find the interests of the following sums for 1 year, at the given rates per cent. per annum.

	Exercises. £	s.	d.		*Answers.* £	s.	d.
1.	774	11	3	at 5	38	14	6¾
2.	539	12	6	5½	29	13	7
3.	288	16	0	6½	18	15	5¼
4.	468	16	8	3½	16	8	2¾
5.	254	14	8	5¼	13	7	[illegible]
6.	876	12	6	5¾	50	8	1½

	Exercises. £	s.	d.			*Answers.* £	s.	d.
7.	589	13	4	at	$3\frac{3}{4}$	22	2	3
8.	376	12	8		4	15	1	$3\frac{3}{4}$
9.	175	8	2		9	15	15	$8\frac{3}{4}$
10.	286	4	7		$7\frac{7}{8}$	22	10	$9\frac{3}{4}$
11.	637	11	$4\frac{1}{2}$		$5\frac{5}{8}$	37	3	10
12.	411	1	10		11	45	4	$4\frac{3}{4}$
13.	2617	7	3		$3\frac{1}{2}$	91	12	$1\frac{3}{4}$
14.	899	10	$7\frac{1}{2}$		$4\frac{3}{4}$	42	14	$6\frac{1}{4}$
15.	937	5	11		$4\frac{1}{2}$	42	3	$6\frac{3}{4}$
16.	534	3	$9\frac{1}{2}$		$7\frac{1}{4}$	38	14	7
17.	39	16	1		$5\frac{1}{8}$	2	0	$9\frac{1}{2}$
18.	671	19	$5\frac{1}{2}$		$4\frac{1}{4}$	28	11	2

RULE H. *To find the interest of a given principal for any other time than a year;* (1.) Find the interest for a year by rule I.: (2.) as one year is to the given time, so is the interest for one year to the interest required.

The work may often be abbreviated by finding the interest for months or fractional parts of a year, by the method of aliquot parts. In using this method, the answer will often be found with more ease, or with a greater degree of correctness by multiplying by the rate; then multiplying or taking aliquot parts for the time; and last of all dividing by 100.

Exam. 4. Required the interest of £99 2 4 for $2\frac{3}{4}$ years at 4 per cent. per annum.

In this example the interest for 1 year is first found, which is £3 19 $3\frac{1}{2}$ nearly. This is then multiplied by $2\frac{3}{4}$, the number of years. It might have been done by multiplying by 3, and subtracting a fourth of £3 19 $3\frac{1}{2}$.—The formal analogy would have been: as 1 y. : $2\frac{3}{4}$ y. :: £3 19 $3\frac{1}{2}$: £10 18 $0\frac{3}{4}$.

	£99	2	4	
			4	
100)	396	9	4	
	£3	19	$3\frac{1}{2}$,	int. for 1 year.
			$2\frac{3}{4}$	
	7	18	7	
	1	19	$7\frac{3}{4}$	
	0	19	10	
	£10	18	$0\frac{3}{4}$,	*Answer.*

Exam. 5. Required the interest of £179 12 11 for 1 year and 7 months, at 5 per cent. per annum.

The interest for 1 year is found, by the method already explained, to be £8 19 $7\frac{3}{4}$. The rest of the work by aliquot parts, is as follows:—

	£8	19	7¾	=interest for 1 year
6 months=½ year	4	9	9¾	=..... 6 months
1 month =⅙ of 6 mo.	0	14	11¾	=..... 1 month
	£14	4	5¼	=interest required.

In this method the division by 100 is delayed till the end of the operation: every thing else is as before. By this means the error that often arises from neglecting the remainders in the division by 100, is done away, and the answer thus found is more nearly true, than that which in many cases would be obtained by the other method.*

Or thus:

	£179	12	4
			5
	898	4	7
6 months=½ year	449	2	3½
1 month =⅙ of 6	74	17	0½
100)	1422	3	44
	£14	4	5¼

Exercises. Find the interests of the following principals for thee given times, and at the given rates per cent. per annum.

	Exercises.				*Answers.*		
	£	s.	d.		£	s.	d.
19.	812	10	10	for 2 y. 5 m. at 4¾	93	5	5½
20.	719	18	4	..... 8½ m. at 5¼	26	15	5¼
21.	419	7	9	..... 1 y. 10 m. at 4⅜	33	12	9¼
22.	493	16	8	..... 1 y. 8 m. at 6	49	7	8
23.	824	18	9	..... 10 m. at 6	41	4	11¼
24.	427	8	8	..... 1 y. 5 m. at 5¾	34	16	4¼
25.	792	12	3	..... 1 y. 10 m. at 6½	94	9	0¾
26.	250	18	4	..... 1 y. 9 m. at 5	21	19	1¼
27.	651	0	0	..... 7 m. at 4½	17	1	9¼
28.	780	1	4½	..... 1 y. 10 m. at 5½	78	13	1¾
29.	193	18	2½	..... 2 y. at 5⅜	20	16	11
30.	584	18	8	..... 1 y. 9 m. at 3⅛	31	19	9¼
31.	1964	7	6	..... 5 m. at 6	49	2	2¼
32.	365	4	10½	..... 2 y. 4½ m. at 5⅝	48	15	10½

* *The interest of a sum for any number of months at 6 per cent. per annum, may be very easily found by multiplying the sum by half the number of months, and dividing the result by* 100: *and hence the interest at other rates may be derived by means of aliquot parts.* Thus, to find the interest of £250 for 10 months, at 4 per cent. per annum, we multiply £250 by 5; and dividing the product by 100, we find the interest at 6 per cent. to be £12 10 0. We then take from this a third of itself, and there remains £8 6 8, the interest required. This method is neat and short, but is seldom applicable to real business. Other contractions in calculating interest for months, are liable to the same objection.

Exam. 6. Required the interest of £342 11 8 for 86 days, at 4 per cent. per annum.

In this exercise the interest for a year is found to be £13 14 0¾ nearly; and as 365 days : 86 days : : £13 14 0¾ : £3 4 7 nearly, the interest required.

Exam. 7. Required the interest of £429 17 4, from June 29, 1818, till Feb. 12, 1819, at 5 per cent. per annum.

The number of days from the 29th of June, till the 12th of February following, is found, by the method shown in page 51, to be 228; and the interest of £429 17 4 for a year, is found, by the method already explained, to be £21 9 10½. Then, as 365 days : 228 days : : £21 9 10½ : £13 8 6¼, the interest required.

It will readily be seen, that this and all similar questions may be wrought by Compound Proportion. The terms will be arranged thus:—

As £100 : £5 } : : £429 17 4 : £13 8 6¼, *Answer.*
365 days : 228 days }

Find the interests of the following sums for the proposed times, and at the assigned rates per cent. per annum.

	Exercises.						*Answers.*		
	£.	s.	d.	days.			£	s.	d.
33.	456	10	0	for 31,	at 5	.....	1	18	9¼
34.	729	16	3	 73,	5¾	.....	8	7	10½
35.	1000	0	0	 1,	6	.....	0	3	3⅔
36.	698	14	8	 89,	5	.....	8	10	4½
37.	788	14	10	196,	6½	.....	27	10	7¼
38.	381	1	8	264,	6	.....	16	10	9
39.	447	12	6	from July 8,	till Dec. 26,	at 5¾	12	1	2
40.	487	18	9	..... Ap. 21,	..... Sept. 4,	5½	9	19	11¾
41.	474	15	3	..... June 20,	..... Nov. 8,	6	11	0	1
42.	943	1	8	..... May 1,	..... Oct. 21,	5¼	23	9	4
43.	364	[illegible]	8	..... Aug. 24,	..... Jan. 1,	6	7	15	8

RULE III. *To find the interest of a given sum for any number of days,* multiply the principal by twice the rate, and the product by the days, and divide the result by 73,000.

The division by 73,000 may be performed by the following rule:—*Below the dividend write one-third of itself, one-tenth of that third, and one-tenth of that tenth, rejecting shillings and remainders: then add the four lines together, divide the sum by* 100,000 (*or cut off five figures*), *and reject a farthing for each* £10 *in the result.**

* The exact correction is a farthing for £10 8 4½. The reason of this rule will appear from performing the operation indicated in it on 73,000, the result being 100,040. Now, 10, the excess of this above 100,000, is contained in it 10,001 times, and 10,001 farthings are £10 8 4½.

The division by 73,000 may also be facilitated by cutting off the ciphers, and using the table given in page 35.

Exam. 8. What is the interest of £372 10 10, from February 12, till December 17, 1824, at 4½ per cent. per annum?

1824 being a leap year, the number of days is found to be 309; and the product of this, of the given principal, and of 9, (twice the given rate,) being found, as in the margin, is £1036077, nearly; which being divided by 73,000, the quotient is £14 3 10¼, the interest required.—The division by 73,000 by the second mode will stand as in the margin, below the answer as found by the first method.—After cutting off five figures, we have £14 remaining to the left. Then, according to the method shown at the bottom of page 113, we divide the number expressed by the next two figures by 5, and obtain for quotient 3 shillings with the remainder 4.—Prefixing this to the next figure, we have 44; and rejecting 2, for a twenty-fifth of 44, we have remaining 42 farthings or 10½d. We then correct the result by rejecting 1 farthing, and we find the same result as before.

```
                £372 10 10
                         9
                ----------
                £3352 17  6, or
                £3353  0  0, nearly.
                  309
                ------
                 30177
               10059
               -------
73000)1036077
               -------
          £14  3 10¼, Answ.

          Or thus:
               1036077
  ⅓ ..... 345359
                34536
                 3453
               --------
               14,19425
               --------
               14  3 10½
Correc. ........ 0¼
               --------
          £14  3 10¼, Answ.
```

The reason of the rule will be evident from the operation by Compound Proportion, if, instead of £100 and the rate ⅌ cent. their doubles be employed. Thus, we should have, in this exercise,

$$\left.\begin{array}{l}\text{As £200 : £9}\\ \text{365 days : 309 days}\end{array}\right\} :: \text{£372 10 10 : £14 3 10}\tfrac{1}{4};$$

and in working this, we should, by the rule for Compound Proportion, multiply together the principal, the days, and twice the rate, and divide the product by 365 × 200, or 73000.

It is easy to see, that when the rate is 5 per cent. since the double of 5 is 10, it will be sufficient to divide the product of the principal and days by 7300.

	Exercises.								*Answers.*		
	£	s.	d.						£	s.	d.
44.	648	15	6	from June 2,	till	Nov. 25,	at	5	15	12	10
45.	1120	10	0	—— Mar. 23,	——	Nov. 2,	——	6	41	5	2
46.	688	18	4	—— Mar. 10,	——	Aug. 25,	——	6	19	[illegible]	6

Exercises.	£	s.	d.		*Answers.* £	s.	d.
47.	884	8	8	from Mar. 3, till Oct. 28, at 5	28	19	1½
48.	4868	15	0	.....June 8, ... Nov. 1, ... 6½	126	11	9
49.	597	10	8	.....May 6, ... Aug. 21, ... 5½	9	12	8¼

The following rule will be found very easy and practicable in Interest and Discount:

RULE IV.* *To find the interest of a given principal for any number of days, at 4 per cent. per annum;* (1.) multiply the principal by the days: (2.) to the product add one-tenth of itself: (3.) from the sum take four times the same product, wanting the three last figures: (4.) divide what remains by 10,000 (or cut off four figures;) the quotient will be the answer nearly.

When the interest is large, reject a farthing for each £10 contained in it.

For other rates than 4 per cent. increase or diminish the product of the principal and days, by the method of aliquot parts, and then proceed by the rule.

Exam. 9. Required the interest of £8985 14 for 12 days, at 4 ₱ cent. ₱ annum.

Here the product of the principal and days is £107828 nearly, and the tenth of this (found by setting each figure one place nearer the right hand side, and increasing the unit figure by 1, because 28 is nearly 30,) being added to it, the sum is 118611. After this, we multiply 107 by 4, and increase the product by 3, (carried for 4 times 8, the first of the figures cut off:) the result, 431, is then subtracted, and the remainder, 118180, divided by 10,000, in the way pointed out in the last example: the quotient is £11 16 4¼, from which, because it is nearly £10, a farthing is subtracted, and the remainder, £11 16 4, is the interest required.—

£8985 14	
12	
107828 8	
10783	
118611	
431	
11,8180	
£11 16 4¼	
¼	
£11 16 4,	*Answ.*

* The following method of finding interest for days at 6 per cent. per annum, may be found useful, when the interest to be found is not very large.

Divide the product of the principal and days by 100; take one-third of the quotient for shillings, and one-sixth of the remainder for farthings; and from the sum thus obtained, reject a penny for each six shillings contained in it; the remainder will be the interest required, very nearly.

Another correction may sometimes be made, by adding to the result obtained by the preceding part of the rule, a penny for each six shillings contained in the first correction, or a penny for each £20 in the entire interest, will give nearly the same correction.

Had the rate been 5 ℔ cent. we must have increased £107828 8, by one-fourth of itself, and then have added to the result one-tenth of itself, &c.

With respect to *the reason* of this easy and expeditious rule, the reader who has studied decimal fractions, will find by the last rule, that the interest of £1, for 1 day, at 4 ℔ cent. ℔ annum, is £8÷73000, or £·0001096, nearly, or £·00011—£·0000004, nearly: and it will appear on a little consideration, that the operation by the rule is nothing else than multiplying by ·00011 and ·0000004, and taking the difference of the results. The correction is necessary, because the decimal ·0001096 is not the exact interest of one pound for a day.

	Exercises.							*Answers.*		
	£	s.	d.					£	s.	d.
50.	773	6	8	from June 9,	till Dec. 6,	at 4	.....	15	5	1
51.	593	12	6	..... May 12,	..... Oct. 29,	 4	.....	11	1	2¼
52.	374	5	0	..... April 1,	..... Dec. 29,	 4	.....	11	3	1½
53.	246	18	10	..... Mar. 14,	..... June 8,	 6	.....	3	9	9¾
54.	511	16	8	..... May 13,	..... Sept. 29,	 6	.....	11	13	10¾
55.	176	11	4½	..... Mar. 17,	..... Aug. 25,	..... 5½	.....	4	5	8

The calculation of interest on *accounts current*, affords a useful application of the preceding principles. An ACCOUNT CURRENT contains a statement of the mercantile transactions of one person with another, when immediate payments are not made. It is usually written on two pages, in the manner of a ledger account; the page towards the left hand containing every payment made by the merchant who furnishes the account, and the other page every thing that is paid to him.—At the foot of pages 156 and 157, is a specimen of this kind of account, and the following is the method of calculating the interest on it, at 4 ℔ cent. ℔ annum.

			£	s.	d.	days.	Dr.	Cr.
Feb.	11,	To	186	7	6	× 303	...56472	
	26,	To	214	10	0	× 288	...61776	
June	20,	To	415	8	4	× 173	...71867	
March	24,	By	166	13	4	× 261	.....	43500
April	6,	By	347	18	0	× 248	.....	86279
Sept.	26,	By	200	0	0	× 75	.....	15000
							190115	144779
							144779	
							45336	
						$\frac{1}{10}$	 4534	
							49870	
						45 × 4 ÷ 1	 181	
							4,9689	
							£4 19 4½, interest	

For rightly understanding this calculation, it is necessary to consider, that the account is made up till the 10th of December, and the interest calculated on it till that date. We place in a column, as above, all the sums on the debit side, and then all those on the credit side, prefixing to both their dates, and to the former the word *to*, and to the latter *by*, for the sake of distinction. We next find, successively, the number of days that elapse between Feb. 11 and Dec. 10, between Feb. 26 and Dec. 10, &c. and place them in the next column. A debit column and a credit one are then formed, and all the sums on the debit side are multiplied by the corresponding numbers of days, and the products are placed in the debit column. In like manner, the products of the sums on the credit side by the days which follow them, are placed in the credit column. The sums of the two columns are then taken, and the debit side is found by subtraction to exceed the other side by 45336; which by means of rule IV. (or of rule III.) gives £4 19 4½, the interest due on the entire account. This is placed on the debit side of the account; and then the sum of all on the credit side is taken from the sum of all on the debit side, and the remainder £106 13 10½, is placed on the credit side, and is the sum due by the person to whom the account is furnished. It is scarcely necessary to say, that the two last lines, in Italics, form the *answer* of the account, being found by the calculator.

It may be proper to require the pupil not only to perform the calculation of the interest on the following accounts current, but also to write the accounts out in proper form on a sheet of paper, after the manner of the specimen given at the foot of this page and the succeeding. The answers are in the lines which are printed in Italics.

Exercises 57, 58, 59. Required the principal and interest due on each of the following accounts current, till the date at the end of each, the first at 6, the second at 5, and the third at 6 ⅌ cent. ⅌ an.

Dr. Mr. J. Fox, in Account Current, with S. Bell. *Cr.*

		£	s.	d.			£	s.	d.
1824.					1824.				
May 19,	To goods ...	512	12	6	June 13,	By cash	400	18	0
Aug. 23,	To tea	273	8	0	Nov. 8,	By wheat ...	680	0	0
Oct. 4,	To goods ...	186	10	0	Dec. 1,	By bill	73	5	8
Nov. 18,	To sugar ...	272	5	0	1825.				
1825.	*To balance*	10	4	11	Jan. 18,	*By balance*	100	16	9
Jan. 18,	*of interest*					*to new ac.*			
		£1255	0	5			£1255	0	5

Ex. 56. *Dr.* Mr. John Jardine, Newry, in Account

		£	s.	d.
1824.				
Feb. 11,	To balance by account furnished	186	7	6
26,	To amount of sugar	214	10	0
June 20,	To amount of rum and sugar	415	8	4
Dec. 10,	*To interest due on this account*	4	19	4½
		821	5	2½

Dr. Mr. C. JOHNS, Dublin, in Account Current with THOMAS LINN & Co. Belfast. *Cr.*

1824.		£	s.	d.	1824.		£	s.	d.
Sept. 3,	To balance	1280	3	11	Sept. 19,	By sugar	1510	10	0
Dec. 21,	To linen ...	793	18	0	1825.				
1825.					Mar. 20,	By goods	1248	8	0
Jan. 27,	To goods...	1040	5	0	May 25,	By bill.....	912	16	8
Mar. 26,	To linen ...	838	14	2	June 26,	*By balance to new ac.*	306	16	3½
June 26,	*To balance of interest*	25	9	10½					
		£3978	10	11½			£3978	10	11½

Dr. Mr. T. HART, in Account Current with H. ORR. *Cr.*

1824.		£	s.	d.	1824.		£	s.	d.
Feb. 9,	To linen ...	768	8	9	April 1,	By sales of flaxseed	419	4	8
May 7,	To lawn....	436	17	6	June 27,	By sugar.....	500	0	0
July 8,	To goods..	948	5	10	Sept. 11,	By bill on Ash & Co. London	1533	12	6
Aug. 18,	To linen...	673	11	0	Oct. 29,	*By balance to new ac.*	408	16	9
Oct. 29,	*To balance of interest.*	34	10	10					
		£2861	13	11			£2861	13	11

Ex. 60. On the fifth of January, 1822, the public funded debt of Great Britain and Ireland, consisted of

£534,355,086 6 11 at 3 ℔ cent. ℔ annum;
29,547,003 19 3 ... 3½ .....
75,947,763 19 4 ... 4 .....
and, 153,056,763 7 9 ... 5 .....

Required the annual interest of the whole. *Answ.* £27,755,546 9 1½.

The following rules serve for the resolution of the remaining cases of Interest. As these cases are of minor importance, the rules are given without proof, and without illustration by examples. They are easily proved however, by the principles of proportion; and the pupil will find it easy to apply them in the resolution of the subjoined exercises.

Current, with CHARLES CAULFIELD, Belfast. *Cr.*

1824.		£	s.	d.
Mar. 24,	By amount of flour.....	166	13	4
April 6,	By cash.....	347	18	0
Sept. 26,	By bill on Cavan & Co. Dublin.....	200	0	0
Dec. 10,	*By balance to your debit in a new account*	106	13	10½
		82[illegible]	5	2½

RULE V. *To find what principal, in a given time, would produce a given interest, at a given rate per cent. per annum;*

As rate : £100 } : : interest : principal.
Given time : 1 year }

Ex. 61. How much money must be lent on the 2d of April, at 6 ₩ cent. ₩ annum, to bring in for interest £24, on the 18th of November following? *Answ.* £634 15 7¾.

62. What principal, at 5 ₩ cent. ₩ annum, will bring a yearly income of £341 5? *Answ.* £6825.

63. What principal lent from the 24th of March, 1824, till the 17th of November, 1825, at 4½ ₩ cent. ₩ annum, will gain £1200? *Answ.* £16141 10 3½.

64. What principal at 5 ₩ cent. ₩ annum, would be equivalent to the funded national debt of Great Britain and Ireland?—*(See the answer to question* 60 *of this rule.) Answ.* £555,110,929 2 6.

RULE VI. *To find what principal, in a given time, would increase to a given amount, at a given rate per cent. per annum;* (1.) To the product of the time and rate, add the product of £100 and 1 year in the same name as the given time: (2.) Then, as the sum is to the abovementioned product of £100 and 1 year, so is the amount to the principal.

65. What principal lent on the 1st of January, 1824, at 5½ ₩ cent. ₩ annum, would amount to £1000, on the 29th of September in the same year? *Answ.* £960 12 6½.

66. What sum must be lent at simple interest, at 4 ₩ cent. ₩ annum, that the amount, at the end of 2 years, 10 months, may be £627 18 6? *Answ.* £564 0 1.

RULE VII. *To find the time in which, at a given rate, per cent. per annum, a given principal would produce a given interest:*

As principal : £100 } : : 1 year : time required.
rate : interest }

67. In what time will £460 amount to £500, at 4½ ₩ cent. ₩ annum? *Answ.* 1 year, 340 days.

68. How long must £2000 be lent at simple interest, at 8½ per cent. per annum, to amount to £2280? *Answ.* 4 years.

69. How long must £887 5 0 be lent at 5¼ per cent. per annum, at simple interest, to gain £120? *Answ.* 2 years, 210 days.

RULE VIII. *To find at what rate a given principal would gain a given interest in a given time:*

As given time : 1 year } : : interest : rate.
principal : £100 }

70. At what rate per cent. per annum, simple interest, will £1500 amount to £1860, in 4½ years? *Answ.* £5 3 8½.

71. If a merchant, with a capital of £5000, gain £2000 in $2\frac{3}{4}$ years, at what rate per cent. per annum, simple interest, has he gained? *Answ.* £14 10 11, nearly.

72. If £1 amount to £1 2 9 in $3\frac{1}{2}$ years, at simple interest, at what rate per cent. per annum must it have been lent? *Answ.* £4 4 $7\frac{4}{11}$.

DISCOUNT.

DISCOUNT is an abatement made for advancing money before it becomes due.

The money which is received as the full payment of any debt or bill due some time after, is called its PRESENT WORTH.

RULE. *To find the present worth of a bill or debt,* (1.) find the interest of the debt at the given rate, and for the given time: (2.) consider this interest as discount, and subtract it from the debt to find the present worth.

Exam. 1. Required the present worth of a bill of £170, due at the end of 3 months, at 5 per cent. per annum.

Here, by the method already explained in Interest, the discount is readily found to be £2 2 6; and this being taken from £170, the remainder, £167 17 6, is the present worth.

Exam. 2. What is the present worth of a bill of £39 5, due on the 1st of September, but paid on the 3d of July preceding, discount being allowed at 5 per cent. per annum?

The time here is 60 days, for which the interest of £39 5 is found to be $6/5\frac{1}{2}$; and by subtracting this from £39 5, we have remaining £38 18 $6\frac{1}{2}$, the present worth.

In Great Britain and Ireland three days, called DAYS OF GRACE, *are always allowed after the time a bill is nominally due, before it is legally due.* Thus, suppose a bill were drawn on the 8th of April, at 4 months, it would be due, not on the 8th, but on the 11th of August.

It may be remarked, that if, without the days of grace, a bill should appear to be due on the thirty-first of a month which contains only thirty days, the last day of that month is to be taken, and not the first of the next; and consequently, the third of the next month will be the day on which, by the addition of the days of grace, the bill will be really due. Thus, a bill drawn on the thirty-first of August, at three months, would be due on the third of December. In like manner, a bill, which, without the addition of the days of grace, would be due on the twenty-ninth, thirtieth, or thirty-first of February, if that month contained so many days, would be really due on the third of March.—It may be farther remarked, that bills which fall due on Sunday, are paid in Britain on Saturday, but in Ireland on Monday.

Exam. 3. Required the present worth of a bill of £77, drawn 8th March, at 6 months, and discounted 3d June, at 5 per cent. per annum.

By counting forward 6 months and 3 days from the 8th of March, we find this bill to be due on the 11th of September. The number of days from the 3d of June till this date is 100, and the interest of £77 for 100 days, at 5 per cent. per annum, is found by any of the methods formerly explained, to be £1 1 1¼, and consequently, the present worth is £75 18 10¾.

Exercises. Required the present worths of the following bills, at the given rates ℔ cent. ℔ annum:

	Exercises. £	s.	d.	*Drawn,*			*Discounted,*			*Answers.* £	s.	d.
1.	416	3	4,	Mar. 1,	at	7 mos.	June 9,	at	4	410	16	7¼
2.	533	6	8,	Sept. 5,	.....	5	Nov. 12,	.....	4½ ...	527	10	11¼
3.	218	11	8,	Aug. 14,	.....	4	Oct. 3,	.....	4	216	15	8¾
4.	607	3	4,	May 22,	.....	5	July 10,	.....	5½	597	7	6½
5.	895	12	0,	Jan. 5,	.....	11	May 9,	.....	5	869	9	4½
6.	284	8	8,	Sep. 25,	.....	5	Nov. 30,	.....	6¼	280	1	0
7.	588	12	8,	Mar. 6,	.....	6	June 11,	.....	6	579	18	6
8.	486	18	8,	Mar. 25,	.....	10	June 19,	.....	5	472	1	2
9.	875	5	8,	Feb. 25,	.....	7	June 4,	.....	5	861	7	6
10.	388	2	6,	Dec. 8,	.....	6	Mar. 25,	.....	6	383	2	11¾
11.	1000	0	0,	Feb. 16,	.....	11	Sep. 12,	.....	5½	980	11	2¾
12.	568	12	9,	Apr. 27,	.....	7	June 3,	.....	5	554	12	4
13.	447	12	6,	June 23,	.....	6	July 8,	.....	5¾	435	11	4
14.	511	3	4,	Mar. 31,	.....	7	May 8,	.....	6½	494	17	5¼
15.	649	13	4,	Nov. 9,	.....	9	April 19,	.....	5½	638	8	2

Ex. 16. Required the discount of £284 13 0½, for one year, at 5½ per cent. per annum. *Answ.* £15 13 1½.

17. What is the discount of £549 9 5½, for 32 days, at 5 per cent. per annum? *Answ.* £2 8 2.

18. What is the present worth of £970 18 4, due at the end of 19 months, at 4¾ per cent. per annum? *Answ.* £897 17 11.

The rule which has been given above for the calculation of discount, is that which is always employed by merchants. It is founded however on a principle radically false; and always *gives the discount too large, and consequently the present worth too small, by the interest of the true discount.* This will appear manifest, if we consider, that *the true present worth of any debt, is such a sum as would, if lent at interest at the assigned rate, amount to that debt at the time at which it would have been due:* and consequently the discount, or the difference between the present worth and the debt, should be, not the interest of the debt, but the interest of the present worth; and therefore the interest of the debt will exceed the true discount, that is, the interest of the present worth, by the interest of that discount.

The true present worth will be found by the following analogy:—*As the amount of £100 for the given time and at the proposed rate, is to £100,*

so is the debt to its true present worth; and the present worth being subtracted from the debt, the remainder is the discount.

The reason of this rule will be evident from the consideration, that £100 is the present worth of its amount regarded as a debt: and consequently the analogy given above will become simply this: as the amount of £100 considered as a debt, is to £100, the present worth of that debt, so is any other debt to its present worth. It is obvious also, that for the two first terms of the analogy, we might use the amount of *any sum whatever*, and that sum itself; but it is generally more simple and easy to employ £100 and its amount.

To exemplify this rule, let it be required to find the true present worth of £200, due at the end of a year, at 5 per cent. per annum.—In this case, the amount of £100 being £105, we have, by the rule, this analogy:—as £105 : £100 : : £200 : £190 9 6$\frac{1}{4}$, the present worth required. By the common rule the result would have been £190: consequently the error is 9/6$\frac{1}{4}$.

Again, let it be required to find the true present worth of £463, for 7 months, at 5 per cent. per annum. Here the amount of £100 is £102 18 4; and therefore, as £102 18 4 : £100 : : £463 : £449 17 6$\frac{3}{4}$ nearly. By the common method the result would be £449 9 11, and the error 7/7$\frac{3}{4}$.

This question, and all similar ones, may be very easily wrought by the following rule:—*Multiply the months by the rate, and add the product to* 1200; *then, as the sum is to* 1200, *so is the debt to its true present worth.* Thus, in the preceding example we should have this analogy: as 1235 : 1200 : : £463 : 449 17 6$\frac{3}{4}$. The reason of this rule may be thus shown: as 12 months : 7 months : : £5 : £$\frac{35}{12}$, the interest of £100 for 7 months. Then, as £100$\frac{35}{12}$: £100, or by reduction of both to twelfths, as 1235 : 1200 : : &c.

As another example, let it be required to find the correct present worth of £512, due on the 19th of September, but paid on the 8th of May preceding.

Here, the number of days being 134, we have this analogy: as 365 days : 134 days : : £5 : £1 16 8$\frac{1}{3}$, the interest of £100 for 134 days: and then, as £101 16 8$\frac{1}{3}$: £100 : : £512 : £502 15 5$\frac{1}{4}$, *answer.*

This question, and all others in which it is required to find the *correct* present worth of a sum for a given number of days, may be wrought more easily and more accurately by the following rule:—*Multiply the days by the rate, and add the product to* 36500 (=365×100:) *then, as this sum is to* 36500, *so is the debt to its true present worth.* Thus, in the present example 134×5=670, and 36500+670=37170: then, as 37170 : 36500 : : £512 : £502 15 5, the present worth.

This rule depends on the same principle as the last; and the reason of it may be thus illustrated. As 365 days : 134 days : : £5 : £$\frac{670}{365}$, the interest of £100 for 134 days, at 5 per cent. per annum. Then, as £100$\frac{670}{365}$: £100; or by reduction to three-hundred-and-sixty-fifths, as 37170 : 36500 : : the debt to its present worth.—Both these rules are in reality the same as rule VI. in Interest.*

* Unless the time be great, the true present worth may be readily derived by approximation from that found by the common method, by finding the interest of the interest first found, and *adding* it to the present worth found by the common method; then by

The following examples and considerations will perhaps be useful in showing the falsity of the common method of discount.

If a person have a bill for £100 payable at the end of a year, at 5 per cent. he will receive, according to the common method of discount, only £95 for it; and were he to lend this sum for a year at the same rate, instead of £100, to which it obviously should amount, he would receive only £99 15. The true present worth is £95 4 $9\frac{1}{7}$, and consequently the error $4/9\frac{1}{7}$. Again, had the same bill been payable at the end of two years, the present worth, by the common method, would have been £90, while it should be £90 18 $2\frac{2}{11}$. The error is consequently $18/2\frac{2}{11}$, and £90, instead of amounting to £100 at the end of two years, would amount to no more than £99. Had the time been four years, the present worths would have been £80, and £83 6 8, and the error £3 6 8. The amount also of the present worth, £80, would be £96, and consequently £4 less than it should be. If the time had been ten years, the present worths would have been £50 and £66 13 4, where the error is £16 13 4; and the amount of the present worth £50 would be £75 instead of £100. Finally, were the time 20 years, the present worth, according to the common method, would be *nothing*, while it should be £50: and were the time greater than 20 years, the present worth would be unassignable, as it would appear to be less than nothing; or if any meaning could be attached to the result of the operation, it would be, that the person who held the bill, instead of receiving any thing for it, would be required *to pay* something to get it off his hands.

From these examples, it will appear how very erroneous the common method of computing discount is, when the time is long.* In every case, indeed, the discounter of the bill has a greater rate of interest for his money, than the nominal rate; and the longer the time,

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finding the interest of this last interest, and *subtracting* it from the approximate present worth; and so on by adding and subtracting alternately, the interest of the last interest till the correction becomes so small, that it would be unnecessary to carry the operation farther. When the time is very short, the true result will often be obtained as easily in this way, as by the principles above explained.

As an example, let it be required to find the present worth of a bill of £140, due at the end of 6 months, at 4 per cent. per annum.

Here the interest of £140 is £2 16; that of £2 16 is $1/1\frac{1}{2}$, and that of $1/1\frac{1}{2}$ is a farthing. Then, by subtracting £2 16 from £140; by adding $1/1\frac{1}{2}$ to the remainder, and lastly, by subtracting a farthing from that result, we find the present worth to be £137 5 $1\frac{1}{2}$, which is correct.

* It might be shown algebraically, that the error in the common method of calculating the discount or present worth of a given debt, at a given rate, is nearly proportional to the square of the time, when the time is small, or more properly when the discount is small compared with the debt. Thus, at 5 per cent. per annum, the error on a bill of £1000, for 2 months, is nearly $1/4\frac{1}{2}$, while for 4 months it is nearly $5/5\frac{1}{2}$, or very nearly 4 times $1/4\frac{1}{2}$.

It might be shown in nearly the same manner, that the error in the sums to which the present worth of a given sum found by the common method, would amount at the given rate, would be exactly proportional to the squares of the times. Thus, in the examples in the text in this page, it appeared, that in case of a bill of £100, payable in a year, at 5 per cent. per annum, the amount would be £99 15, while, if it were payable in two years, the amount would be only £90 the error being in the one case 5/, and in the other, £1, or 4 times 5/

the greater is this rate. Thus, to recur to the last series of examples, since by paying £95 at present, the discounter will be entitled to £100 at the end of a year, he obviously gains £5 on £95; and therefore, as £95 : £5 : : £100 : £5 5 $3\frac{3}{19}$, his gain per cent. In like manner, if the time were two years, the gain per cent. would be found to be £5 11 $1\frac{1}{3}$; if four years, £6 5; if 10 years, 10 per cent.; and if 19 years, cent. per cent.

It is true indeed, that when the time is short, as it generally is in real business, the difference between the results found by the two methods, is inconsiderable; and therefore the common method, the calculation for which is so easy, may be employed without much error. Still however, the principle is false, as it gives profits to the discounter, which are not proportional to the times. It may be said, indeed, that those who keep money for the purpose of discounting, are entitled to more than the simple common rate. This may be true; yet at the same time, it does not prove the correctness of the principle on which this mode of computation depends; since, if the discounter is to have a greater rate, it should be some fixed rate, and not a variable one, depending on the time the bill has to run.

Should it be thought advisable for the learner to work Discount in the correct method, the exercises at the beginning of this article will serve his purpose as well as any others; and the following are their answers by that method:—

True Answers.

	£	s.	d.		£	s.	d.		£	s.	d.
Ex. 1.	410	17	$11\frac{1}{2}$	Ex. 7.	580	1	$0\frac{1}{2}$	Ex. 13.	435	17	8
2.	527	12	$2\frac{1}{4}$	8.	472	9	$11\frac{3}{4}$	14.	495	7	$6\frac{1}{4}$
3.	216	16	$0\frac{1}{4}$	9.	861	11	$10\frac{1}{4}$	15.	638	12	0
4.	597	10	$7\frac{3}{4}$	10.	383	4	$2\frac{3}{4}$	16.	14	16	$9\frac{1}{2}$
5.	870	4	$2\frac{1}{4}$	11.	980	18	$7\frac{3}{4}$	17.	2	7	$11\frac{1}{2}$
6.	280	2	$3\frac{3}{4}$	12.	554	19	1	18.	903	0	$0\frac{3}{4}$

COMMISSION, INSURANCE, &c.

COMMISSION is the sum which a merchant charges for buying or selling goods for another.

BROKERAGE is a smaller allowance of the same nature, paid usually for negotiating bills, or transacting other money concerns.

INSURANCE or ASSURANCE is a contract by which one party, on being paid a certain sum or premium by another, on account of property that is exposed to risk, engages in case of loss, to pay to the owner of the property the sum insured on it.

RULE I. *To compute the commission, brokerage, insurance, or any other allowance on a given sum, at a given rate per*

cent.; multiply the sum by the rate per cent. and divide the product by 100: or, as £100 are to the rate per cent. so is the given sum to the allowance required.

The work may often be abbreviated by the method of aliquot parts.

Exam. 1. Find the commission on £791 11 8, at $2\frac{1}{2}$ ℘ cent.

Here, by multiplying by $2\frac{1}{2}$, and dividing by 100, we find for the answer £19 15 $9\frac{1}{2}$. The same result would be obtained by dividing the given sum by 40, since $2\frac{1}{2}$ is a fortieth of 100.

Exam. 2. Find the brokerage on £829 14 6, at 3/6 ℘ cent.

Here, aliquot parts are taken for 3/6, and the result being divided by 100, the quotient is £1 9 $0\frac{1}{2}$, the brokerage required. In like manner, had the rate been £$\frac{1}{2}$, £$\frac{3}{4}$, or any other fractional rate ℘ cent. aliquot parts must have been taken for the rate, and the result divided by 100.

	£	s.	d.
	829	14	6
2/6=£$\frac{1}{8}$	103	14	$3\frac{3}{4}$
1/0=£$\frac{1}{20}$	41	9	$8\frac{3}{4}$
100)	145	4	$0\frac{1}{2}$
	£1	9	$0\frac{1}{2}$

Exam. 3. Required the premium of insurance on £512 9 4, at £6 16 6 per cent.

Here, by multiplying by 6, taking parts for 16/6, and dividing by 100, the insurance is found to be £34 19 $6\frac{1}{4}$.—The result, in this particular exercise, might be found more easily by multiplying by 6 and dividing by 100, by which means, the insurance at £6 ℘ cent. would be found; and thence the insurance at £6 16 6 ℘ cent. would be deduced by adding to the result an eighth of itself, and a tenth of that eighth; because £6 16 6 is six times £1 2 9, the best parts for which are 2/6=$\frac{1}{8}$ of £1, and 3d.=$\frac{1}{10}$ of 2/6.

	£	s.	d.
	512	9	4
			6
	3074	16	0
12/=$\frac{1}{10}$ of £6	307	9	$7\frac{1}{4}$
4/=$\frac{1}{3}$ of 12/	102	9	$10\frac{1}{2}$
6d.=$\frac{1}{8}$ of 4/	12	16	$2\frac{3}{4}$
100)	3497	11	$8\frac{1}{2}$
	£34	19	$6\frac{1}{4}$

RULE II. *To find how much must be insured on property worth a given sum, so that, in case of loss, both the value of the property and the premium of insurance may be repaid:* (1.) subtract the rate from £100; (2.) as the remainder is to £100, so is the value of the property to the sum to be insured.

Exam. 4. How much must be insured, at $8\frac{1}{2}$ ℘ cent. on an adventure of £600, that, in case of loss, not only the value of the adventure, but also the premium of insurance, may be repaid?

Here, as £100—£$8\frac{1}{2}$: £100 :: £600 : £655 14 9. The truth

of this operation is proved by finding the premium on £655 14 9, at $8\frac{1}{2}$ ℘ cent. This is found to be £55 14 9. Hence, in case of the adventure being lost, the owner will receive not only £600, the value of the adventure, but also £55 14 9, the premium; and thus he will sustain no loss whatever. The reason will appear manifest from considering, that in receiving £100 which had been insured at $8\frac{1}{2}$ ℘ cent. the owner would receive but £$91\frac{1}{2}$ in lieu of the property, £$8\frac{1}{2}$ having been paid for the insurance.

Ex. 1. What is the commission on £942 16 3, at $4\frac{1}{2}$ ℘ cent.? *Answ.* £42 8 $6\frac{1}{4}$.

2. Required the brokerage on £946 18 10, at 5/6 ℘ cent. *Answ.* 2 12 1.

3. Required the commission on £569 14 9, at $7\frac{1}{2}$ ℘ cent. *Answ.* £42 14 $7\frac{1}{4}$.

4. What is the premium of insurance on £675 11 8, at £5 13 9 ℘ cent.? *Answ.* £38 8 $5\frac{3}{4}$.

5. What is the expense of insuring a vessel and cargo, worth £3649 8 0, at $3\frac{1}{2}$ ℘ cent.? *Answ.* £118 12 $1\frac{1}{4}$.

6. Required the premium of insurance on £1486 13 9, at £2 16 $10\frac{1}{2}$ ℘ cent. *Answ.* £42 5 $6\frac{3}{4}$.

7. What must be the sum insured, at $5\frac{3}{4}$ ℘ cent. on goods worth £1938 12 6, that, in case of loss, the owner may be repaid both the value of the goods and the premium of insurance? *Answ.* £2056 17 11.

8. What is the brokerage on £681 4 10, at 5/3 ℘ cent. *Answ.* £1 15 $9\frac{1}{4}$.

9. At £2 5 6 ℘ cent. what will be the cost of insuring goods worth £1560, that, in case of loss, the owner may be entitled to the value of the goods and the premium? *Answ.* £36 6 $3\frac{3}{4}$.

10. Required the commission on £863 12 6, at $2\frac{1}{2}$ ℘ cent. *Answ.* £21 11 $9\frac{3}{4}$.

11. Add to £579 16 10 the commission on itself at $7\frac{1}{2}$ ℘ cent. and find the insurance of the sum at $4\frac{3}{4}$ ℘ cent. *Answ.* £27 5 5.

12. At $4\frac{3}{4}$ guineas ℘ cent. what will be the premium of insuring property worth £592 6 8, so that, in case of loss, the owner may be entitled to the premium and the value of the property? *Answ.* £31 1 $10\frac{1}{2}$.

13. At $5\frac{1}{2}$ guineas ℘ cent. how much must be insured on goods worth £813 11 3, so that, in case of loss, the owner may receive the value of the goods and the premium? *Answ.* £863 8 6.

The following accounts of mercantile transactions will serve as exercises in calculating insurance, &c.; and may be found useful, particularly for pupils who are intended for business in the counting-house. The learner should be required to write out the fifteenth, seventeenth, and eighteenth, in proper form, and to find the neat proceeds.

An ACCOUNT SALES is an account of goods sold by a factor. It is furnished by him to his employer, and contains the quantity and price of the goods sold, the charges attending the sales, and the neat proceeds,

or the sum which the owner of the goods is to receive, after all charges are deducted.

An INVOICE is an account containing the quantity, prime cost, and charges of goods sent from one person to another, usually by sea.

The following account contains the names of the purchasers, the quantities sold, and the rates. The amounts of the several quantities are left for the pupil to find and set in the money columns. The sum of the several amounts will be found to be £1146 16 10. The several charges, and the commission on £1146 16 10, are then added together, and the sum, £116 8 9, taken from the gross amount, £1146 16 10. The remainder, £1030 8 1, is the neat proceeds.

Ex. 14. Account sales butter received per the Dolphin, from Cork, sold per order and on account of John Evans, Esq. Belfast.

			firkins	lbs.				
1824.								
Jan.	16	Jos. Hyman	5	312	at 2/1			
	...	Elizabeth Naylor ..	5	318	...			
	20	John Hughes	2	128	2/2			
	25	Wm. Roberts	4	251	2/0			
Feb.	3	M. A. Hynds	19	1200	1/9			
	...	Aaron Moreno	20	1274	...			
	5	Eml. Emanuel	10	646	...			
	...	Leah Levy	10	634	...			
	...	Mathew Shannon ..	10	636	...			
	6	S. Touzalin & Co...	10	642	...			
	13	Soloman Marks	22	1402	...			
	...	Thos. Hill	20	1266	...			
	...	Eml. Emanuel	10	628	...			
	15	J. C. Clarke	25	1603	22½d.			
	16	M. A. Hyams	3	187	1/9			
	...	Jos. Isaacs & Co....	15	920	22½d.			
Mar.	15	B. Daly & Co.....	10	647	...			
			200	12694	£	1146	16	10

CHARGES.

Freight and primage, £22 1, wharfage and landing, £6 5.	£28	6	0			
Advertising, 17/6, drayage and negro hire 25/	... 2	2	6			
Commission on £1146 16 10, at 7½ per cent.				116	8	9
Neat proceeds, to credit of J. Evans, Esq.			£	1030	8	1

Errors excepted.

MORROW, NEVIN, and EVANS

Kingston, Jamaica, Sept. 14, 1824.

Ex. 15. Account sales of soap received per the Waterloo from Belfast, sold by order, and on account of David Sanford, Esq. 1827, March 2. To John Panillo, 2055 lbs. at £5 per 100 lbs.; to A. Maxwell, 437 lbs. at £4;* March 23, to A. Maxwell, 1976 lbs. at £3 3 4; to William Moliere, 2431 lbs. at £3 2 6; April 1, to Marcusa Hyams, 1198 lbs. at £3 10; May 23, to D. S. Davis, 1963 lbs. at £3; to A. Grant & Co. 463 lbs. at £3; June 18, to John Angus, 3692 lbs. at £2 13 4; June 26, to Carvallo and Rivers, 4091 lbs. at £2 13 4; June 28, to J. Antonio, 2372 lbs. at £2 13 4; Aug. 22, to John Berry, 279 lbs. at £3; to R. M'Cleland, 1997 lbs. at £3 10; Sept. 4, to Leah Levy, 3988 lbs. at £3 6 8; Sept. 10, to Sarah Myers, 3194 lbs. at £3 6 8.—CHARGES—Advertising for sale, Drayage, and Negro hire, £2 10; Commission on £ , at 7½ per cent.—*Kingston, Jamaica, Sept.* 12, 1824. (Signed) Mason and Newsam. *Answ.* £887 6.

16. INVOICE of 3 whole and 40 half barrels beef, and 20 half barrels pork, shipped by John Marsden & Co. on board the Rosanna, Robert Bishop, master, for Glasgow, on account and risk of Mr. Andrew Blakely, merchant, Greenock.

	3 barrels Mess Beef at £4 11 0 per.....						
	40 half do. do. do. ..... 2 5 6.....						
A. B.	20 do. do. Pork ..... 2 12 0.....						
	CHARGES.			£	156	13	0
	Paid carriage and shipping 7/2, quayage 1/6.....	£0	8	8			
	Stamp 3/4, Bills of Lading 1/.....	0	4	4			
	Commission on £ at 2½ per cent.....					11	8
	Errors excepted.				[illegible]	[illegible]	[illegible]
	Belfast, Dec. 4, 1824.						

In the preceding invoice, the letters (A. B.) in the margin, are the letters with which the barrels were marked. The blanks are left to be filled up by the learner.—The following invoices are to be written out in proper form by the pupil, and the amount of the cost and charges found.

Ex. 17. INVOICE of 100 firkins butter, shipped by Pollock and Blackwell in the Henry of Belfast, James M'Millan, master, for Jamaica, to address of Messrs. Mackay and Dunn, Kingston, by order and for account of John Maxwell & Co. Belfast., 100 firkins, (marked I. M.) neat 55 c. 1 q. 17 lbs. at £5 17 per cwt.—CHARGES.—Duty 4 per cent. (on the prime cost:) Cooperage, Iron Hoops, Nails, Cartage, Shipping, Inspection, &c. £16 5: Commission at 2½ per cent. on £353 6 3. (Signed) Pollock and Blackwell. *Cork, June* 28, 1824. *Answ.* £362 2 11.

18. INVOICE of 10 bags sea island cotton, shipped on board the

* Per 100 lbs; and so in all the rest.

Downshire, William Garnett, master, for Belfast, by order, and for account and risk of James Morton, merchant there. (The bags marked P. B. and their number and weight as follows:) No. 132, 1 cwt. 3 q. 26 lbs.; No. 290, 1 c. 3 q. 21 lbs.; No. 382, 1 c. 3 q. 20 lbs.; No. 12, 1 c. 3 q. 26 lbs.; No. 125, 1 c. 3 q. 27 lbs.; No. 250, 1 c. 3 q. 23 lbs.; No. 286, 1 c. 3 q. 22 lbs.; No. 149, 1 c. 3 q. 21 lbs.; No. 225, 2 c. 0 q. 2 lbs.; No. 385, 2 c. 0 q. 1 lb.; draft 1 lb. per bag, tare 2 q. 22 lbs.: price 2/3 per lb. neat.—CHARGES—Entry, Town duty, &c. 10/4; Cartage and Porterage, 10/7; Bills of Lading, 3/6; Insurance on £256, at 1¼ guinea per cent.; Policy on £300, at 3/ per cent.; Commission on £243 3 10, at 2½ per cent. (Signed) Croker & Co. *Liverpool, May* 1, 1824. *Answ.* £249 5 5.

The *draft* mentioned in the preceding invoice, is a deduction on cotton wool, of the same nature as tare, tret, &c.; and the *policy* is an expense attending the effecting of an insurance, and charged on even hundreds of pounds.

EQUATION OF PAYMENTS.

WHEN one person owes another several debts payable at different times, the rule which determines the just time for a single payment of the whole, is called EQUATION OF PAYMENTS.

RULE I. (1.) Multiply each debt by the time that must elapse before it will become due. (2.) then divide the sum of the products thus obtained by the sum of the debts, and the quotient will be the time required.

When the products are taken, all the times, and likewise all the debts, must evidently be in the same denomination.

Exam. 1. If one person owe to another £300 payable in 4 months, £500 payable in 6 months, and £400 payable in 10½ months; in what time may the whole be paid without loss to either party?

Here, $300 \times 4 + 500 \times 6 + 400 \times 10\frac{1}{2} = 8400$, and $300 + 500 + 400 = 1200$: then $8400 \div 1200 = 7$. Hence, the time required, according to the preceding rule, is 7 months.

The work may be proved, according to the principle on which the preceding rule is founded, by finding the interest of the first debt at any assumed rate for 3 months, the interest of the second at the same rate for 1 month, and the interest of the third at the same rate still for 3½ months: the last of these interests will be found to be exactly equal to the sum of the other two. The times used in the proof, are the differences between the equated time, and the times at which the several debts are due.

Exam. 2. If a person owe £100 payable at present, and £600 payable in 7 months; at what time may both be justly paid at a single payment?

Here, $100 \times 0 + 600 \times 7 = 4200$, and $100 + 600 = 700$: then $4200 \div 700 = 6$, the months required.

Ex. 1. Required the equated time for the payment of two debts, one of £350 due at the end of 8 months, and another of £600 due at 13 months. *Answ.* $11\frac{3}{19}$ months.

2. If a debt of £45 be payable at 6 months, another of £70 at 11 months, and a third of £75 at 13 months; what is the equated time for the payment of the whole? *Answ.* $10\frac{23}{38}$ months.

3. If a debt of £1200 be payable, one-half at 18 months, one-fourth at 15 months, one-sixth at 10 months, and the remainder at 3 months, what is the equated time for the payment of the whole? *Answ.* $14\frac{2}{3}$ months.

4. What is the equated time for the payment of four debts, the first for £120 due at 1 month, the second for £135 due at 7 months, the third for £160 due at 10 months, and the fourth for £100 due at 1 year? *Answ.* $7\frac{22}{103}$ months.

5. What is the just time for the payment of two debts, the one of £139 6 due at the end of 7 months, and the other of £170 12 6 due at the end of $2\frac{3}{4}$ years? *Answ.* $21\frac{856}{1771}$ months.

6. If a debt be payable, one-third at present, one-fourth at 6 months, one-fifth at 12 months, and the remainder at 18 months; what is the equated time for the payment of the whole? *Answ.* $7\frac{4}{5}$ months.

The rule above given, is that which is generally preferred in working questions in Equation of Payments. No rule in Arithmetic, however, has given origin to so warm disputes; some writers arguing strongly in support of its accuracy, and others entirely condemning the principles from which it is deduced. The following rule is used by those who consider the preceding one erroneous:—

RULE II. (1.) Find the present worths of the several debts at the given or common rate of interest: (2.) then find, by rule VII. page 158, in what time, at the same rate, the sum of the present worths thus found, would amount to the sum of the debts. The result thus obtained will be the time required.

To renew the dispute on this unimportant subject, and to echo the arguments that have been advanced on both sides of the question, is inconsistent with the plan of this work. It may suffice to say, that the principle on which the first rule depends, is, that the interest of the money, the payment of which is delayed beyond the time at which it is due, is to be equal to the interest of that which is to be paid, before it becomes due, and that this principle is liable to the same objections as the common rule for Discount. In the second rule, on the contrary, the result depends on the *present* values of the debt; and it appears to be an obvious principle, that all such transactions and agreements should be regulated in conformity to the *present value* of the money, and the *improvement* of which it is susceptible. This latter principle, which is

entirely neglected in the use of the first rule, is acted on in the second in using the rate of interest. When the times are short, however, the difference of the results by the two rules is small, and the first being easier in its application, may be employed with as much propriety as the common rule for Discount.

The following are the answers of the exercises according to the second rule; the first at 5, and the rest at 6 per cent. per annum :—

1. 11 mos. 4 days	3. 14 mos. 17 days	5. 20 mos. 16½ days
2. 10 mos. 17 days	4. 7 mos. 13 days	6. 7 mos. 17 days.

PROFIT AND LOSS.

THAT branch of Arithmetic which treats of the gains or losses on mercantile transactions, is called PROFIT AND LOSS.

The method of performing the first example and the first six exercises, is so obvious as not to require a formal rule.

Exam. 1. If a hundred weight of rice be bought for £2 8, and sold at 6½d. per pound, what is the gain?

Here, the hundred weight being bought for £2 8, and sold for £3 0 8, the gain is evidently 12/8, the difference between them.

112 lbs. at 6½d.

6d.= ½ of 1/0 | 56
½d.= 1/12 of 6d. | 4 8

60/8, or

Selling price	£3	0	8
First cost....	2	8	0
Gain.....	£0	12	8, *Answer.*

Ex 1. If a merchant purchase 93 c. 3 q. 12 lbs. of rozin, at 9/4 per cwt. and pay for charges £3 4 6; what does he gain by selling 27 c. 2 q. at 12/4 per cwt.; 29 c. 1 q. 20 lbs. at 12/8 per cwt.; and the remainder at 12/9 per cwt.? *Answ.* £12 2 3¼.

This exercise may be wrought by finding the price of 93 c. 3 q. 12 lbs. at 9/4 per cwt. and increasing the result by £3 4 6; then by finding the price of 27 c. 2 q. at 12/4; of 29 c. 1 q. 20 lbs. at 12/8; and of the remainder, 36 c. 3 q. 20 lbs. at 12/9: and finally, by subtracting the result formerly obtained from the sum of these prices. It may be wrought more easily however, by finding the amount of 27 c. 2 q. at 3/0, (the difference between 12/4 and 9/4;) of 29 c. 1 q. 20 lbs. at 3/4, (the difference between 12/8 and 9/4;) and of 36 c. 3 q. 20 lbs. at 3/5, (the difference between 12/9 and 9/4:) and finally, by taking £3 4 6 from the sum of these.

2. If a piece of linen containing 25½ yards, cost £3 8 3, what is gained by selling it at 3/9½ per yard? *Answ.* £1 8 5¼.

3. If a pipe of wine containing 138 gallons, cost £113 15, what is gained by selling it at 18/6 per gallon? *Answ.* £13 18.

4. If a chest of tea containing 113 lbs. be sold at 7/10 per lb.: what is the gain, the first cost and charges being £37 10? *Answ.* £6 15 2.

5. If 246 c. 1 q. 11 lbs. (long weight,) of pork be bought at £2 4 8 ℔ cwt. and the charges amount to £41; what is the gain or loss, the neat proceeds being £504? *Answ.* Loss, £87 3 3.

6. If a roll of tobacco containing 25 lbs. be bought at 3/6 ℔ lb.; what is gained by selling it at $3\frac{1}{4}$d. ℔ ounce, a pound and a half being lost by weighing it out in small quantities, and by drying? *Answ.* 14 $\frac{1}{4}$.

RULE I. *From the prime cost and the selling price, to find the gain or loss per cent.;* as the prime cost is to the gain or loss on that cost, so are £100 to the gain or loss per cent.

RULE II. *To find how a commodity must be sold to gain or lose a certain rate per cent.;* as £100 are to the gain or loss per cent. so is the prime cost to the gain or loss on that cost; and from this and the prime cost, the selling price will be found by addition or subtraction.

It should be particularly remarked, that *by the gain or loss per cent. is to be understood the sum that would be gained or lost at the given rates,* not on a hundred pounds' worth sold, but *on a hundred pounds laid out in prime cost, and in charges, if there be any.*

Exam. 2. If tea be bought at 6/6 ℔ lb. and sold at 7/4 ℔ lb. what is the gain ℔ cent.?

Here, 7/4 [illegible] 0d. the gain on 6/6; then, as 6/6 (the first cost of 1 lb.) : 10d. (the gain on 6/6) : : £100 (regarded as first cost) : £$12\frac{2}{3}$, or £12 16 $4\frac{1}{13}$, the gain ℔ cent. or the gain on £100.

Exam. 3. How must nutmegs, which cost 17/6 ℔ lb. be sold to gain 16 ℔ cent.?

As £100 (regarded as first cost) : £16 (the gain on £100) : : 17/6 (the first cost of 1 lb.) : $2/9\frac{2}{3}$, the gain ℔ lb. which being added to 17/6, the amount is £1 0 $3\frac{2}{3}$, the required rate.

Exam. 4. If cotton wool be bought at 3/1 ℔ lb. and sold at 2/11 ℔ lb. what is the loss ℔ cent.?

In this exercise there is a loss of 2d. upon the first cost, 3/1: therefore, as 3/1 : 2d. : : £100 : £$5\frac{1}{4}$ or £5 8 $1\frac{1}{4}$, the loss ℔ cent.

Exam. 5. If sheet lead, which cost £2 2 6 ℔ cwt. be sold at a loss of 7 ℔ cent.; at what rate ℔ cwt. is it sold?

As £100 : £7 (the loss on £100) : : £2 2 6 : $2/11\frac{3}{10}$, the loss ℔ cwt. which being taken from £2 2 6, the remainder, £1 19 $6\frac{3}{10}$, is the selling price required.

This question may also be wrought thus: as £100 : £93 : : £2 2 6 : £1 19 $6\frac{3}{10}$, the selling price as before. The reason of

this latter mode of operation is, that since there is a loss of 7 ℘ cent. what cost £100, is sold for £100—£7, or £93.

Ex. 7. If whiskey which cost 6/8 ℘ gallon, be sold for 7/10 ℘ gallon, what is gained ℘ cent.? *Answ.* $17\frac{1}{2}$.

8. How must wine which cost 15/ ℘ gallon, be sold to gain $21\frac{1}{4}$ ℘ cent.? *Answ.* $18/2\frac{1}{4}$.

9. If paper which cost 19/2 ℘ ream, be sold for 17/11 ℘ ream, how much is lost ℘ cent.? *Answ.* £6 10 $5\frac{5}{23}$.

10. How must linen which cost $3/1\frac{1}{2}$ per yard, be sold to gain 16 per cent.? *Answ.* $3/7\frac{1}{2}$.

11. If rum cost 14/6 per gallon, and be sold for 18/ per gallon, what is gained per cent.? *Answ.* £24 2 $9\frac{3}{29}$.

12. If broadcloth cost £1 3 6 per yard, how much is gained per cent. by selling one parcel at £1 8 0 per yard; and how much by selling another at £1 9 0 per yard? *Answ.* £19 2 $11\frac{34}{47}$, and £23 8 $1\frac{1}{47}$.

13. If hemp cost £48 7 6 per ton, and be sold for £45 10 per ton, how much per cent. is lost, and how much is lost in the sale of 39 tons, 17 c.? *Answ.* £5 18 $10\frac{46}{129}$, and £114 11 $4\frac{1}{2}$.

14. Isinglass which cost 8/6 per lb. is sold at a loss of 11 per cent.: at what rate is it sold, and how much is lost on the sale of 17 c. 1 q. at the same rate? *Answ.* $7/6\frac{38}{50}$, and £90 6 $5\frac{1}{25}$.

15. How must bees' wax which cost £14 5 per cwt. be sold to gain 21 per cent. and how much must be sold at that rate to gain £100? *Answ.* £17 4 $10\frac{1}{5}$, and 33 c. 1 q. $18\frac{1}{2}$ lbs.

16. If paper which cost £1 2 9 per ream, be sold at £1 5 6 per ream, how much is gained per cent.? *Answ.* £12 1 $9\frac{6}{91}$.

17. If soap cost £3 5 per cwt. how must it be sold to gain 10 per cent. and how to gain 20 and 30 per cent.? *Answ.* £3 11 6, £3 18, and £4 4 6.

18. How must butter which cost £5 14 10 per cwt. be sold to gain 19 per cent.? *Answ.* £6 16 $7\frac{4}{50}$.

19. Bought 2048 yards of linen at $3/2\frac{1}{2}$ per yard, and sold the whole for £360 19 9: required the whole gain, and the gain per cent. *Answ.* £32 9 1, and £9 17 $6\frac{3}{4}$.

20. Bought 125 c. 3 q. 12 lbs. of flour at £1 7 $10\frac{1}{2}$ per cwt and sold it at £1 10 $8\frac{1}{2}$ per cwt.: required the whole gain and the gain per cent. *Answ.* £17 16 $7\frac{1}{7}$, and £$10\frac{11}{69}$.

21. Bought 2688 yards of cambric at 8/8 per yard; and sold one-fourth at 10/2, one-third at $10/11\frac{1}{2}$, and the remainder at $11/4\frac{1}{2}$ per yard. Required the whole gain, and the gain per cent. *Answ.* £304 14 8, and £26 3 $2\frac{3}{4}$.

The following rule, the reason of which will appear evident from the exercises, will solve many questions that are usually proposed in Profit and Loss.

RULE III. *From the gain per cent. and the selling price, to find the first cost;* as ℘100, together with the gain per cent.

or diminished by the loss per cent. are to £100, so is the selling price to the prime cost.

Exam. 6. What was the first cost of flaxseed, which being sold at £3 10 6 per hogshead, the seller clears 13 per cent.?

As £113 : £100 : : £3 10 6 : £3 2 $4\frac{76}{113}$, the first cost required. In this example it is evident, that what cost £100, is sold for £113; and the analogy used above is no more than this: as the selling price, £113, is to its first cost, £100, so is the selling price, £3 10 6, to the corresponding first cost, £3 2 $4\frac{76}{113}$.

Exam. 7. If a merchant, by selling pearl ashes at £4 14 6 per cwt. lose 18 per cent.: what was the prime cost?

As £100—£18 : £100 : : £4 14 6 : £5 15 $2\frac{38}{41}$, the first cost required. In this example it is evident, that what cost £100, must have been sold for £82; and therefore the preceding analogy is simply this: as the selling price, £82, is to the corresponding first cost, £100, so is the selling price, £4 14 6, to the corresponding first cost, £5 15 $2\frac{38}{41}$.

Ex. 22. If 11 per cent. be lost by selling 128 yards of broadcloth for £98 18 8, what was the prime cost per yard? *Ans.* $17/4\frac{1}{2}$.

23. Suppose a book to be sold for £4 19 9, and 17 per cent. to be gained; what was the first cost? *Answ.* £4 5 $3\frac{1}{13}$.

24. If 13 per cent. be gained by selling tea at 7/4 per lb. what was the first cost, and what is gained by the sale of 349 lbs. at the same rate? *Answ.* $6/5\frac{88}{113}$, and £14 14 $5\frac{1}{4}$

25. What was the first cos of tar which being sold at 17/4 per barrel, the merchant loses 9 per cent.; and how much is lost on the sale of 63 barrels? *Answ.* $19/0\frac{4}{7}$, and £5 8.

26. If £13 13 be lost by the sale of 112 lbs. of raw silk at £2 8 per lb. what was the prime cost, and what is the loss per cent.? *Answ.* £2 10 $5\frac{1}{4}$, and $4\frac{223}{288}$.

Exam. 8. If 7 per cent. be gained by selling linen at 2/9 per yard, how must it be sold to gain 25 per cent.?

This exercise might be wrought by finding the first cost, and thence the selling price, as in the preceding examples. It will be wrought more easily however by this analogy: as £107 : £125 : : $2/9 : 3/2\frac{89}{107}$, the price required. The reason of this will be manifest, if it be considered, that what cost £100, is in the one case sold for £107, and in the other for £125; and therefore the above analogy is no more than this: as the selling price of one hundred pounds' worth in the one case, is to its selling price in the other, so is the selling price of a yard in the first case, to its selling price in the other. The mode of working the following exercise depends on the same principle.

Exam. 9. If 15 per cent. be gained by selling molasses at £2 10 per cwt. how much is gained per cent. by selling it at £2 5 6 per cwt.?

As £2 10 : £2 5 6 : : £115 : £104 13; and £104 13—£100 =£4 13, *answ.*

Ex. 27. If 6 per cent. be gained by selling broadcloth at 19/4½ per yard, how much per cent. would be gained by selling the same at £1 2 9 per yard? *Answ.* £24 9 3$\frac{15}{31}$.

28. If by selling muslin at 3/1½ per yard, 2 per cent. be lost, how must it be sold to gain 25 per cent.? *Answ.* 3/11$\frac{163}{196}$.

29. If 17 per cent. be gained by selling flour at £1 5 8 per cwt. how much per cent. is gained or lost by selling it at £1 0 9 per cwt.? *Answ.* £5 8 2$\frac{74}{77}$, lost.

30. Bought 21,300 yards of linen at 2/9½ per yard, and paid for various charges £88 15 1; and sold one-third at 3/0, one-third at 3/4, and one-third at 3/2 per yard: required the whole gain, and the gain per cent. *Answ.* £310 12 5, and £10 2 10¾.

EXCHANGE.

THE object of EXCHANGE is to find how much of the money of one country is equivalent to a given sum of the money of another.

By the PAR OF EXCHANGE between two countries, is meant the intrinsic value of the money of the one compared with that of the other, and estimated by the weight and fineness of the coins.*

The COURSE OF EXCHANGE at any particular time, is the sum of the money of one country, which at that time is given for a fixed sum of the money of another country.—This is seldom at par, but is continually varying according to the circumstances of trade.†

* The par continues the same, so long as the coins in each country are of the same weight, fineness, and nominal value. There is an exception to this, when the standard coins are of different metals. In this case the par varies as the comparative values of the metals vary. Thus, gold is the standard coin in this country, but in most of the Continent silver is the standard. Now, from the gold mines in America having of late years become less productive, and the silver ones more productive than formerly, the comparative value of silver has fallen in a small degree; so that the par between Holland and England is now about 11½ florins for the pound sterling, though it was formerly only 11; and a similar reduction has taken place in other countries under similar circumstances.

† The variation in the course of exchange depends on several circumstances, which, as well as several other questions connected with exchanges, it does not accord with the plan of this Publication to explain, such discussions belonging properly to Political Economy. Suffice it to say, that whenever, in consequence of a deficient crop in any country, such quantities of corn and other provisions are to be imported, as will exceed the value of the exports to the places from which these articles were brought; or when, in consequence of war, subsidies are to be paid to foreign powers, or remittances are to be made abroad for the payment of forces on foreign service; or in a word, when any

In foreign exchanges, one place always gives another a *fixed sum or piece of money* for a *variable one*. The former is called the CERTAIN RATE; the latter, the VARIABLE RATE.

Thus, in exchanges with Portugal, London gives for one milree sometimes 65 pence, sometimes 68 pence, &c. In this case, 1 milree is the *fixed price*, and 65 pence, 68 pence, &c. the *variable price*.

All the calculations in Exchange may be performed by the Rule of Proportion: and the operation may often be abbreviated by the method of aliquot parts.

RULE I. or GENERAL RULE. Place as the second term in the analogy, that sum whose value is to be found in the money of another country: make that term of the rate which is of the same kind with it, the first term of the analogy, and the remaining term of the rate the third term; then work the analogy in the usual way.

The tables that precede most of the subdivisions of the remaining part of this article, must be employed in the work of almost every exercise. It may be a matter for consideration, whether they should be committed to memory, or only referred to in the book when necessary. If they be committed, the learner will find considerably more facility in performing the calculations; but most pupils will probably soon forget them.

EXCHANGE WITH AMSTERDAM.

In Amsterdam accounts are kept in florins, stivers, and pennings; and also in pounds, skillings, and grotes, or pence, Flemish.

TABLE. 8 pennings=1 grote or penny Flemish; 2 grotes or pence, or 16 pennings=1 stiver; 20 stivers, or 40 grotes=1 florin or guilder; 6 florins=1 pound.

Also, 12 grotes or pence=1 skilling; 20 skillings=1 pound.

Also, 2½ guilders, or 50 stivers=1 rix dollar.

In exchanges between London and Amsterdam, London gives the

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cause makes the imports from a country greater than the exports to it:—in all these cases, the bills of the foreign state must increase in value, as none of them will be required to be sent abroad to pay the balance; and thus the money in that country will increase in value, and will continue above par, till coin or bullion is remitted to make up the deficiency: or till, by a greater exportation or a smaller importation, or both, on the part of the debtor country, the equilibrium of trade is restored. It is of consequence to observe, that the course of exchange can never differ *very much* from the par, as coin or bullion will be remitted instead of bills, whenever the course of exchange is such, that the value of coin or bullion, and the expense of remitting and insuring it, would be less than the cost of bills. Besides, when the course of exchange is against a country, it affords an inducement to merchants to export to the other country, as the bills which they will get in return will be more valuable, in consequence of the money of the foreign country being above par; and thus they can procure a better and surer market for their commodities, as they will be enabled to sell them at a lower price.

certain, and Amsterdam the *variable* sum. Thus, London for 1 pound receives sometimes more and sometimes less Flemish money.

Exam. 1. Reduce £386 17 4 British to Dutch money, exchange at 37/2 Flemish ℔ pound sterling.

As £1 : £386 17 4 : : 37/2, or as 240d. : 92848d. : : 3568 pennings : 1380340 pennings. By division by 16 and 20, this will become 4313 flor. 11 st. 4 pennings, the answer in florins, &c. but if it be divided by 8, 12, and 20, it will become 718 pounds, 18 sk. 6 grotes, 4 pennings, or, as it may be written, £718 18 6½, the answer in pounds, &c.

This exercise has been wrought without any contraction; and it may perhaps be proper for the learner to proceed thus for some time, as in this way he will better understand what he does. Abbreviations will soon however readily suggest themselves, particularly in the mode of reducing the terms. These are left unexplained, because in far the greater number of instances, the use of aliquot parts is much to be preferred, especially in conjunction with the following rule:

RULE II. *To reduce pounds, skillings, and grotes, Flemish, to florins and stivers;* Multiply the pounds and skillings by 6, adding to the skillings half the grotes or pence before found.

Thus, resuming the last example, and taking aliquot parts, we find £718 18 6½, the same as before, and by multiplying the pounds and skillings by 6, and adding to the skillings 3¼, the half of 6½: the same result is found as before.

£386	17	4	at £1	17	2	
193	8	8	for 10	skillings		
96	14	4	.....	5	.....	
38	13	8¾	.....	2	.....	
3	4	5¾	.....	2	grotes	
£718	18	6½,				
	6					
Flor. 4313	11¼	st. or				
4313 flor. 11 st. 4 pennings						

Exam. 2. Reduce 2475 florins, 10 stivers, 9 pennings to British money, exchange at 36/10 Flemish ℔ pound sterling

As 36s. 10g. : 2475fl. 10st. 9p. : : £1 : £224 0 7.

12	20
442 grotes	49510 stivers
8	16
3536 pen.	792169 pennings

After the preceding reductions, which are performed according to the table, the answer is found by simply dividing the second term by the first, since the third is a unit.

There are two kinds of money in Amsterdam, called *banco*, or bank money, and *currency*, or current money. In the former of these, all bills of exchange are valued and paid. It is of purer

metal than the currency; and hence it bears a premium of 3 or 4, and sometimes 5 ℔ cent.: that is, £100 of bank money is valued at £103, £104, &c. of currency. This premium is called the *Agio.*

The method of reducing bank to current money, and current to bank money, will appear from the two following examples:

Exam. 3. How much *currency* is equivalent to 798 florins, 17 stivers *banco*, agio at 3½ ℔ cent.?

Here the last term is multiplied by 20, to reduce it to stivers. Then, the operation being conducted in the usual way, the result is 16536 stivers, with the remainder 19½, which is multiplied by 16, and divided by 100, to find pennings, because 16 pennings = 1 stiver.—Then, the stivers being divided by 20 to reduce them to florins, the answer is found to be 826 florins, 16 stivers, 3 pennings.—The operation might also have been performed by multiplying the given sum by 3½, dividing the product by 100, and adding the result to the given sum.

```
As 100 : 103½ : : 798 f. 17 st.
                  20
                ------
Stivers 15977  }
        103½   } multiply
-----------
100)1653619½
-----------
   20)16536 3
-----------
Florins 826 16 3, Answer.
```

Exam. 4. Reduce 439 florins, 14 stivers, *currency*, to *banco*, agio at 3¾ ℔ cent.

Here, after the necessary reductions and contractions of the first and second terms, the third is multiplied by 80 by Compound Multiplication; and by dividing the product by 83, the answer is found to be 423 florins, 16 stivers, and 2 pennings.

```
As 103¾ : 100 : : F.439 14
    4       4          10
  ----    ----      -----
5)415   5)400       4397
  ----    ----         8
   83      80      -----
                83)35176(423ff.16s.2p.
                                Answer.
```

Exam. 5. Reduce £125 14 6 British to Dutch *currency*, exchange at 36/3 Flemish ℔ pound sterling, and agio at 4 ℔ cent.

In this example, by the method of aliquot parts, the value in *bank money*, is found to be £227 17 6½. The agio on this (found by multiplying by 4 and dividing by 100) is £9 2 3½, which being added to £227 17 6½, the sum,

```
   £125 14  6 at £1 16  3
     62 17  3 for 10/
     31  8  7½ ---- 5/
      7 17  2 ---- 1/3
   ----------
   £227 17  6½
      9  2  3½ for agio
   ----------
   £236 19 10, Answer.
         6
   ----------
Flor. 1421 19 stivers, Answer.
```

£236 19 10, is the result in pounds, skillings, and grotes, *currency*, which by reduction, according to rule IV. becomes 1421 florins, 19 stivers.

Ex. 6. Reduce 2084 florins, 16 stivers, *currency*, to British money, agio at $4\frac{1}{4}$ ℘ cent. and exchange 37/8 Flemish ℘ pound sterling.

By multiplication by 20 and 2, the given sum is reduced to 83392 grotes or pence: then, as $104\frac{1}{4} : 100 :: 83392$d. : 79992d. the value of the given sum in bank money; and, as 37/8 : 79992d. :: £1 : £176 19 $5\frac{1}{2}$, which is the answer. The reason of the operation is obvious.

Exercises. Reduce the following sums at the given rates Flemish ℘ pound British.

	Exercises.	*Answers.*
1.	3486 fl. 17 stiv. 1 gr. to British, at 35/3	£329 14 $6\frac{3}{4}$
2.	£345 1 8 to Flemish, at 33/11	3511 fl. 4 st. 7 pen.
3.	4600 fl. 7 stiv. to British, at $35/10\frac{1}{2}$	£427 8 $10\frac{1}{4}$
4.	£159 to Flemish, at 35/3	1681 fl. 8 st. 1 gr.
5.	£235 to Flemish, at 35/6	2502 fl. 15 st.
6.	£722 18 4 to Flemish, at 34/6	7482 fl. 3 st. $1\frac{1}{2}$ gr.
7.	2406 fl. $17\frac{1}{2}$ st. to British, at 35/4.....	£227 1 $3\frac{1}{4}$
8.	5409 fl. 5 st. 5 pen. to British, at 34/7	£521 7 6
9.	6270 fl. 11 st. 1 pen. to British, at 34/10...	£598 12 6
10.	2617 fl. $17\frac{1}{2}$ st. to British, at 35/4	£246 19 $4\frac{1}{2}$

Ex. 11. What [illegible]e value, in Flemish money, of goods sold in England for £2[illegible]9 $6\frac{1}{2}$, exchange at 34/6 Flemish ℘ pound British? *Answ.* 2[illegible]8 fl. 17 st. 4 pen.

12. Required the value, in British money, of a bill for 7767 fl. $17\frac{1}{2}$ st. exchange between England and Holland at $35/7\frac{1}{2}$ Flemish ℘ pound British. *Answ.* £726 16 $4\frac{1}{2}$.

13. Reduce 1863 rix dollars,* $48\frac{1}{2}$ stivers, to British money, exchange between England and Holland at $35/1\frac{1}{2}$ Flemish ℘ pound sterling. *Answ.* £442 4 $5\frac{1}{2}$.

14. Reduce £279 9 5 British to florins, &c. *currency*, exchange at 37/10 Flemish ℘ pound British, and agio at $3\frac{3}{8}$ ℘ cent. *Answ.* 3279 fl. 1 st. nearly.

15. Reduce 5179 florins, 16 stivers, *currency*, to British money, exchange at 36/11 Flemish ℘ pound sterling, and agio at $4\frac{1}{8}$ ℘ cent. *Answ.* £449 3 $5\frac{3}{4}$.

---

* Rix dollars being reduced to stivers by multiplying them by 50, and conversely, stivers to rix dollars by dividing them by 50, we can as readily reduce rix dollars, or find the value of rix dollars, as we can manage florins.

EXCHANGE WITH HAMBURGH.

Accounts are kept in Hamburgh in marks, schillings, and pfennings; and also in pounds, skillings, and pence Flemish.

TABLE. 6 pfennings=1 grote, or penny Flemish; 12 grotes or pence=1 skilling; 20 skillings=1 pound.

Also, 12 pfennings, or 2 grotes or pence Flemish=1 schilling, Hamburgh money; 16 schillings=1 mark.*

Hence, 1 skilling Flemish=6 schillings Hambro' money, and the schilling is equal to the stiver.

In Hamburgh, as in Amsterdam, there are two kinds of money, *banco*, or bank money, in which exchanges are reckoned, and *currency*. The agio on the former is high, varying from 18 to 25 ℘ cent.

Exam. 7. Reduce £318 7 9 British to Hambro' money, exchange at 35/6 Flemish ℘ pound British.

As £1 : £318 7 9 : : 35/6, or, by reduction, as 240d. : 76413d. : : 426 grotes : 135633 grotes, which being divided, according to the table, by 2 and by 16, is reduced to 4238 marks, 8½ schillings, or 4238 m. 8 sch. 6 pf.

This may also be wrought in the following manner:—In 35/6, the rate Flemish, multiply the skillings by 6, and add to the product half the grotes; the sum 213 is the schillings, Hambro' money, which are equivalent to 35/6 Flemish, as is evident from the table. Then by multiplying this by 318 for the pounds of the given sum, and taking aliquot parts for 7/9, we find for the answer 67816 schillings, 6½ pfennings, or, by reduction, 4238 m. 8 sch. 6½ pf. the same as before.

Exam. 8. Reduce 5127 marks, 5 schillings, Hambro', to British money, exchange at 36/2 Flemish ℘ pound sterling.

As 36/2 : 5127 m. 5 sch. or, by reduction, as 217 schillings : 82037 schillings : : £1 : £378 1, *Answer*.

Exam. 9. Reduce £86 17 1 British to Hambro' *currency*, exchange at 35/8 Flemish per pound sterling, and agio at 22 per cent.

This being wrought as the 7th example, there will result 18586 schillings, 9½ pfennings, the value in bank money. Then for currency, multiply by 22 and divide by 100, and there will result for the agio 4089 sch. 1 pf. which being added to the bank money already found, the sum, 22675 sch. 10½ pf. or 1417 m. 3 sch. 10½ pf. is the sum required in *currency*.

Exam. 10. Reduce 7854 marks, 7 schillings, Hambro' *cur-*

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* Also 2 marks=1 dollar of exchange, 3 marks=1 rix dollar.—Hamburgh money was formerly distinguished by the word *lubs*, from Lubec, the place where it was coined. Thus, in most books on Arithmetic, we find *mark lubs* instead of mark, &c. This however appears now to be laid aside among men of business, the word *Hambro'* being generally used in [illegible]ce.—By the examples it will be seen, that London gives the *certain*, and Hambu[illegible] *uncertain* price

rency, to British money, exchange at 34/11 Flemish per pound sterling, and agio at 21 per cent.

The given sum is equivalent to 125671 schillings; then, as 121 : 100 : : 125671 sch. : 103860$\frac{1}{2}$ schillings nearly, *banco;* and, again, as 34/11, or its equivalent, 209$\frac{1}{2}$ schillings : 103860$\frac{1}{2}$ schillings : : £1 : £495 15 0$\frac{3}{4}$ British, *Answer.*

Exercises. Reduce the following sums at the given rates, Flemish per pound British:

	Exercises.	*Answers.*
16.	7563 m. 8 sch. 8 pf. to British, at 34/6...	£584 12 5
17.	£275 12 6 to marks, &c. at 35/3	3643 m. 6 sch. 1$\frac{3}{8}$ g.
18.	£346 18 10$\frac{1}{2}$ to marks, &c. at 36/1$\frac{1}{2}$	4700 m.
19.	2468 m. 5 sch. to British, at 33/8	£195 10 2$\frac{1}{4}$
20.	£496 12 8 to Hambro' at 35/6	6611 m. 6 sch. 1$\frac{4}{5}$ g.
21.	6487 m. 13 sch. 4 pf. to British at 35/1$\frac{1}{2}$..	£492 11 0$\frac{1}{2}$
22.	£543 16 10 to Hambro' at 35/8	7273 m. 14 sch.
23.	8236 m. 10 sch. to British, at 35/11	£611 10 8$\frac{1}{2}$

Ex. 24. Reduce £428 14 9 British to Hambro' *currency*, exchange at 36/4 Flemish per pound British, and agio at 22$\frac{1}{2}$ per cent. *Answ.* 7155 m. 14 sch. 0 g. 4 pf.

25. Reduce 5364 m. 13 sch. Hambro' *currency* to British money, exchange at 35/9 Flemish per pound British, and agio at 23 per cent. *Answ.* £325 6 10$\frac{1}{2}$.

26. Required the value, in Hambro' bank money, of a bill for £721 13 10$\frac{1}{4}$ British, exchange between England and Hamburgh at 35/3 Flemish per pound British. *Answ.* 9539 m. 13 sch. 1$\frac{11}{12}$ g.

27. Reduce 6748 m. 11 sch. Hambro' *currency*, to British money, exchange at 36/10 Flemish per pound sterling, and agio at 23 per cent. *Answ.* £397 4 7$\frac{1}{6}$.

EXCHANGE WITH FRANCE.

Accounts are kept in France in *francs* and *centimes*. They were formerly kept in *livres*, *sous*, and *deniers*.

The *livres*, *sous*, and *deniers* were formerly used exclusively in accounts in France, but now the francs and centimes are employed, except in small transactions. The livre also had formerly the name of *franc;* but in a coinage during the time of the Republic, by some mistake in the mint, the 5 livre pieces were made too heavy, each being worth 101$\frac{1}{4}$ sous instead of 100, and the error made in the one coin was extended to the rest. The coin corresponding to the former livre was called the *franc*, and the name *livre* appropriated exclusively to the old coin; and to facilitate calculations, the old division, (similar to the divisions of our coins) was laid aside, and the franc was divided decimally, as will appear from the first of the following tables.

TABLES I. 10 centimes=1 decime; 100 centimes, or 10 decimes=1 franc.

II. 12 deniers = 1 sou; 20 sous = 1 livre; 3 livres = 1 ecu, or crown Tournois; 81 livres = 80 francs.

In exchange with France, England gives the *certain* for the *variable* price. The great advantage of the *decimal division* of money, will be felt in the work of the examples and exercises in exchange with France and Portugal.

Exam. 11. Reduce £274 5 9 British to francs, &c. exchange at 23 francs, 57 centimes ℔ pound sterling.

As £1 : 274 5 9 : : 23 f. 57 c. or, by reduction, as 240d. : 65829 d. : : 2357 centimes : 646496 centimes, or 6464 francs, 96 centimes.

This and other exercises of the same kind, may be wrought by the method shown in the margin, the division by 100 being performed merely by cutting off the two figures.

```
  2357
   274   5   9
  ----------
  9428
 16499
 4714
    589¼ for 5/
     59 ..... 0/6
     29½ ..... 0/3
 ---------
 646495¾ cents, or
 6464 f. 95¾ c.
```

Exam. 12. Reduce 2867 fr. 48 centimes to British money, exchange at 24 f. 36 c. ℔ pound sterling.

As 2436 centimes : 286748 centimes : : £1 : £117 14 3, *Answ.*

Exercises. Reduce the following sums at the given rates, French, ℔ pound British:

	Exercises.	*Answers.*
28.	£484 18 4 to French, at 23 f. 49 c.	11390 f. 69¼ c.
29.	9564 f. 33 c. to British, at 23 f. 57 c.	£405 15 8
30.	£675 18 3 to French, at 23 f. 15 c.	15647 f. 37 c.
31.	4260 f. 75 c. to British, at 23 f. 50 c.	£181 6 2
32.	6499 f. 67 c. to British, at 23 f. 40 c.	£277 15 3¼
33.	£868 17 6 to French, at 23 f. 60 c.	20505 f. 45 c.
34.	21167 f. 83 c. to British, at 23 f. 34 c.	£906 18 8
35.	£468 19 10½ to French, at 24 f. 1 c.	11260 f. 54 c.

Ex. 36. Required the value, in French money, of a bill for £483 16 11, exchange at 24 francs 37 centimes ℔ pound British. *Answ.* 11791 f. 34 c.

37. Required the value, in British money, of goods sold for 75647 francs, 18 centimes, exchange at 24 f. 51 c. per pound sterling. *Answ.* £3086 7 7¼.

EXCHANGE WITH PORTUGAL.

In Portugal, accounts are kept in *milrees* and *rees*.

TABLE. 1000 rees = 1 milree.

Also 400 rees = 1 crusado, and 4800 rees = 1 moidore: consequently, $2\frac{1}{2}$ crusados = 1 milree.

Exam. 13. Reduce £64 10 10 British to milrees, &c., exchange at $5/3\frac{1}{2}$ sterling per milree.

As $5/3\frac{1}{2}$: £64 10 10 : : 1 milree : 243 milrees, 937 rees, *Answ.*

Exam. 14. Reduce 5236 milrees, 266 rees, to British money exchange at 5/4 per milree.

As 1000 rees : 5236266 rees : : 64d. : £1396 6 9, *Answer.*

This and similar questions may be very easily wrought as in the margin, by the method of aliquot parts, and by decimals.

$4/ = \frac{1}{5}$
$1/4 = \frac{1}{8}$ of $4/$

5236·266 at 5/4

1047·2532
349·0844
1396·3376, or
£1396 6 9, *Answ.*

Exercises. Reduce the following sums at the given rates per milree:

	Exercises.	*Answers.*
38.	£479 17 6 to Portuguese, at $64\frac{1}{2}$d	1785 m. 581 r.
39.	2486 milrees, 560 rees, to British, at $63\frac{1}{4}$d.....	£655 6 $2\frac{3}{4}$
40.	594 m. 480 r. to British, at $64\frac{1}{8}$d	£158 16 9
41.	£512 8 9 to Portuguese, at $62\frac{3}{8}$d	1971 m. 703 r.

Ex. 42. If goods worth £236 14 $8\frac{1}{2}$ British be consigned to Portugal, what is their value in Portuguese money, at 59d. per milree? *Answ.* 962 m. 990 r.

43. If port wine be bought for 3245 milrees, 435 rees, what is the cost in British money, at 61d. per milree, 6 per cent. being paid for insurance? *Answ.* £874 7 $5\frac{3}{4}$.

EXCHANGE WITH SPAIN.

In Spain there are two kinds of money called *plate* and *vellon*; and accounts are kept in both in *piastres*, *reals*, and *maravedies*. The piastre is also called the *pezza*, the *dollar of exchange*; and the *piece of eight*, or of $\frac{8}{8}$.

Plate money is more valuable than vellon in the ratio of 32 to 17. Thus, 17 reals plate are equivalent to 32 reals vellon. Plate only is used in exchanges with England.

Hence, as 17 is to 32, so is any sum, plate, to the sum, vellon, equal to it; and as 32 is to 17, so is any sum, vellon, to the equivalent sum, plate.

TABLE. 34 maravedies = 1 real; 8 reals = piastre.

Also 4 piastres = 1 pistole of exchange; 375 maravedies = 1 ducat.

Exam. 15. Reduce £406 3 9 British to Spanish money, exchange at $39\frac{1}{2}$d. per piastre.

As $39\frac{1}{2}$: £406 3 9, or by reduction, as 79 halfpence : 194970 halfpence : : 1 piastre : 2467 piast. 7 reals, 27 mar.—Here England gives the *uncertain* price.

Exam. 16. Reduce 7511 piast. 5 reals to British money, exchange at $40\frac{1}{2}$d. per piastre.

As 1 p. : 7511 p. 5 r. : : $40\frac{1}{2}$d. : £1267 11 $8\frac{3}{4}$.

The second mode would have been wrought rather more easily by first finding the value at £1 per piastre, (See RULE IX. page 132.) This value would have been £7511 12 6; 5 reals being five-eighths of a piastre.

Or thus :—

7511 p. 5 r. at $3/4\frac{1}{2}$.

£1251	16	8	for 3/4
15	12	$11\frac{1}{2}$	... $\frac{1}{2}$d.
	1	$8\frac{1}{4}$	... 4 reals.
		5	... 1 .....
£1267	11	$8\frac{3}{4}$, *Answer.*	

Exercises. Reduce the following sums at the given rates per piastre :

Exercises.	*Answers.*
44. 936 p. 4 r. 15 m. to British at $38\frac{1}{2}$d.....	£150 4 $9\frac{1}{4}$
45. £167 15 4 to Spanish, at $39\frac{1}{4}$d.....	1025 p. 6 r. 23 m.
46. £809 9 8 to Spanish, at $40\frac{3}{4}$d.....	4767 p. 4 r. 2 m.
47. 5318 p. 5 r. 24 m. to British at $38\frac{3}{4}$d.....	£858 15 0

Ex. 48. Reduce £263 18 9 British to Spanish money, vellon, exchange at $41\frac{3}{4}$d. per piastre, plate. *Answ.* 2855 p. 7 r. 31 m.

49. Reduce 8429 piastres, 7 reals, vellon, to British money, exchange at $40\frac{3}{4}$d. per piastre, plate. *Answ.* £760 7 $9\frac{1}{2}$.

50. Required the value, in Spanish money, of a bill for £363 18 $10\frac{1}{2}$ British, exchange at $38\frac{3}{4}$d. per piastre. *Answ.* 2254 p. 0 r. 28 m.

51. Reduce 1927 piastres, 3 reals, vellon, to British money, exchange at $39\frac{3}{4}$d. per piastre, plate. *Answ.* £169 11 $8\frac{3}{4}$.

EXCHANGE WITH AMERICA AND THE WEST INDIES.

In the United States, accounts are pretty generally kept in dollars, which are subdivided each into 100 cents, or into 10 dimes, each containing 10 cents. In several parts of the United States, however, and in the British possessions in America, accounts are kept, as in Britain and Ireland, in pounds, shillings, and pence.

American pounds, shillings, &c. are generally distinguished by the term *currency* annexed. These pounds, in consequence of the scarcity of coins, and the use of paper money instead of them, are of much smaller value than pounds British.

When exchanges with America are made in pounds, the calculations are conducted in the same way as in exchanges between England and Ireland : but when they are made in dollars, the calculations proceed in the same manner as in exchanges with France or Portugal.

Exam. 17. Reduce £946 17 10 British, to American *currency*, at 77 ℔ cent.

As £100 : £177 : : £946 17 10 : £1675 19 11½, currency. The answer might also be found by taking aliquot parts; or by multiplying by 77, dividing the product by 100, and adding the quotient to the given sum.

Exam. 18. Reduce £796 13 6 American currency, to British money, exchange at 69 ℔ cent.

As £169 : £100 : : £796 13 6 : £471 8 1¼, British, *Answer*.

Exam. 19. Reduce 2746 dollars, 30 cents, to British money, exchange at 4/3½ British, ℔ dollar.

As 100 cents : 274630 cents : : 4/3½ : £589 6 2½, British, *Answ.* The answer might also be found very easily by the method of aliquot parts.

Exam. 20. Reduce £492 3 4 British, to dollars, &c. exchange at 4/7 British ℔ dollar.

As 4/7 : £492 3 4 : : 1 dollar : 2147 dollars, 64 cents, *Answer*.

Exercises.	*Answers.*
52. Reduce £382 8 Brit. to Amer. cur. at 71, &c.	£653 18 1
53. ..... 5611 dol. 42 cents, to Brit. at 4/5½ ℔ d.	1250 17 7
54. ..... £846 10 9 currency, to Brit. at 66, &c.	£509 19 3
55. ..... £936 5 4 Brit. to dollars at 4/6½ ℔ d.	4123 d. 1 c.

Ex. 56. What sum, American currency, is a bill for £367 4 0 British, exchange between England and America at 52 ℔ cent.? *Answ.* £558 2 10½.

57. Reduce 7593 dollars, 70 cents, to British money, exchange between England and America at 4/7 ℔ dollar. *Answ.* £1740 4 5½.

If the pupil be made fully acquainted with what precedes respecting exchanges, he will find little difficulty, with the assistance of tables, in applying the same principles to similar cases, which it would exceed the limits of the present Publication to illustrate individually. To facilitate this, the following tables are annexed, which will be found to contain what is most useful and necessary on the subject.

Leghorn. 12 denari* di pezza = 1 soldo di pezza: 20 soldi di pezza = 1 pezza of 8 reals.

Also 12 denari di lira = 1 soldo di lira; 20 soldi di lira = 1 lira; 5¾ lire, moneta buona = 1 pezza of 8 reals.

Genoa. The same table as for Leghorn serves for Genoa. Besides this, 4 lire and 12 soldi = 1 scudio di cambio, or crown of exchange; 10 lire and 14 soldi = 1 scudio d'oro marche, or gold crown.

Naples. 10 grains = 1 carlin; 10 carlins, or 100 grains = 1 ducat regno.

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* *Denari* is the plural of *denaro*, *soldi* of *soldo*, *lire* of *lira*, and *pezze* of *pezza*.

VENICE. 12 denari = 1 soldo; 20 soldi = 1 lira; 6 lire and 4 soldi = 1 ducat current, or of account; 8 lire = 1 ducat effective.

PETERSBURGH. 100 copecs = 1 ruble.

VIENNA. 4 pfenings = 1 creutzer; 60 creutzers = 1 florin; 90 creutzers, or 1½ florin = 1 rix dollar of account.

STOCKHOLM. 12 fenings, or oers = 1 skilling; 48 skillings = 1 rix dollar.

COPENHAGEN. 12 pfenings = 1 skilling; 16 skillings = 1 mark; 6 marks Danish, (or 3 marks Hambro') = 1 rix dollar.

The following exercises, which the pupil will find to be easily resolved, will serve to illustrate these tables.

Ex. 58. Reduce 2467 pezze, 12 soldi, 6 denari, of Leghorn, to British money, exchange at 50 pence sterling ℔ p. *Ans.* £514 1 9¼.

59. Reduce £467 14 8 British to pezze, &c. of Leghorn, exchange at 49 pence ℔ pezza. *Answ.* 2290 p. 18 s. 9d.

60. Reduce £947 8 6 British to pezze, &c. of Genoa, exchange at 47 pence sterling ℔ pezza. *Answ.* 4837 p. 18 s. 3 d.

61. Reduce 4263 pezze, 16 soldi, 8 denari of Genoa to British money, exchange at 44½d. sterling ℔ pezza. *Answ.* £790 11 8½.

62. Reduce £786 4 8 British to Naples money, exchange at 41½d. sterling, ℔ ducat regno. *Answ.* 4546 d. 8 c. 9 gr.

63. Reduce 3794 ducats, 4 carlins, 5 grains, Naples currency, to British money, exchange at 40¾d. ℔ ducat. *Answ.* £644 5 3¾.

64. Reduce £170 10 British to lire, &c. of Venice, exchange at 47 pence sterling ℔ lira. *Answ.* 870 l. 12 s. 9 d.

65. Reduce 793 lire, 12 soldi, and 6 denari of Venice, to British money, exchange at 45 pence sterling ℔ lira. *Answ.* £148 16 1⅛.

66. Reduce £190 19 3 British to rubles, &c. exchange at 3/1½ sterling ℔ ruble. *Answ.* 1222 r. 16 c.

67. Reduce 31416 rubles, 20 copecs, to British money, exchange at 3/4½ sterling ℔ ruble. *Answ.* £5301 9 8.

68. Reduce 255 11 8 British to rix dollars, &c. of Vienna, exchange at 4/4 sterling ℔ rix dollar. *Answ.* 1179 rix d. 55 cr. 1 pf.

69. Reduce 867 rix dollars, 25 creutzers, of Vienna, to British money, exchange at 4/7 ℔ rix dollar. *Answ.* £198 15 0¼.

70. Reduce 1045 6 8 British to Swedish money, exchange at 4/3 ℔ rix dollar. *Answ.* 4919 rix d. 10 sk. 4 f.

71. Reduce 1318 rix dollars, 20 skillings, 6 fenings, Swedish, to British money, exchange at 4/6 sterling ℔ rix d. *Answ.* £296 12 11.

72. Reduce £313 9 7 British to Danish money, exchange at 4/4½ per rix dollar. *Answ.* 1433 rix d. 4 sk. 7 pf.

73. Reduce 576 rix dollars, 4 Danish marks, 4 skillings, to Brit. money, exchange at 4/4 sterling per rix d. *Ans.* £124 19 0¾.

74. In 1795, the values of the imports and exports of Petersburgh were 31,767,952 and 23,019,175 rubles respectively. Required the values in British money, exchange between Petersburgh and London at 2/11½ per ruble. *Answ.* £4,699,009 11 4, and £3,404,919 12 8½.

75. At the end of the reign of Louis XIV. the values of the imports and exports of France, were 71,000,000, and 105,000,000 livres respectively; and at the period of the French Revolution, they had respectively increased to 380,000,000, and 424,000,000 livres. Reduce these several values to British money, exchange at 25 livres, 9 sous, 9 deniers, per pound sterling. *Answ.* £2,785,679 5 1; £4,119,666 10 0$\frac{3}{4}$; £14,909,269 4 11$\frac{3}{4}$; and £16,635,605 13 9$\frac{1}{4}$.

76. In 1784, the value of the exports from Spain to America was computed at £4,348,078 British; and the value of the imports was, in money and jewels, £9,291,237, and in merchandise £3,343,936. Required the value of each of these sums in Spanish money, exchange at 38$\frac{1}{8}$d. British per dollar. *Answ.* 27,104,901$\frac{8}{11}$ dol.; 57,919,399$\frac{47}{77}$ dol.; and 20,845,315$\frac{37}{77}$ dol.

ARBITRATION OF EXCHANGES.

When the courses of exchange between the first and the second, the second and the third, the third and the fourth, &c. of any number of places, are given; the method of finding the course of exchange between the first place and the last, corresponding to these courses, or of valuing any sum of the money of the first place in that of the last through the medium of the others, is called ARBITRATION OF EXCHANGES.

As the actual course of exchange between the first place and the last is almost always, from various circumstances, different from the arbitrated course, this method is of use in enabling a merchant in one place to discover whether he should draw and remit directly between his own place and another, or circuitously through other places.

All the operations may be performed by one or more analogies in the Rule of Proportion.

The method by the Rule of Proportion is easy and intelligible, when there are only three places concerned, or in what has been termed SIMPLE ARBITRATION. But when more than three places are concerned, or in what has been called COMPOUND ARBITRATION, the following rule, usually called the CHAIN RULE, is generally preferable.

Rule (1.) Let all the quantities of the same kind be reduced to the same denomination, if they be not so already. (2.) Let a blank* be left for the required quantity, and to

* The interrogative mark (?) may properly be placed in the blank.

This rule will be found considerably more easy in its application, than those usually given for the same purpose in books on Arithmetic. It is also expressed in general terms, so as to serve not only for the purposes of exchange, but for the comparison of weights and measures, and for any other uses to which it can be applied.

the right of it place as *consequent* the term to which it is to be equivalent: then, below the blank, place as *antecedent* the other given term which is of the same kind as the last consequent, and to the right of it place as consequent the term which is equivalent to it. (3.) Proceed thus, till all the terms are arranged in two vertical columns: then, divide the continual product of the consequents by the continual product of the antecedents, and the quotient will be the result required.

The operation may often be much abridged by striking out any antecedent and consequent that are equal, and by reducing to their lowest terms such as admit a common divisor.*

Exam. 21. When the exchange between England and America is at 80 per cent. and between England and Amsterdam at 36/4 Flemish per pound British; what is the arbitrated course of exchange between America and Amsterdam?

Here the American money is in pounds, and also the English. Then the Flemish being reduced to pence, the answer is found in pence Flemish, and becomes, by reduction, 20/2$\frac{2}{9}$ Flemish per pound American.

Flemish ? =£1 American.
American £180=£100 English.
English £1 =436d. Flemish.

$$\frac{100 \times 436}{180} = \frac{20 \times 109}{9} = 242\tfrac{2}{9}\text{d.} = 20/2\tfrac{2}{9}$$

This question might have been solved by a single analogy. Thus, as £180 : £100 :: 436d. : 242$\frac{2}{9}$d. In this mode, we multiply and divide by the same quantities as in working by the chain rule; and the same is the case in all applications of this rule. This not only shows the correctness of the rule, but also that it is nothing else than Simple or Compound Proportion exhibited in a different, and, in many cases, in a more convenient form. The learner will perceive, that, in the arrangement of the terms by the chain rule, the first antecedent and the last consequent are always of the same kind.

Ex. 77. What is the value of a franc in American money, exchange between England and America at 50 per cent. and between England and France at 24 francs, 87 centimes, per pound sterling? *Answ.* 14$\frac{324}{529}$d.

Exam. 22. Suppose a merchant in England owes a merchant in Portugal £572, whether is it better for the Portuguese mer-

* The application of logarithms greatly facilitates, in many cases, the operations by the chain rule. In using them in this way, the contraction above mentioned is generally of little use, unless when terms can be rejected.

chant to have a direct remittance from London to Lisbon at 68d. per milree, or a circular remittance through Amsterdam and Paris, exchange between London and Amsterdam being at 37/3 Flemish per pound sterling; between Amsterdam and Paris at 56 pence Flemish for 3 francs; and between Paris and Lisbon at 460 rees for 3 francs, an expense of $1\frac{1}{2}$ per cent. being incurred in the circular course?

Portuguese	?	= £572	English.
English	£1	= 37/3	Flemish = 447d.
Flemish	56d.	= 3	Francs.
Francs	3	= 460	rees, Portuguese.
	100	$98\frac{1}{2}$	

$$\frac{572 \times 447 \times 3 \times 460 \times 98\frac{1}{2}}{56 \times 3 \times 100} = \frac{143 \times 447 \times 46 \times 197}{28 \times 10}$$

$$= \frac{143 \times 447 \times 23 \times 197}{14 \times 10} = 2068757 \text{ rees} = 2068 \text{ milrees, } 757 \text{ rees,}$$

the sum Portuguese by the circular course. Again, as 68d. : £572 : : 1 milree : 2018 milrees, 824 rees, the sum Portuguese by direct remittance. Hence the circular exchange will be more advantageous to the Portuguese merchant, as by it he will receive 49 milrees, 933 rees more than by the direct. Below the antecedents and consequents we place 100 and $98\frac{1}{2}$, and multiply by them, to modify the result according to the loss ℔ cent.; $98\frac{1}{2}$ being equal to $100 - 1\frac{1}{2}$.

Ex. 78. Suppose, that a merchant in Russia owes a merchant in London 12000 rubles, and that the course of exchange between London and Petersburgh is $36\frac{1}{2}$d. ℔ ruble: it is required to find how much more or less profitable it is for the London merchant to draw directly on Petersburgh, or to draw through Paris, Amsterdam, Hamburgh, and Vienna, the course of exchange between London and Paris being at 24 f. 55 c. ℔ pound sterling; between Paris and Amsterdam at 55 grotes Flemish for 3 francs; between Amsterdam and Hamburgh at 33 stivers for 2 marks Hambro'; between Hamburgh and Vienna at 100 rix dollars Hambro' for 299 rix dollars of Vienna; and between Vienna and Petersburgh at 185 creutzers ℔ ruble. *Answ.* The direct remittance is better by £10 7 $11\frac{1}{2}$.

The following useful table, with the remarks which are subjoined to it, will form a proper conclusion to this article. It will be easily understood by a reference to the tables already given.

"TABLE of the intrinsic par of exchange between London and the principal places in Europe, 'gold against gold, and silver against silver,' calculated from assays lately made both in London and Paris:"

	GOLD.	SILVER.
Amsterdam, banco	35/10·8	37/1·7
Rotterdam, cur. ...	11 flor. 4 st.....	11 flor. 14 st.
Hamburgh, banco...	34/2·4 Flemish.....	35/1 Flemish
Paris, old coins ...	25 liv. 9 s. 9 den...	25 liv. 9 s. 9 den.
....., new coins ...	25 francs, 26 c.....	24 f. 87 c.
Genoa	45·52 pence, ster....	45·82 pence, ster.
Leghorn	49·05	46·57 .....
Naples	42	41¼ .....
Lisbon	66·5	68·4 .....
Madrid and Cadiz	36·05	39 .....
Venice	46·38	48·9 .....

In addition to the information contained in the preceding table, it may be remarked, that the American dollar, and the rix dollars of Vienna, Copenhagen, and Stockholm, are each equivalent to about 4/6 British at par.

The rubles in use in Russia prior to 1762, were of different values, from 3/7¼ to 3/9 British, valued according to their weights; but those coined since that time have been made lighter, and their values have been about 3/2 or 3/3 British.

The East India coin which is most frequently mentioned, is the *rupee*. Of this there are two kinds the *current rupee*, and the *sicca rupee*. The value of the former, according to the mint price of silver in England, is 21d., or more accurately, 21·177d.; and that of the latter 24½d., or more accurately, 24·566d.—The current is to the sicca rupee, as 100 to 116.—In India, the market price of rupees is generally much higher than their intrinsic value, the current rupee being worth about 2/, and the sicca rupee nearly 2/6. A *lack* of rupees is 100,000 rupees, or about £10,000 British, and a *crore* is 100 lacks.

DIVISION INTO PROPORTIONAL PARTS.

RULE. *To divide a given quantity into parts which shall have to each other given ratios:* as the sum of the numbers expressing the ratios, is to any one of these numbers, so is the entire quantity to be divided, to the part corresponding to the number used as the second term of the analogy.

The operation is proved by adding the several results together: if the sum be equal to the quantity to be divided, the work is right.

When all the parts, except one, have been determined, that one may be found by adding the rest together, and taking the sum from the number to be divided. In this case the operation will be proved by finding that part by the general rule: the agreement of the two results will prove the accuracy of the work.

The reason of this rule, which is of frequent use in Arithmetic, and other branches of the Mathematics, is obvious from the principles delivered in page 82.

Exam. A farm of 97 a. 3 r. 5 p. is to be divided into two parts, such that the one may be three-fourths of the other. What are the parts?

Here, the parts are evidently in the ratio of 4 and 3, the sum of which is 7. Therefore, as 7 : 4 : : 97 a. 3 r. 5 p. : 55 a. 3 r. 20 p., the greater part; and as 7 : 3 : : 97 a. 3 r. 5 p. : 41 a. 3 r. 25 p., the less part. The sum of these parts is 97 a. 3 r. 5 p., which proves the operation.

Ex. 1. Suppose a traveller to proceed from Belfast for Killarney at the rate of 6 miles ℘ hour, and another at the same time from Killarney for Belfast at the rate of 5 miles ℘ hour: where will they meet, the distance between the two places being 224 miles? *Answ.* $122\frac{2}{11}$ miles from Belfast.

2. Divide 398 into three parts which will be to one another as the numbers 5, 7, and 11. *Answ.* $86\frac{12}{23}$, $121\frac{3}{23}$, and $190\frac{8}{23}$.

3. Divide 80 miles into four parts in the ratio of 10, 9, 8, and 7. *Answ.* $23\frac{9}{17}$, $21\frac{3}{17}$, $18\frac{14}{17}$, and $16\frac{6}{17}$.

4. Divide £5000 among three persons in such a manner, that the share of the second may be one-half greater than that of the first, and the share of the third, one half greater than that of the second. *Answ.* £1052 12 $7\frac{1}{19}$, £1578 18 $11\frac{7}{19}$, and £2368 8 $5\frac{1}{19}$.

In this exercise it is easy to see, that the parts will be as the numbers 1, $1\frac{1}{2}$, and $2\frac{1}{4}$, or as 4, 6, and 9.

5. Pure water is composed of two gases, or kinds of air, called oxygen and hydrogen, in such proportions, that the weight of the former is to that of the latter as 15 to 2 Required the weight of each contained in a cubic foot, or 1000 ounces, avoirdupois weight, of water. *Answ.* $882\frac{6}{17}$ oz., and $117\frac{11}{17}$ oz.

6. How much copper and how much tin will be required to make a cannon weighing 16 c. 1 q. 20 lbs., gun-metal being composed of 100 parts of copper, and 11 of tin? *Answ.* 14 c. 3 q. $5\frac{73}{111}$ lbs., and 1 c. 2 q. $14\frac{38}{111}$ lbs.

7. The British standard gold for coinage consists of 11 parts of pure gold, and 1 part of alloy; (usually a mixture of silver and copper:) how much pure gold and how much alloy are contained in a guinea? (See exercise 77, page 80.) *Answ.* 4 dwts. $22\frac{1}{4}$ gr. & $10\frac{3}{4}$ grs.

8. The British silver coin consists of 37 parts of silver and 3 of copper; how much of each does the half crown (2/6) contain, each pound, troy weight, being coined into 66 shillings? *Answ.* 8 dwts. $9\frac{9}{11}$ grs., and $16\frac{4}{11}$ grs.

9. How much tin and copper are contained in a bell weighing 150 lbs, bell-metal being composed of three parts of copper and 1 of tin? *Answ.* $112\frac{1}{2}$ lbs., and $37\frac{1}{2}$ lbs.

10. Pewter is composed of 112 parts of tin, 15 of lead, and 6 of brass; how much of each ingredient is requisite to make a ton of pewter? *Answ.* 16 c. 3 q. $10\frac{6}{19}$ lbs., 2 c. 1 q. $0\frac{12}{19}$ lb. and 3 q. $17\frac{1}{19}$ lbs.

11. Proof spirits are composed of 48 parts of alcohol, or pure spirit, and 52 parts of water. How much of each of these is contained in 84 gallons of proof spirits? *Answ.* $40\frac{6}{25}$ gal. and $43\frac{17}{25}$ g.

12. 76 parts of nitre, 14 of charcoal, and 10 of sulphur, compose gun-powder: how much of these ingredients will be requisite to form a hundred weight of powder? *Answ.* 3 q. $1\frac{9}{25}$ lb. $15\frac{17}{25}$ lbs., and $11\frac{1}{5}$ lbs.

The following questions are illustrative of some applications of this rule which are often very useful to the Mathematical student. They are not useful however, nor even intelligible, to the mere Arithmetical pupil. By him, therefore, they should be omitted.

Exam. 2. Required the natural sine of 54° 35′ 43″, those of 54° 35′ and 54° 36′ being ·8149593 and ·8151278.

Here the difference of the two sines is ·0001685; then, as 1′ or 60″, (the difference of the arcs whose sines are given): 43″ (the excess of the intermediate arc above the less,) : : ·0001685 : ·0001208, the part of the difference corresponding to 43″; and the sum of this and the less sine is ·8150801, the sine required. The ciphers might have been rejected, and 1685 used as a whole number, and the result 1208 added to the smaller sine, as if it had been a whole number.—Had the second of the given numbers been less than the first (as is the case in the cosines,) the result of the analogy must evidently have been subtracted.

Ex. 13. The logarithm of 3·1415 being ·4971371, and that of 3·1416, ·4971509, required the logarithm of 3·141593. *Answ.* ·4971499.

14. The logarithmic tangent of 23° 27′ is 9·6372646, and that of 23° 28′ is 9·6376106: required that of 23° 27′ 54″. *Answ.* 9·6375760.

15. Required the sun's declination at Greenwich on the 10th of July 1818, at 35 minutes past 7 o'clock in the evening, his declination on the 10th at noon being 22° 18′ 47″, and on the 11th at noon 22° 11′ 10″ *Answ.* 22° 16′ 23″.

16. The moon passed the meridian of Greenwich, on the 10th of October 1818, at 36 minutes past 9 o'clock in the evening, and on the 11th at 23 minutes past 10; at what time did she pass the meridian of Mexico, in longitude 99° 5′ 15″ W.? *Answ.* 49 minutes past 9 o'clock. on the 10th.

17. On the 19th of August, 1818, the declination of Venus, at noon, at Greenwich, was 2° 0′ S., and on the 25th at noon 5° 5′ S. Required the declination on the 23d at noon, in the island of Owyhee, in longitude 155° 58′ 45″ W. *Answ.* 4° 16′ 41″ S.

FELLOWSHIP.

FELLOWSHIP is the method of determining the respective gains or losses of the partners in a mercantile company.

Fellowship is usually distinguished into two kinds, *Simple* and *Compound*, or *Single* and *Double*.

In SIMPLE OR SINGLE FELLOWSHIP the stocks or sums contributed by the several partners, all continue in trade for the same time.

In COMPOUND OR DOUBLE FELLOWSHIP the stocks continue in trade for different periods.

SIMPLE FELLOWSHIP AND BANKRUPTCY.

RULE I. As the whole stock is to the whole gain or loss, so is the stock of any partner to his gain or loss.

In the same way, the estate of a bankrupt may be divided among his creditors by this analogy: As the sum of all the claims on the estate is to its value, so is the claim of any creditor, to his dividend or share of the estate.

This rule is merely a particular application of that contained in the last article, and therefore requires no separate illustration. The method of proof is also the same.

Exam. 1. Three merchants, A, B, and C, form a joint capital, of which A contributes £700, B £1000, and C £1600. What is the share of each in a gain of £880?

Here, the sum of the stocks is £3300, the whole capital. Then, as £3300 : £880, or by contraction, as 15 : 4 : : £700 : £186 13 4, A's share; and as 15 : 4 : : £1000 : £266 13 4, B's share; and lastly, as 15 : 4 : : £1600 : £426 13 4, C's share. The sum of these shares is exactly £880, which proves the operation to be correct.

Exam. 2. A bankrupt owes to A £900; to B £860; to C £640; to D £150; to E £70; and to F £30; but his whole estate amounts only to £1250. Required the share of each creditor.

Here the sum of the debts is £2650. Then, as £2650 : £1250, or by contraction, as 53 : 25 : : £900 : £424 10 6¾, A's dividend; and as 53 : 25 : : £860 : £405 13 2½, B's dividend. In the same manner we find C's share to be £301 17 8¾; D's £70 15 1; E's £33 0 4½; and F's £14 3 0¼. The sum of all these is £1249 19 11¾, a farthing being lost by neglecting the remainders.

In the division of a bankrupt's estate, it is usual first to find *how much in the pound* he can pay, that is, how much the creditors will receive for each pound of their respective claims. Thus, resuming

the same example, we have this analogy: as £2650 : £1250, or as 53 : 25 : : £1 : 9/5 nearly, the sum that each creditor is to receive in the pound. Then, by using the method of aliquot parts as in the margin, A's share will be found to be £423 15 0, agreeing nearly with the result already found, the difference arising from taking 9/5 instead of $9/5\frac{11}{53}$. In the same way the rest of the dividends would be found.

£900 at 9/5 ₩ pound.

5/ = £$\frac{1}{4}$.....	225
4/ = £$\frac{1}{5}$.....	180
5d = $\frac{1}{12}$ of 5/.	18 15
	£423 15 0

The following rule, which may be readily employed by a person who has not learned a regular course of decimal fractions, will be found to be preferable, in a considerable degree, to that given above, in the more complicated questions in Fellowship; and it is evident, that it is equally applicable in every case of dividing into parts in a given ratio.

Rule II. (1.) Reduce the whole stock and gain to the same denomination, if they be not so already: (2.) To the latter annex six ciphers, or more if the stocks be large, and divide by the former; the quotient will be a decimal: (3.) Multiply this decimal by the several stocks successively, taking aliquot parts for shillings and pence; and from each product cut off as many figures as there were ciphers annexed: (4.) Value the figures cut off by the method shown in Reduction of decimals, problem II.

Exam. 3. If a bankrupt, whose property amounts to £2100, owe to A £826 12, to B £1263 9 6, to C £724 15 10, to D £1000, and to E £242 16 4; how much can he pay in the pound, and what is the dividend of each creditor?

In this example, the sum of the debts is £4057 13 8; and the pence in this and in £2100 are 973844 and 504000. Annexing ciphers to the latter, and dividing by the former, we obtain ·5175367, the value of which is $10/4\frac{1}{4}$ nearly, the sum which he can pay in the pound. We then, as in the margin, multiply ·5175367 by the first debt £826 12, and obtain for A's dividend £427 15 11. By proceeding in a similar manner, we should find for B's dividend £653 17 $10\frac{3}{4}$, for C's £375 2 $1\frac{1}{2}$; for D's £517 10 $8\frac{3}{4}$; and for E's £125 13 4. The sum of all these is £2100, which proves the correctness of the operation.—The learner who has studied decimal fractions,

	·5175367
	£826 12
	31052202
	10350734
	41402936
10/ = £$\frac{1}{2}$	 2587683
2/ = £$\frac{1}{10}$	 517537
	£427·7958362, or
	£427 15 11.

will see that in finding the decimal, besides other contractions, figures may often be cut from the divisor, and a smaller number of ciphers annexed than are prescribed in the rule.

Several of the following questions are expressed briefly, and it may be a useful exercise for the pupil to write them out at large.

Ex. 1. A's and B's stocks are £375 and £425 respectively: required the share of each in a gain of £240. *Answ.* A's £112 10, B's £127 10.

2. Three merchants, A, B, and C, enter into partnership: A puts into the joint stock £329, B £289 10, and C £317. Required the share of each in a gain of £583 12 6. *Answ.* A's £205 5 0¼, B's £180 12 2, C's £197 15 3½.

3. A's stock £1750, B's £1250: whole gain £565 12. *Answ.* A's £329 18 8, B's £235 13 4.

4. A's stock £349 16 7, B's £520: whole gain £346 18 9. *Answ.* A's £139 10 7¾, B's £207 8 1¼.

5. A's stock £384 18 10½, B's £186 17 4, C's £811 17 6½: whole gain £396 13 7. *Answ.* A's £110 7 1½, B's £53 11 5¼, C's £232 15 0¼.

6. A's stock £348 16 6, B's £804 11 4, C's £621 12 2: whole gain £795 18 8. *Answ.* A's £156 8 4¼, B's £360 15 6¾ C's £278 14 9.

7. B's stock £595 12 8, C's £701 11 6: whole gain £588 1 8. *Answ.* B's £270 0 7, C's £318 1 2.

8. X's stock £448 19 3, Y's £582 13 4, Z's £261 1 4: whole gain £718 18. *Answ.* X's £249 13 7, Y's £324 0 8½ Z's £145 3 8½.

9. M's stock £475 15 8, N's £346 12 4, O's £396 17 6: whole gain £279 4 10. *Answ.* M's £108 19 3½, N's £79 7 7¾, O's £90 17 10¼.

10. A's stock £178 18 8, B's £236 15 8, C's £493 18 8, D's £213 17 6: whole gain £583 10 9. *Answ.* A's £92 18 8¼, B's £122 19 7½, C's £256 10 9½, D's £111 1 7¾.

11. A debtor, the value of whose effects is only £1075 12 6, owes to A £586 13 7, to B £348 10, to C £674 5, and to D £1000. What is the dividend of each, and how much is paid in the pound? *Answ.* A's £241 16 8, B's £143 13 1, C's £277 18 7½, D's £412 4 1½; and 8/2¾ in the pound.

12. A, B, C, and D enter into partnership, and A puts in 987 gallons of whiskey at 9/10½ ₩ gallon; B, 789 gallons of rum at 16/1½; C, 238 gallons of brandy at £1 4 9, and 183 gallons of geneva at £1 1 11; and D 497 gallons of wine at 18/4½: and they contribute besides £120 in money, in equal shares. Required the share of each in a gain of £318 10 8. *Answ.* A's £75 1 4½, B's £96 13 2¾, C's £76 3 9¾, D's £70 12 3.

13. A, B, and C, enter into partnership with a joint stock of £7500, of which £3600 belong to A, £3000 to B, and the remainder to C. At the end of a year, the gain is found to be £1679 4. Required the share of this gain which each is to re-

ceive, a clear salary of £511 17 6 ℔ annum being allowed to C as acting partner. *Answ.* A's £560 6 3¾, B's £466 18 7¼, C's £651 19 1.

14. If two persons purchase a house jointly for £2000, and afterwards let it for the yearly rent of £183 6 8; what share of the yearly profit rent is each to receive, the one having contributed £850, and the other £1150 of the purchase money, and the ground rent being £44 8 ℔ year? *Answ.* £59 0 11¼, and £79 17 8¾.

COMPOUND FELLOWSHIP.

RULE (1.) Let all the times be of the same denomination, and multiply each stock by the time of its continuance in trade: (2.) Then using the products as stocks, proceed according to either of the rules for Simple Fellowship.

Exam. A and B enter into partnership: A contributes £600 for 13 months, and B £800 for 10 months. Required the share of each in a gain of £650.

Here the products are £7800 and £8000, the sum of which is £15800. Then, as £15800 : £650 : £7800 : : or, by contracting the first and second terms, as £316 : £13 : : £7800 : £320 17 8¾, A's share: and as £316 : £13 : : £8000 : £329 2 3¼, B's share. The sum of these is £650, which proves the correctness of the operation.

£600 × 13 = £7800
800 × 10 = 8000
15800

In proceeding by RULE II. of Simple Fellowship, we divide 650 with ciphers annexed by 15800, or 13 with ciphers annexed by 316, and we find for quotient ·04113924; and multiplying this successively by 7800 and 8000, and cutting eight figures from the products, we obtain £320·886072 and £329·11392, or £320 17 8¾ and £329 2 3¼, the same as before.

The reason of this rule will be evident from the consideration, that a stock of £600 for 13 months, would be the same as 13 times £600 for 1 month; and one of £800 for 10 months, the same as 10 times £800 for 1 month. Hence, if these increased stocks be employed, it is evident, that since the times are then to be regarded as equal, the operation will proceed in the same manner as those in Simple Fellowship.

Ex. 1. A's stock £280 for 5 months, B's £266 13 4 for 6 months: whole gain £331 12 6. *Answ.* A's gain £154 15 2, B's £176 17 4.

2. A's stock £170 for 8 months, B's £280 for 6 months: whole gain £250. *Answ.* A's £111 16 10¼, B's £138 3 1¾.

3. A's stock £248 12 6 for 10 months, B's £670 for 3 months, C's £512 7 6 for 6 months: whole gain £439 18 8. *Ans.* A's £144 9 7¼, B's £116 16 1, C's £178 12 11¾.

4. C's stock £178 6 8 for 18 months, D's £287 17 6 for 12 months, E's £536 5 for 10 months: whole gain £370. *Answ.* C's £103 18 9¼, D's £92 8 6½, E's £173 12 8¼.

5. A's stock £485 18 4 for 1 year, B's 279 10 for 9 months, C's £675 11 8 for 8 months: whole gain £386 15. *Answ.* A's £163 19 11, B's 70 14 11½, C's £152 0 1¼.

6. A's stock £576 15 for 11 months, B's £365 4 10½ for 15 months, C's £582 6 8 for 9 months: whole gain £568 15. *Answ.* A's £211 9 1¾, B's £182 12 1¾, C's £174 13 8½.

7. M's stock £1038 13 9 for 5 months, N's £692 9 2 for 9 months, O's £1384 18 4 for 6 months: whole gain £686 1 2 *Answ.* M's £180 10 10, N's £216 13, O's £288 17 4.

8. Three merchants A, B, and C, entered into partnership, and on the 1st of March each contributed £1000. On the 3d of May A took out £300; on the 8th of June B put in £360, and on the 20th of August C withdrew £280. On the 1st of September A put in £450; and on the 16th of October each took out £180. On the 8th of January of the following year, on making up accounts, it is found, that they have gained £1250. How is this gain to be divided among them? *Answ.* A's share £384 1 11¾, B's £512 12 1½, C's £353 5 10¾.

ALLIGATION.

ALLIGATION is a rule which is chiefly employed in calculations respecting the compounding or combining of articles of different kinds.

This rule has its name from a Latin word, which signifies *to bind*, because in the practical application of the rule, the quantities are usually *linked* or connected together by lines. It is a rule which is of little practical utility; being principally used in the solution of questions, which are of rare occurrence in real transactions. Besides, every thing that can be effected by this rule can be done in general in a better and easier way by Algebra. Hence this article will be more circumscribed in its limits than it might otherwise have been. The following is the principal problem in this rule, and indeed the only one that belongs to it exclusively.

To find in what proportions, quantities of given values must be taken, to form a compound of a given value.

RULE. (1.) Let the rates of the ingredients, all in the same denomination, be written in a line; and let the mean rate in the same denomination be written above them. (2.) Take two of the rates, one of which is greater, and the other less, than the mean rate, and write the difference between each of them and the mean rate, below the other. (3.) Proceed thus with the rates two by two, if there be

more than two, till one or more differences stand below each. (4.) Then, if only one difference stand below any rate, it will be the quantity required at that rate; but if there be more than one, their sum will be the required quantity.

The connecting or *linking* of the rates with crooked or curved lines, in the use of this rule, is attended with little advantage. Should that method be preferred, however, it can present no difficulty, as each rate less than the mean rate is to be connected with one greater, and each greater with one less, and the differences are to be set below the rate to which the line directs.

Exam. 1. In what ratios must two kinds of flour, worth 2½d. and 3¾d. ℔ lb. respectively, be taken, to make a mixture worth 3¼d. ℔ lb.?

Here the mean rate, 13 farthings, is set above the other rates, 10 farthings and 15 farthings. Then, the difference between 10 and the mean is set below 15, and the difference between 15 and the mean below 10. Hence we find, that the quantities must be in the ratio of 2 to 3; that is, for every 2 lbs., or 2 cwts. at 2½d. ℔ lb., 3 lbs., or 3 cwts. at 3¾d. ℔ lb. must be taken, to form a compound worth 3¼d. ℔ lb.

	13	
10,	15	
2,	3.	

The correctness of this operation, and of the principle on which it depends, will appear manifest from the consideration, that in selling 3 lbs. at 3¼d. per lb., instead of 3¾d., there is a loss of 1½d.; but in selling 2 lbs. at 3¼d. per lb., which cost only 2½d. per lb., there is a gain of 1½d.: and therefore, the gain on the one quantity balancing the loss on the other, the value of the compound must be exactly the mean rate.

Exam. 2. In what proportions must wines worth 10/, 14/, 17/, and 18/, ℔ gallon respectively, be mixed, so that the compound may be worth 16/ ℔ gallon?

Here by setting the difference between 10 and 16 below 17, and the difference between 17 and 16 below 10; and likewise by setting the difference between 14 and 16 below 18, and the difference between 18 and 16 below 14, we find, that for 1 gallon at 10/, we must take 2 at 14/, 6 at 17/, and 2 at 18/. By using 10/ with 18/, and 14/ with 17/, a second answer is obtained, from which it appears, that if 2 gallons at 10/, 1 at 14/, 2 at 17/, and 6 at 18/, be mixed together, the compound will also be worth 16/ ℔ gallon. It is scarcely necessary to say, that any quantities in the same ratios will serve the same purpose.

		16		
10,	14,	17,	18	
1,	2,	6,	2;	or
2,	1,	2,	6	

With respect to the reason of the operation, it is obvious from what was said respecting the preceding exercise, that 1 gallon at 10/, and 6

at 17/ each, would make a mixture worth 16/ per gallon, and likewise, that a mixture of 2 gallons at 14/, and 2 at 18/, would be worth the same per gallon; and it is evident, that both mixtures taken together must make a mixture of the same value per gallon also; and in the same way every operation in this rule may be explained.

From these principles it is also manifest, that if we should multiply or divide 1 and 6 by any number, and 2 and 2 by any number, we should still have results that would satisfy the conditions of the question. Thus, multiplying the former by 3, and dividing the latter by 2, we find for answer 3 gallons at 10/, 1 at 14/, 18 at 17/, and 1 at 18/.—Different answers may also be found by connecting the rates differently. Thus, by using 10 and 17 we get 1 and 6, and by connecting 10 and 18, we have 2 and 6; and then by using 14 and 17 we get 1 and 2. After this, by the requisite addition, we find 3, 1, 8, 6 for the required quantities; and it is obvious, that by a still farther application of these principles, different answers might be found without limit.

		16	
10,	14,	17,	18
1	1	6	6
2		2	
3,	1,	8,	6

The correctness of these results is proved by adding together the prices of 3 gallons at 10/, of 1 at 14/, of 8 at 17/, and of 6 at 18/. This will be found to be 288/, which divided by 3+1+8+6 gallons, gives exactly 16/ for the mean rate; and thus the proof may be conducted in every case.—This method of proof has been generally made a separate case of this rule, and called with no great propriety *Alligation Medial*. The operation according to the preceding rule, is usually called *Alligation Alternate*.

Exam. 3. How much linen at 2/ and at 2/5 ℔ yard, must be taken with 216 yards at 3/4, that the whole may be worth 2/6 ℔ yard at an average?

Here by taking the differences, &c. as in the margin, we find that the quantities may be in the ratio of 10, 10, and 7. Then as 7 : 10 : . 216 : 308⅔. It appears therefore, that 308⅔ yards at 2/, the same quantity at 2/5, and 216 yards at 3/4, will compose a parcel worth 2/6 ℔ yard, at an average: and various other answers might be found.—This question belongs to what is usually called *Alligation Partial*.

	30	
24,	29,	40
10	10	6
		1
10,	10,	7

Exam. 4. What quantities of tea, worth 8/, 7/6, and 6/6 ℔ lb. respectively, must be mixed together, to form a parcel containing 112lbs. worth 7/4 ℔ lb.?

Here, in finding the ratios, to make the first two terms different, the differences between 88 and 90, and 88 and 78, are set down twice. In this way it is found, that the quantities may be as 10, 20, and 12, or, by halving each term, as 5, 10, and 6. Hence, by the method of dividing into parts in a given ratio (see page 191,) as 21, the sum of these,

	88	
96,	90,	78
10	10	8
	10	2
		2
10,	20,	12

: 5 : : 112 lbs.; or by contracting, as 3 : 5 : : 16 lbs.: 26⅔ lbs.; and as 3 : 10 : : 16 lbs. : 53⅓ lbs.; and lastly, as 3 : 6 : : 16 lbs. : 32 lbs. It appears therefore, that 26⅔ lbs. at 8/, 53⅓ lbs. at 7/6, and 32 lbs. at 6/6, will form a compound of 112 lbs., worth 7/4 ℔ lb.—This question belongs to what has generally been called *Alligation Total.* This name for this particular case of Alligation, as well as those already mentioned for the other cases, is properly falling into disuse.

As many questions in this rule admit of several answers, the pupil should prove his results in working the following exercises, particularly when his answers may differ from those here given.

Ex. 1. In what proportions must sugars, worth 13d., 11½d., and 9d. ℔ lb. respectively, be compounded, that the mixture may be worth 10½d ℔ lb.? *Answ.* 3, 3, 7; 1, 2, 3; &c.

2. How much water must be added to a cask of spirits containing 84 gallons, worth 13/6 ℔ gallon, to reduce the value to 11/4½ ℔ gallon? *Answ.* 15$\frac{9}{13}$ gallons.

3. What quantities of three different kinds of raisins worth 11d., 15d., and 22d. ℔ lb. respectively, must be mixed together, to fill a cask containing 200 lbs., and to be worth 16½d. ℔ lb. *Answ.* 61⅓ lbs.; 61⅓ lbs.; and 77⅓ lbs.

4. How much land worth 17/6 ℔ acre, must be added to a farm containing 51 a. 2 r. 20 p., worth £1 14 6 ℔ acre, to reduce the average value of both together to £1 2 9? *Answ.* 115 a. 2 r. 6⅔p.

5. A box of linen, containing 1200 yards, worth at an average 3/ ℔ yard, consists of two kinds, one worth 2/8⅓, and the other worth 3/9½ ℔ yard. How much of each kind does it contain? *Answ.* 876⅓ yards, and 323 $\frac{1}{13}$ yards.

6. How much spirits, at 14/ ℔ gallon, must be added to a mixture consisting of 41 gallons at 9/6, and 59 gallons at 10/8, to make the compound worth 11/6 ℔ gallon? *Answ.* 52$\frac{7}{15}$ gallons.

7. How much first flour worth £1 11 6 per cwt. second flour wort £1 9, third flour worth £1 7 6, and fourth flour worth 17/6, must be taken, to form a ton worth £25 16 8? *Ans.* 2 c. 1 q. 20½ lbs.; 7 c. 1 q. 5½ lbs.; 4 c. 3 q. 13 lbs.; and 5 c. 1 q. 17 lbs.

8. A gentleman's labourers consist of men at 1/4, and women at 11 d. per day; and the amount of the wages of the whole is the same as if each of them had 1/2½. Required the number of the men, the number of the women being 21. *Answ.* 49.

INVOLUTION.

A POWER of any number is the product obtained by the continual multiplication of that number, repeated a certain number of times as factor.

A number, in relation to any power of it, is called the ROOT of that power.

When the proposed number is used *twice* as factor, the product is called the SECOND POWER, or the SQUARE,* of that number; when *three times*, the THIRD POWER or CUBE; when *four times*, the FOURTH POWER; when *five times*, the FIFTH POWER, &c.

Powers are often denoted by writing after the proposed number, and a little higher, the number which shows how often the proposed number is repeated as factor. This number is called the INDEX, or the EXPONENT, of the power.

Thus, $5 \times 5 = 25$, is the second power, or the square of 5, and may be written 5^2, where 2 is the index; $7 \times 7 \times 7 \times 7 = 2401$, is the fourth power of 7, and may be written 7^4, where 4 is the index, &c.

The method of finding any assigned power of a given number, or, as it is also expressed, the method of *raising* a number to any proposed power, is called INVOLUTION.

From the preceding definition of a power we have the following rule for Involution:

RULE. *To find any assigned power of a given number; or to raise a given number to any proposed power:* find the continual product of the given number repeated as factor, as often as there are units in the index of the proposed power.

The process may often be abbreviated by multiplying together powers already found. In this case, the index of the power thus found is equal to the *sum* of the indices of the powers multiplied together.

---

* The second power of a number is very improperly called the *square* of the number. The square of a *line*, or the square described on a line, is a geometrical figure having four sides each equal to the proposed line, and having its adjacent sides perpendicular to each other: but this is evidently by no means applicable to an abstract number. The mistake has arisen from the circumstance, that the *area* of a square is found *numerically* by multiplying the number expressing the length of the side, by itself, which is the same as the process by which the second power of that number is determined. It might be shown in a similar manner, that the third power of a number is with equal impropriety called the *cube* of that number, the cube being, not a power of a number, but a solid body. The terms *square* and *cube*, however, as well as *square* root and *cube* root, have been so long and so generally used in this improper sense, that it would perhaps be vain to attempt to correct the error any farther than by frequently using the legitimate expressions, *second power*, *second root*, &c. instead of them.

When the given number is either wholly or partly a decimal, the operation may often be much abbreviated, by the rule for contracting the multiplication of decimals given in page 118.

Exam. 1. Required the fifth power of 23.

Here, by multiplying 23 by itself, we find 529 for the second power of 23. By multiplying this by 23, we find 12167 for the third power. By proceeding in like manner, we find the fourth power to be 279841, and the fifth to be 6436343.

Multiply..... $\left\{\begin{array}{r} 23 = \text{1st power.} \\ 23 \end{array}\right.$

Multiply..... $\left\{\begin{array}{r} 529 = \text{2d} \\ 23 \end{array}\right.$

Multiply.... $\left\{\begin{array}{r} 12167 = \text{3d} \\ 23 \end{array}\right.$

Multiply... $\left\{\begin{array}{r} 279841 = \text{4th} \\ 23 \end{array}\right.$

Answ. 6436343 = 5th

The same result might also have been found by multiplying the second power 529 by itself, and the product 279841, which is the fourth power, by 23. The same result would also be obtained by multiplying the third power by the second.

In like manner, if it were required to find the twelfth power of a number; multiply the number by itself to find the second power; the second power by itself to find the fourth power; the fourth by itself to find the eighth; and, lastly, the eighth by the fourth to find the twelfth.

Exam. 2. Required the fifth power of $\frac{3}{8}$.

The fifth power of 3 is 243, and the fifth power of 8, 32768: the answer therefore is $\frac{243}{32768}$. *The reason* of this is evident from Multiplication of Fractions.

Exam. 3. What is the third power of $1\frac{1}{4}$?

This, by reduction to an improper fraction, becomes $\frac{5}{4}$; and by involving the numerator and denominator each to the third power, we find for answer $\frac{125}{64}$, or $1\frac{61}{64}$.—Each of the last examples might have been wrought by reducing the fractions to decimals, and then working by the general rule.

Exam. 4. Required the sixth power 1·12, true to five places of decimals.

By raising this to the third power, in the way already shown, we find 1·404928, and this being multiplied by itself, as in the margin, we find for the sixth power 1·973822.

```
1·404928 = 3d power.
8 29404·1
---------
  1404928
   561971
     5620
     1264
       28
       11
---------
1·973822 = 6th
```

Exercises. Involve the following numbers to the powers denoted by their respective indices:

Ex.	Answ.	Ex.	Answ.
1. 678^3	459684	11. $(\frac{3}{5})^7$	$\frac{1092724}{4783125}$
2. 119^3	1685159	12. $4{\cdot}367^4$	363·691179
3. 75^4	31640625	13. $1{\cdot}03^{17}$	1·652848
4. 86^5	4704270176	14. $1{\cdot}035^{18}$	1·857489
5. 9^9	387420489	15. $1{\cdot}04^{15}$†	1·800943
6. 7^{12}	13841287201	16. $1{\cdot}0475^{24}$	3·045767
7. 11^{10}	25937424601	17. $1{\cdot}05^{31}$	4·538039
8. $(\frac{2}{3})^{6}$*	$\frac{64}{729}$	18. $1{\cdot}055^{26}$	4·0231290
9. $(2\frac{2}{3})^5$	$157\frac{283}{1024}$	19. $1{\cdot}06^{34}$	7·251025
10. $(3\frac{3}{7})^4$	$116\frac{1134}{2401}$	20. $1{\cdot}07^{32}$	8·7152708

EVOLUTION, OR THE EXTRACTION OF ROOTS.

EVOLUTION is the method of finding an assigned root of a given number.‡

The INDEX of a root is a fraction whose denominator denotes the order of the root, and whose numerator is a unit. The root of a number is also expressed by prefixing to the number the sign, $\sqrt{}$ § with the number above it which denotes the order of the root. In case of the square or second root, however, the number 2 is omitted.

Thus, the fourth root of 10 is denoted by $10^{\frac{1}{4}}$, or $\sqrt[4]{10}$, and means a number whose fourth power is 10; and the second, or square root of 7 is written $7^{\frac{1}{2}}$ or $\sqrt{7}$, and means a number, such, that if it be multiplied by itself, the product will be 7.

To facilitate the extraction of the square and cube roots, it may be proper for the pupil to be familiar with the following tables:

TABLE I. The square of $1 = 1$; of $2 = 4$; of $3 = 9$; of $4 = 16$; of $5 = 25$; of $6 = 36$; of $7 = 49$; of $8 = 64$; of $9 = 81$.

TABLE II. The cube of $1 = 1$; of $2 = 8$; of $3 = 27$; of $4 = 64$; of $5 = 125$; of $6 = 216$; of $7 = 343$; of $8 = 512$; of $9 = 729$

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* The brackets enclosing this fraction, which in this use of them constitute the algebraic *vinculum* in one of its forms, denote, that the entire fraction, and not its numerator alone, is to be involved to the sixth power. The object in the next exercise, in like manner, is to find the fifth power of $2\frac{2}{3}$.

† Perhaps the easiest mode of working this exercise will be to find the sixteenth power, and divide it by 1·04. In like manner, in the 17th exercise, the thirty-second power may be divided by 1·05.

It is unnecessary to inform the Mathematical student, that involution, when great accuracy is not required, is immensely facilitated by the use of logarithms.

‡ Evolution may also be defined to be the method of finding a number, the continual product of which repeated a given number of times as factor, will amount to a given number.

§ This sign is the letter *r*, the initial of the Latin word *radix*, *a root*, changed in form by rapidity in making it, and by its appropriation to a particular use.

EXTRACTION OF THE SECOND, OR SQUARE ROOT.

RULE I. *To extract the second, or square root of a given number:* (1.) Commencing at the unit figure, cut off periods of two figures each, till all the figures of the given number are exhausted.* (2.) The first figure of the required root will be the square root of the first period, or of the greatest square contained in it, if it be not a square itself. (3.) Subtract the square of this figure from the first period; to the remainder annex the next period for a dividend; and, for part of a divisor, double the part of the root already obtained. (4.) Try how often this part of the divisor is contained in the dividend wanting the last figure, and annex the figure thus found to the parts of the root and of the divisor already determined. (5.) Then multiply and subtract, as in Division; to the remainder bring down the next period; and, adding to the divisor the figure of the root last found, proceed as before. (6.) Continue the process till all the figures in the given number have been used; and if any thing remain, proceed in the same manner to find decimals, adding, to find each figure, two ciphers, or if the given number end in an interminate decimal, the two figures that would next arise from its continuation.

Exam. 1. Required the square root of 365.

```
     3'65(19·1049
     1
     ——
29) 265
 9  261
———————
381 ) 400
  1   381
——   ——
38204 ) 190000
     4  152816
————————————
382089 ) 3718400
      9  3438801
————————————
382098 )  279599
```

Here, by placing a separating mark between 3 and 6, the given number is divided into the two periods, 3 and 65. 1, the root of the greatest integral square contained in 3, is then put in the quotient, and its square taken from the first period. To the remainder the next period is brought down, which makes for dividend 265. The first part of the divisor is found by doubling 1, the first part of the quotient. In finding, in the next place, what figure must be annexed to the part of the root already found, though 2 would be contained 13 times in 26, yet we try 9, as we know the next figure cannot be greater than 9. We annex 9, therefore, to the parts of the root and of the divisor al-

* In dividing a decimal, or a number consisting of a whole number with a decimal, into periods, the division must also commence at the unit figure or the decimal point, and must be continued both ways, if there be a whole number, and if there be an odd figure at the end of the decimal, a cipher, or if it be a periodical decimal, the figure that would next arise from its continuation, must be annexed. Thus, 417·245 will be divided thus, 4'17'·24'50; 41·66666, &c. thus, 41'·66' 66'66; and ·567 thus, ·56'70, &c.

ready found, and multiplying 29 by 9, we find for product 261, and for remainder 4. Hence, we have for root 19, and for remainder 4. Now, to find decimal figures, a point is put in the quotient, two ciphers annexed to the remainder, and 9, the figure last found, added to 29, the former divisor. We have then for dividend 4·00, and for part of a divisor 38. This part of the divisor is contained once in 40; and therefore the first figure of the decimal is 1, which is also annexed to 38. In working for the next figure, we have the divisor 382, which not being contained in 190, a cipher is annexed to the parts of the root and divisor already found, and two ciphers annexed to the dividend to find another figure. The rest of the work proceeds in the same manner, and the root, true to four places of decimals, is found to be 19·1049. The truth of this result is proved by multiplying 19·1049 by itself, and adding the remainder to the product; as the result will be exactly 365.

In this, as well as in every other case in extracting roots, in which there is a remainder, after all the significant figures have been used, the fractional part of the root would be an interminate decimal, differing from the interminate decimals which we have seen already, in its not repeating or circulating, and thus presenting no law by which it can be continued.* In this case, in any practical application, the extraction is to be carried on, till as many decimal figures are obtained as the degree of accuracy necessary in the result may require.

The principle on which the preceding rule depends, is, that *the square of the sum of two numbers is equal to the squares of the numbers with twice their product.* Thus, the square of 34 is equal to the squares of 30 and of 4 with twice the product of 30 and 4; that is, to $900+2\times30\times4+16=1156$. Hence, in extracting the second root of 1156, we separate it into two parts, 1100 and 56. Then 1100 contains 900, the square of 30, with the remainder 200; the first part of the root therefore is 30, and the remainder $200+56$ or 256. Now, according to the principle abovementioned, this remainder must be twice the product of 30 and the part of the root still to be found, together with the square of that part. Now, dividing 256 by 60, the double of 30, we find for quotient 4. Then, this part being added to 60 the sum is 64, which being multiplied by 4, the product, 256, is evidently twice the product of 30 and 4, together with the square of 4.—In the same manner the operation may be illustrated in every case. The rule is best demonstrated, however, by Algebra.

RULE II. *When the root is to be extracted to many figures, the process may be much contracted by the following rule:* Find, by rule I., half, or one more than half, the number of figures required: then to the remainder annex one figure instead of

* Continued fractions, as will appear hereafter, show the law of continuation of *square* roots: in respect to *other* roots, however, there is no method known which shows the law of their continuation.

two, and having found the divisor in the usual way, proceed according to the contracted method of dividing decimals.

Thus, suppose it had been required, in the preceding example to find the answer true to nine places of decimals; these and the two places of whole numbers are eleven figures in all; and, therefore, before commencing the contraction, it is necessary to find six figures. These have been already found to be 19·1049. Taking therefore 382098 and 279599, the divisor and remainder already found, and also the quotient 19·1049, the continuation of the preceding work will stand as in the margin, a cipher being added to the dividend, according to the rule. The first figure that results from the division is 7, which, in working the operation at full length, must have been annexed to the divisor: we therefore carry 5, for 7 times 7, to 56, the product of 7 and 8. After this, the work proceeds exactly as in the contracted mode of dividing decimals. By this means the root is found to be 19·104973174.

```
382098)2795990(19·104973174
   ····  2674691
         -------
         121299
         114630
         ------
           6669
           3821
           ----
           2848
           2675
           ----
            173
            153
            ---
             20
```

RULE III. *To extract the root of a vulgar fraction*, reduce it to its simplest form, if it be not so already, and extract the roots of both terms, if they be complete powers: otherwise divide the root of their product by the denominator.

The root may also be found by reducing the fraction to a decimal, if it be not such already, and taking the root of the decimal.

Thus, the second root of $\frac{1}{4}$ is $\frac{1}{2}$. This result may be obtained either by taking the roots of both terms, or by reducing the given fraction to the decimal ·25, the second root of which is ·5, or $\frac{5}{10}$, or $\frac{1}{2}$, the same as before.

In like manner, the root of $2\frac{1}{4}$, or $\frac{9}{4}$, is $\frac{3}{2}$, or $1\frac{1}{2}$. This result might also be obtained by extracting the root of 2·25. This would be found to be 1·5, or $1\frac{1}{2}$, as before.

Again, if it be required to find the second root of $\frac{5}{7}$; let the square root of 35 ($=5\times7$,) which will be found to be 5·9160798, be divided by the denominator 7, and there will result ·84515425, the root required. The same result would be obtained by extracting the root of ·71′42′85′71, the decimal equivalent to the given fraction.

The more advanced pupil may sometimes find the following contractions useful:—1. *If the denominator be an exact square, and the numerator*

not, divide the square root of the numerator by the square root of the denominator. 2. *If the numerator be an exact square and the denominator not*, divide the product of the square roots of the numerator and denominator by the denominator.

The following rule, which is only a particular application of the general rule to be given hereafter for finding the roots of powers in general, may be found useful in carrying a root out to a great number of figures, after it has been carried to a considerable number by the common rule:

(1.) *Find the square of the part of the root already found.* (2.) *Then make the sum of the given number and three times this square the first term of an analogy; the sum of this square and three times the given number, the second term; and the part of the root already found, the third term: the fourth term will be the required root true to nearly three times the number of figures in the part before found.*

Thus, the square root of 2 is found by the common method to be 1·414, the square of which is 1·999396. Then, as $3 \times 1{\cdot}999396 + 2 : 3 \times 2 + 1{\cdot}999396 :: 1{\cdot}414 : 1{\cdot}41421356237$, which is true, except the last figure; and if we should repeat the operation, using this number and its square, the root would be found true to about thirty places.

We might employ this rule from the beginning of the operation, by estimating the root as nearly as possible. It will in general be easier, however, to find the leading figures of the root by the common method. That method also, contracted in the way already shown, will for the most part be preferable to this, when the root is not required to be carried beyond ten or twelve figures.

Exercises. Required the square roots of the following numbers:

Exercises.	*Answers.*	*Exercises.*	*Answers.*
1. 5	2·236068	16. 33	5·7445626465
2. ·5	·7071068	17. 333	18·24828759
3. ·7	·8366600	18. 666	25·806975801
4. ·07	·2645751	19. $\frac{5}{9}$	·74535599250
5. ·06	·2449490	20. $\frac{4}{11}$	·60302268915
6. ·006	·0774597	21. $\frac{2}{3}$	·81649658093
7. 785	28·01785	22. $1\frac{13}{16}$	1·16′, or $1\frac{1}{6}$
8. 78·5	8·8600226	23. $11\frac{1}{9}$	3·3′, or $3\frac{1}{3}$
9. 562	23·70653918	24. $\frac{11}{13}$	·82915619759
10. $\frac{3}{8}$	·6123724357	25. $6\frac{1}{7}$	2·4784787961
11. $13\frac{1}{4}$	3·633180425	26. $794\frac{1}{2}$	28·181554251
12. 1728	41·56921938	27. 3333333	1825·7417671
13. $\frac{3}{50}$	·19364916731	28. $105\frac{3}{11}$	10·269106361
14. $1\frac{3}{8}$	1·172603940	29. $25381\frac{2}{3}$	159·31624734
15. $1\frac{3}{50}$	1·018577439	30. $57000\frac{7}{17}$	238·74727068442

31. 123456789 ..... 11111·1110605555
32. 987654321 ..... 31426·968052932
33. $207\frac{13}{36}$ ..... 14·411607975672
34. 822650 ..... 907·00055126775
35. $34967\frac{4}{17}$ ..... 186·99513848809
36. $29524^2 + 244^2$ ..... 29525·008247247

EXTRACTION OF THE THIRD, OR CUBE ROOT.

RULE I. *To extract the third, or cube root of a given number:* (1.) Find by trial a number nearly equal to the required root, and call it the *assumed root:* (2.) Find the cube of this root: (3.) Then as twice this cube added to the given number, is to twice the given number added to the same cube, so is the assumed root to the true root, nearly: (4.) By employing the approximate root thus found, and repeating the process, a number still nearer the true root will be obtained; and thus the process may be repeated as often as may be thought necessary.*

The operation is proved by involving the root, when found, to the third power: and the more nearly the result agrees with the given number, the more nearly correct is the root.

In the use of this rule, each result will generally be true to between two and three times the number of figures to which the assumed root was true. It may be observed also, that *each result is too nearly equal to the assumed root;* or, in other words, the correction is too small, as if the assumed root be too great, the result is also too great; otherwise, it is too small.

Exam. 1. Required the third or cube root of 34567.

Here, by cutting off three figures towards the right hand, we find for the first *period* 34, the root of which is above 3, since the third power of 3 is only 27; and hence, since the root must have

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* This rule, as well as the approximative rule already given for the extraction of the square root, is only a particular application of the general rule to be given hereafter, for the extraction of roots. The rule which is commonly employed in extracting the cube root is here subjoined for the use of any who may be disposed to prefer it. The rule given above, however, abridges the labour very greatly, as will appear by the application of both rules to any particular exercise. The labour by the common rule, indeed, of finding the cube root of a number true even to seven or eight figures, is so very great as to render the operation formidable in a high degree, and to make any shorter method very desirable. The following is the

Common Rule. (1.) Commencing at the units, divide the given number into periods of three figures each. (2.) The first figure of the required root will be the cube root of the first period, or of the greatest cube contained in it, if it be not a cube itself. (3.) Subtract the third power of this figure from the first period, and to the remainder annex the next period for a dividend, and for part of the divisor take 300 times the square of the part of the root already obtained. (4.) Try how often this part of the divisor is contained in the dividend, and annex the figure thus found to the part of the root already determined. (5.) Then, to find the complete divisor, add to the part already found 30 times the last figure of the root multiplied by the part of the root before it, and add also the square of the last figure. (6.) Then multiply and subtract as in Division; to the remainder bring down the next period, and proceed as before. (7.) Repeat the operation, till all the figures in the given number have been used; and, if any thing remain, continue the operation in the same manner to find decimals, adding, to find each figure, three ciphers: or, if there be an interminate decimal, the three figures that would next arise from its continuation.

two figures, (one for each period,) it must be greater than 30.* We may suppose it therefore to be about 32 or 33. The cube of the former we find to be 32768, and that of the latter 35937. Hence, the given number differing from each by pretty nearly the same quantity, we may suppose its root to be about 32·5. The cube of this, is found to be 34328·125. The first part of the remaining work may conveniently stand as in the margin. The work is then to proceed as in the Rule of Proportion, and the result is found to be 32·5752101, which is the required result, true except the last figure, the true result being 32·57521043, &c., as would be found by repeating the operation with 32·575, as the *assumed root*.

34328·125	34567
2	2
68656·250	69134
34567	34328·125

As 103223·250 : 103462·125 : : 32·5 : 32·5752101, *Answ.*

Exam. 2. Required the cube root of 100.

The root here is evidently between 4 and 5, but nearer the latter, as 100 differs less from 125, the third power of 5, than from 64, the third power of 4. Let then 4·6, the cube of which is 97·336, be assumed. After this, the work may stand as in the margin; and the result will be 4·64158, the required root nearly. By repeating the operation with 4·64, as the assumed root, there will result 4·641588833 for the required root still more correctly.

97·336	100
·2	2
194·672	200
100	97·336

As 294·672 : 297·336 : : 4·6 : 4·64158

Exam. 3. Required the cube root of 782140.

Here the root proper to be assumed is readily found to be 92, the cube of which is 778688. The work will then stand as in the margin, and there will result 92·13575 for the required root nearly.

778688	782140
2	2
1557376	1564280
782140	778688

As 2339516 : 2342968 : : 92 : 92·13575

By repeating the operation with 92·136, there will be found, for the required root, very nearly true, 92·135747933.

* It is scarcely necessary to remind the Mathematical pupil, that the root proper to be assumed, both in this article and the next, will be found with great ease by means of logarithms. The finding of it will also be facilitated, as in the preceding example, by dividing the given number into periods as in the Square Root, but each consisting of three figures instead of two, and by considering that there must be a figure in the root for each period.

RULE II. *To extract the cube root of a vulgar fraction*, reduce it to a decimal, and then extract the root; or multiply the numerator by the square of the denominator, find the cube root of the product, and divide it by the denominator.

The cube root of a mixed number is generally best found by reducing the fractional part to a decimal, if it be not so already, and then extracting. It may also be found by reducing the given number to an improper fraction, and then working according to the preceding directions.

Exercises. Find the cube roots of the following numbers:

Exercises.	*Answers.*	*Exercises.*	*Answers.*
1. 123	4·973190	6. 1234567	107·276572
2. 517	8·025957	7. 44·6	3·546323
3. 900	9·654894	8. $\frac{4}{15}$	·64365958974
4. 123456789	497·9338592	9. $\frac{8}{9}$	·9614997135
5. 12345678	231·120418	10. 376	7·217652

EXTRACTION OF ROOTS IN GENERAL.

The following article is intended only for the Mathematical or the more advanced Arithmetical pupil. For others, besides it being in a considerable degree difficult, it is seldom useful.

RULE. *To extract any root whatever:* (1.) Call the index of the given power n; and find by trial a number nearly equal to the required root, and call it the *assumed root.* (2.) Raise the assumed root to the power whose index is n. (3.) Then, as $n+1$ times this power added to $n-1$ times the given number, is to $n-1$ times this power added to $n+1$ times the given number, so is the assumed root to the true root nearly. (4.) The number thus found may be employed as a new *assumed root*, and the operation repeated to find a result still nearer the true root.

For the mode of investigating this rule, which is perhaps the best and most convenient that has been discovered, see the tenth of Dr. Hutton's Tracts, where it was first given.

Exam. 1. Required the 365th root of 1·06.

Here we may take 1 for the assumed root, the 365th power of which is 1; and n being 365, we have $n+1=366$, and $n-1=364$. The work will then proceed in the following manner, and the answer is found to be 1·0001596.

$$1 \times 366 = 366 \qquad 1 \times 364 = 364$$
$$1{\cdot}06 \times 364 = 385{\cdot}84 \qquad 1{\cdot}06 \times 366 = 387{\cdot}96$$

As 751·84 : 751·96 :: 1 : 1·0001596.

In extracting the fourth root, we may either use the preceding rule, or we may extract the second root of the given number, and the second root of the result. In extracting the sixth root also, we may either use

the preceding rule, or we may extract the third root of the given number, and the second root of the result: and in this way we may proceed in every case in which the index of the root to be extracted is a composite number. When the index is a prime number, however, the root must be found by the general rule.

Exam. 2. Find the value of $11^{\frac{2}{3}}$.

This expression means the third root of the second power of 11, and therefore we may extract the cube root of 121 for the required result. The answer might also be obtained by the general rule by taking 5 for the assumed root, and $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$, for n. The pupil will probably find the former method the more simple. The answer will be found to be 4·946088.

Exercises. Find the roots of the following numbers signified by their several indices:

Exercises.	*Answers.*	*Exercises.*	*Answers.*
1 $987654321^{\frac{1}{7}}$	19·27274	3. $1{\cdot}05^{\frac{1}{12}}$	1·004074
2. $100^{\frac{1}{100}}$	1·047128	4. $9^{\frac{9}{5}}$	52·19591

SERIES.

A SERIES is a succession of quantities, or terms, that depend on one another according to a certain law.

In every series, the first and last terms are called the EXTREMES, and the rest the MEANS.

Writers on Arithmetic usually treat of only two kinds of series, *equidifferent series* and *continual proportionals*. These are of more frequent and general use, than other kinds of series, and on this account claim more particular attention. Quantities in equidifferent series are also said to be in *arithmetical progression*; and continual proportionals are said to be in *geometrical progression*.*

* These names for equidifferent quantities and continual proportionals, are highly improper. Series of both kinds belong equally to Arithmetic and Geometry. The appellations, *arithmetical progression* and *geometrical progression*, should, therefore, be entirely disused, as tending to impress false ideas on the mind respecting the nature of the quantities. The term *proportion* is applied, if possible, still more improperly to equidifferent quantities, as this term is always expressive not of *equality of differences*, but of *equality of ratios*. The latest and best continental writers have accordingly rejected these terms, and substituted more appropriate ones, calling them by the names above given, or others of similar import, such as *progressions by differences*, and *progressions by quotients*. Bonnycastle, in his larger work on Algebra, though he still retains the terms, acknowledges their impropriety. With respect to the name, *continual proportionals*, here applied to the second kind of quantities, it may be observed, that, besides its being perfectly expressive of the nature of such quantities, it has long been thus applied in works on Geometry, and it is equally applicable in Arithmetic.

EQUIDIFFERENT SERIES, OR ARITHMETICAL PROGRESSION.

An EQUIDIFFERENT SERIES is that in which the terms either all increase, or all decrease, by the same quantity.*

This quantity is called the COMMON DIFFERENCE.

Thus, 5, 7, 9, 11, 13, is an equidifferent series, in which the successive terms increase by the addition of the common difference 2: and 20, 17, 14, 11, 8, is another, in which the successive terms decrease by the common difference 3.

The following are the most useful rules for the management of quantities of this kind.

RULE I. *The first term and the common difference being given, to find any other assigned term:* (1.) Multiply the common difference by the number which is equal to the number of the terms preceding the required term: (2.) Then, if it be an increasing series, add the product to the first term; otherwise, subtract it.

Exam. 1. Required the thirty-fifth term of the increasing equidifferent series, whose first term is 7, and common difference 3.

Here, 34 times precede the required term; wherefore, $34 \times 3 + 7 = 109$, is the term required.

The reason of this operation will be manifest from the consideration, that were the series to be continued to the thirty-fifth term, the first term must be increased by 34 additions of the common difference.

Ex. 1. Required the fifty-fourth term of the decreasing equidifferent series whose first term is 100, and common difference $1\frac{1}{4}$. *Answ.* $33\frac{3}{4}$.

2. First term of an increasing series 36, common difference $3\frac{3}{5}$; required the hundredth term. *Answ.* $392\frac{3}{5}$.

3. First term of a decreasing series 329, common difference $\frac{7}{8}$; required the ninty-ninth term. *Answ.* $243\frac{1}{8}$.

RULE II. *The extremes and the number of terms being given, to find the sum of the series:* Multiply the sum of the extremes by the number of terms, and take half the product.

Exam. 2. The first term of an equidifferent series is 1, its last term 312, and the number of terms 193. What is its sum?

Here, $1 + 312 = 313$, and $313 \times 193 = 60409$, the half of which is $30204\frac{1}{2}$, the sum of the series.

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* The definition of equidifferent quantities would stand more correctly, though not so simply, thus:—*An equidifferent series is that in which the terms either all increase, or all decrease, and the difference of any two adjacent terms is the same as that of any other two adjacent terms;* and *the common difference* may defined to be *the difference between two adjacent terms.*

The reason of this rule will be understood from the following property of equidifferent quantities:

In any equidifferent series, the sum of the extremes is equal to the sum of any two terms that are equally distant from them, or to double the middle term, if the number of terms be odd.

Thus, in the series 5, 8, 11, 14, 17, 20, 23, the sum of 5 and 23 is equal to the sum of 8 and 20, and of 11 and 17, and is double of the middle term 14. The reason of this will appear manifest, if it be considered that 8 and 11 respectively exceed the less extreme by the same quantities by which 20 and 17 are respectively less than the other extreme.

Hence, with respect to the sum of this latter series, it is evident that though each term were made 14, half the sum of the extremes, still the *sum* of the whole would be the same; and consequently the sum of the series will be obtained by multiplying half the sum of the extremes by the number of terms, or which is the same by multiplying the sum of the extremes by the number of terms, and taking half the product.

Ex. 4. Given the greater extreme = 1000, the common difference = $2\frac{1}{8}$, and the number of terms 367; required the sum of the series. *Answ.* $221484\frac{1}{2}$.

In working this exercise, the less extreme will be found by rule I. and the sum by rule II.

Ex. 5. Given the greater extreme = 1, the common difference = $\frac{1}{100}$, and the number of terms = 51; required the sum of the series. *Answ.* $38\frac{1}{4}$.

RULE III. *The extremes and the common difference being given, to find the number of terms:* Divide the difference of the extremes by the common difference, and add a unit to the quotient.

The reasons of this rule and of the next will be obvious from comparing them with rule I.

Ex. 6. Given the greater extreme = 500, the less = 70, and the common difference = 10; required the number of terms. *Answ.* 44.

7. Given the less extreme = 3, the greater = 579, and the common difference = 9; what is the sum of the series? *Answ.* 18915.

Here, let the number of terms be found by this rule, and the sum of the series by rule II.

Ex. 8. With the common difference 12, how many equidifferent means can be inserted between the extremes 8 and 1700? *Answ.* 140.

RULE IV. *The extremes and the number of terms being given to find the common difference:* Take a unit from the

number of terms, and by the remainder divide the difference of the extremes.

Ex. 9. Given the extremes = 3 and 300, respectively, and the number of terms = 10; required the common difference. *Answ.* 33.

10. What is the common difference of a series consisting of 1001 terms, the extremes being 1 and 100001? *Answ.* 100.

RULE V. *The extremes being given, to find any assigned number of equidifferent means:* Find the common difference by rule IV, and add it continually to the less extreme, or subtract it from the greater; the several results will be the required terms.

One mean may be found by taking half the sum of the extremes.

Exam. 3. Given the first term = 1, the last = 99, and the number of terms = 8; required the complete series.

By rule IV. the common difference is found to be 14, by the continual addition of which to 1, the entire series is found to be 1, 15, 29, 43, 57, 71, 85, 99.

Ex. 11. Insert 5 equidifferent means between 20 and 30. *Answ.* $21\frac{2}{3}$, $23\frac{1}{3}$, 25, $26\frac{2}{3}$, $28\frac{1}{3}$.

12. Required the several terms of a series the extremes of which are 4 and 49, and the number of terms 6. *Answ.* 4, 13, 22, 31, 40, 49.

RULE VI. *The sum of the series, one extreme, and the number of terms being given, to find the other extreme:* Divide twice the sum of the series by the number of terms, and from the quotient take the given extreme.

The reason of this rule is evident from rule II.*

Ex. 13. Given the first term of an equidifferent series consisting of 24 terms, = 1; required the last term, the sum of the series being = 576. *Answ.* 47.

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* If the greater extreme be denoted by g, the less by l, the common difference by d, the number of terms by n, and the sum of the series by s: then $g = l + (n-1)d$, and $s = \frac{1}{2}n(g+l)$; from which two equations, any three of these quantities being given, the rest can be found. The full resolution of these equations will, among other results, give the rules contained in the text.

The consideration of such equidifferent quantities as are usually, but very improperly said to be in *arithmetical proportion*, has been omitted from its comparative inutility. These are quantities of such a nature, that the differences of the first and second, of the third and fourth, of the fifth and sixth, &c., are equal. Of this kind are the following quantities:—2, 5; 10, 13; 21, 24, &c. The principal property of *four* such quantities is, that, as in the *continued* equidifferent quantities already explained, the sum of the extremes is equal to the sum of the means; whence it follows, that if there be four such quantities, and from the sum of the means either extreme be taken, the remainder will be the other extreme.

14. Given the number of terms = 50, and the sum of the series = 1275; required the greater extreme, the less being $3\frac{3}{4}$. *Answ.* $47\frac{1}{4}$.

15. Required the sum of the first ten thousand numbers in the natural series, 1, 2, 3, 4, &c. *Answ.* 50,005,000.

16. Required the sum of the first ten thousand odd numbers, 1, 3, 5, 7, &c. *Answ.* 100,000,000.

17. Required the sum of the first ten thousand even numbers, 2, 4, 6, 8, &c. *Answ.* 100,010,000.

18. Required the sum of the first ten thousand numbers that are divisible by 3, (3, 6, 9, 12, &c.) *Answ.* 150,015,000.

19. If a person on a journey travel the first day 30 miles, and each succeeding day a quarter of a mile less than he did the day before, how far will he travel in 30 days? *Answ.* $791\frac{1}{4}$ miles.

20. How many strokes does a common clock strike in a year? *Answ.* 56940.

21. If 120 stones be laid in a straight line, each at the distance of a yard and a quarter from the one next it; how far must a person travel who picks them up singly and places them in a heap at the distance of 6 yards from the end of the line, and in its continuation? *Answ.* 8 Irish miles, 4 furl. 35 per. 5 yds.

22. A body falling by its own weight, if it were not resisted by the air, would descend in the first second of time through a space of 16 feet and 1 inch; in the next second, through three times that space; in the third, through five times that space; in the fourth, through seven times, &c. Through what space would it fall at the same rate of increase in a minute? *Answ.* 57900 feet.

CONTINUAL PROPORTIONALS, OR GEOMETRICAL PROGRESSION.

A SERIES OF CONTINUAL PROPORTIONALS is that in which the successive terms all increase by a common multiplier, or all decrease by a common divisor.*

The common multiplier, or common divisor, is called THE RATIO OF THE SERIES, OR THE COMMON RATIO.

Thus, 3, 6, 12, 24, 48, are continual proportionals, in which the successive terms increase by the ratio 2; and 192, 48, 12, 3, $\frac{3}{4}$, &c. are continual proportionals decreasing by the ratio 4.

It may be observed, that we might regard every series of this kind, whether increasing or decreasing, as being produced by multiplication, the ratio in a decreasing series being a proper fraction. Thus, in the

* This definition would stand more correctly, though not so simply, thus:—*A series of continual proportionals is that in which the terms all increase, or all decrease; and the quotient obtained by dividing the greater of any two adjacent terms by the less, is equal to that obtained by dividing the greater of any other two adjacent terms by the less.*

series last given, the ratio or common multiplier might be considered to be $\frac{1}{4}$. In what follows, however, the ratio will be taken always greater than a unit, according to the definition already given.*

The following are the most useful rules for the management of quantities of this kind.

RULE I. *The first term and the ratio being given, to find any other proposed term:* (1.) Raise the ratio to a power whose index is equal to the number of the terms which precede the required term. (2.) Then, if it be an increasing series, multiply the first term by the result before found; otherwise divide it by that result.

Exam. 1. Required the 8th term of the series of continual proportionals, whose first term is 6, and ratio 2.

Here, the 7th power of 2 is found to be 128; which being multiplied by the first term 6, the product is 768, the 8th term.

The reason of this operation will be manifest if it be considered, that in finding the successive terms up to the 8th, the first term must be multiplied by 2, the product by 2, that product by 2, and so on, till the 8th term would be found after seven such multiplications: and it is evident, that the same result will be found by a single multiplication by the 7th power of 2. A similar illustration serves in case of a decreasing series.

Exam. 2. Required the 20th term of the series, whose first term and ratio are each 1·06.

Here, we are to multiply the 19th power of 1·06 by 1·06, or, which is the same, we are to involve 1·06 to the 20th power. This is found by involution, to be 3·207135.

Exam. 3. Required the 6th term of the decreasing series whose first term is 100, and ratio $1\frac{1}{2}$.

The 5th power of $1\frac{1}{2}$, or 1·5, is 7·59375, and 100 being divided by this, the quotient is 13·16872428, the term required.

Ex. 1. Given the first term of an increasing series 12, and its ratio 3; to find the 18th term. *Answ.* 1549681956.

2. The first term of a decreasing series is 1, and the ratio 1·07; required the 14th term. *Answ.* ·4149644.

3. The first term of an increasing series is £194·3, and the ratio 1·05; required the 31st term. *Answ.* £839·75333.

4. Given the first term of a decreasing series = 500, and the ratio = 1·04; to find the 14th term. *Answ.* 300·287.

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* It is proper for the learner to know, that *in a series of continual proportionals, the product of the extremes is equal to the product of any two terms equally distant from them; or, to the second power of the middle term, if the number of terms be odd.* The reason of this is evident, since the greater extreme exceeds the term next it in the same ratio in which the other extreme is less than the term next it.

5. Given the first term of an increasing series = 1, and the ratio = 2; required the 36th term. *Answ.* 34359738368.

6. Given the first term of an increasing series = 1, and the ratio = 3; required the 36th term. *Answ.* 50031545098999707.

RULE II. *To find the sum of a series of continual proportionals;* multiply the greater extreme by the ratio, and divide the difference between the product and the less extreme, by the difference between the ratio and a unit.

When the series is a decreasing one, and the number of terms infinite, divide the product of the ratio and the greatest term by the difference between the ratio and a unit. Or,

Divide the ratio by the difference between it and a unit, and multiply the quotient by the first term.*

Exam. 4. Given the first term of an increasing series = 4, the ratio = 3, and the number of terms = 6; to find the sum of the series.

Here, by rule I. we find the last term to be 972. Multiplying this by the ratio, we obtain 2916: and dividing 2912, the difference between this and the first term, by 2, the difference, between the ratio and a unit, we obtain 1456, the required sum.

The reason of this operation is best shown by Algebra; it may be illustrated in the following manner however: let the terms of the series be placed as in the margin; then let each term be multiplied by the ratio, and let the products be removed each one place towards the right hand. If the upper line be then subtracted from the lower, there will remain 2912 = sum × 2; and consequently the sum is equal to 2912 ÷ 2 = 1456. Now 2916 is evidently the product of the ratio and the greater extreme, and 2912 is the difference between this and the less extreme; also the divisor 2 is the difference between the ratio and a unit: and a similar illustration may be given in any other case. In a decreasing infinite series, the last term is to be regarded as nothing; and hence the reason of the rule for its summation is manifest.

$$4 + 12 + 36 + 108 + 324 + 972 = \text{sum}$$
$$12 + 36 + 108 + 324 + 972 + 2916 = \text{sum} \times 3$$

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* The following rules, which may be illustrated in the same manner as the rule given in the text, may also on some occasions be employed with advantage:

I. *To sum an increasing series:* (1.) Raise the ratio to a power whose index is equal to the number of terms: (2.) Divide the difference between the result and a unit, by the difference between the ratio and a unit: (3.) Multiply the quotient by the first term.

II. *To sum a decreasing series:* (1.) Raise the ratio to a power whose index is equal to the number of terms: (2.) From the power thus obtained, take a unit, and divide the remainder by the same power: (3.) Divide the quotient by the difference between the ratio and a unit, and multiply the result by the first term.

Exam. 5. Required the sum of the series whose least term is 45, and ratio 1·04, the number of terms being 31.

This exercise may be wrought in the same manner as the last; or perhaps more easily by the first rule in the note. Thus, by involution, we find the thirty-first power of 1·04 to be 3·37313. Then 3·37313 — 1 = 2·37313, and 1·04 — 1 = ·04; also 2·37313 ÷ ·04 = 59·32825, the product of which by 45, the first term, is 2669·77125, the required sum.

Exam. 6. Required the value of the interminate decimal ·1′8′.

This is the same as $\frac{18}{100} + \frac{18}{10000} + \frac{18}{1000000} +$ &c., continued without limit, where the ratio is evidently 100. Multiplying the first term therefore by 100, and dividing the result by 100 — 1, we obtain for the sum of the series, or the value of the decimal, $\frac{18}{99}$, or in its lowest terms, $\frac{2}{11}$.

Ex. 7. Required the sum of the series whose first and last terms are 100, and 13·16872428 respectively, and its ratio $1\frac{1}{2}$. *Answ.* 273·66255144.

8. Given the extremes = 1 and 18·42015, and the ratio = 1·06; required the sum of the series. *Answ.* 308·755983.

9. Find the sum of the infinite series whose greatest term is 100, and ratio 1·04. *Answ.* 2600.

10. Given the first term of an increasing series = 6, the ratio = 4, and the number of terms = 8; to find the sum of the series. *Answ.* 131070.

11. Given the first term of a decreasing series = 1, the ratio = 3, and the number of terms = 12; to find the sum of the series. *Answ.* $1\frac{88573}{177147}$, or $1\frac{1}{2}$ nearly.

12. Given the least term of a series = 1, the ratio = $1\frac{1}{2}$, and the number of terms = 16: required the sum of the series. *Answ.* 1311·68167114.

13. Given the least term = $\frac{1}{8}$, the ratio = 4, and the number of terms = 14: required the sum of the series. *Answ.* $11184810\frac{2}{3}$.

14. Given the greatest term = 12, the ratio = $1\frac{1}{5}$, and the number of terms = 12; to find the sum of the series. *Answ.* 53·2706.

RULE III. *The extremes and the number of terms being given, to find the ratio:* Divide the greater extreme by the less, and extract that root of the quotient whose index is one less than the number of terms.

Exam. 7. Given the extremes of a series = 3 and 192, and the number of terms = 7: required the ratio.

Here, 192 ÷ 3 = 64, the 6th root of which is 2, the ratio of the series.—In the use of this rule we must generally employ either logarithms, or the rule given in page 211.

Ex. 15. Given the extremes = 1 and 10, and the number of terms = 9: required the ratio. *Answ.* 1·333521.

RULE IV. *To find any proposed number of mean proportionals between two given numbers:* (1.) Take the two given numbers as extremes; take also the number of terms in the series two greater than the required number of means, and find the ratio by rule III: (2.) Then the product of the ratio and the less extreme will be one of the means; the product of this mean and the ratio will be another; and thus all the means may be found, whatever is their number.

When only one mean is required, it is most easily found by extracting the second root of the product of the extremes.

Exam. 8. Find three mean proportionals between 5 and 1280.

Here, the series would consist of 5 terms, and the extremes are 5 and 1280; and hence the ratio is found, by the last rule, to be 4; and by the repeated multiplication of this and of the first term, the means are found to be 20, 80, and 320.

Exam. 9. Find a mean proportional between 5 and 10.

Here, $5 \times 10 = 50$, the square root of which is 7·0710678, &c. the mean required.

Ex. 16. Find two mean proportionals between 1 and 2. *Answ.* 1·259921, and 1·587401.

17. Find a mean proportional between $\frac{1}{10}$ and 100. *Answ.* 3·16227766.

Ex. 18. If a thrasher agree to work 18 days for a farmer, on condition of receiving two grains of wheat for the first day's work, 6 grains for the second, 18 for the third, &c.: what would be the value of all he would be entitled to receive, supposing 7680 grains to fill a pint, and the wheat to be worth 7 shillings a bushel? *Answ.* £275 17 5½.

19. Suppose a house, having 20 windows, to be sold at the rate of 4/0 for the first of these windows, 6/0 for the second, 9/0 for the third, and so on; the value of each being increased by one half of itself, to find the value of the next; for how much would it be sold? *Answ.* £1329 14 0½.

20. If a father give as a portion to his daughter, aged 19, a farthing for the first year of her life, three farthings for the second, 9 farthings for the third, &c.; to how much does her portion amount? *Answ.* £605,344 10 3¼.

21. It is said that an Indian discovered the game of chess, and showed it to his sovereign, who was so much pleased with it, that he desired the inventor to ask any reward he chose. The inventor then asked one grain of wheat for the first square of the chess table, two for the second, four for the third, and so on; doubling continually to 64, the number of squares. Now, suppose it had been possible for the prince to pay this reward, what would have

been the value of the whole at 12/6 ℔ cwt., 10,000 grains being supposed to weigh a pound avoirdupois?

Answ. £10,293,942,005,418 5 $6\frac{1}{4}$.

22. Find the value of the interminate decimal ·46′3′. *Answ.* $\frac{61}{110}$.
23. Required the value of the infinite decimal ·51′85′. *Answ.* $\frac{14}{27}$.
24. Find the sum of the infinite series $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, &c. *Answ.* 1.
25. Find the sum of the infinite series $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, &c. *Answ.* $\frac{1}{2}$.

HARMONICAL PROPORTION.

It may be proper to subjoin to what has been said respecting equidifferent quantities and continual proportionals, a few observations on *harmonical proportion*, a subject which, though of minor importance, should not be entirely overlooked.

Three or four numbers are said to be in HARMONICAL PROPORTION, when the first is to the last, as the difference of the first and second is to the difference of the last and the last but one.

Thus, 2, 3, and 6, are three numbers in harmonical proportion, 2 being to 6 as 3—2 to 6—3: and 15, 12, 6, and 5, are four numbers in harmonical proportion, since 15 : 5 : : 15—12 : 6—5.

The reciprocals of *three* equidifferent numbers are in harmonical proportion. Thus, $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{8}$, the reciprocals of 2, 5, and 8, are in harmonical proportion: and if these fractions be reduced to equivalent ones having a common denominator, the numerators 20, 8, and 5, will be of the same nature.

To find *four* numbers in harmonical proportion, find *three* such numbers in the manner just shown; and then let the mean and one extreme, and the mean and the other extreme, be multiplied or divided by any numbers whatever, and the four numbers thus found will be the terms required. Thus, taking 20, 8, and 5, the numbers last found, we have, by halving the first term and the mean, and trebling the mean and the third term, 10, 4, 24, and 15, which are in harmonic proportion, since 10 : 15 : : 10—4 : 24—15; and in this manner we may find as many such numbers as we please.*

* It is worthy of remark, that the reciprocals of continual proportionals are also continual proportionals, while the reciprocals of three equidifferent numbers are in harmonical proportion, and, consequently, the reciprocals of three numbers in harmonical proportion are equidifferent numbers. Farther also, if any two numbers be taken as extremes, the harmonical mean, the mean proportional, and the equidifferent mean between them are three continual proportionals. Thus, the three means between 20 and 5, are 8, 10, and $12\frac{1}{2}$: the second of which is a mean proportional between the other two.

POSITION.*

POSITION is a rule by which, from the assumption of one or more false answers to a problem, the true one is obtained.

It admits of two varieties, *Single Position*, and *Double Position*.

In SINGLE POSITION the answer is obtained by *one* assumption: in DOUBLE POSITION it is obtained by *two*.

Single Position may be employed in resolving problems, in which the required number is any how increased or diminished in any given ratio; such as when it is increased or diminished by any part of itself, or when it is multiplied or divided by any number.

Double Position is used, when the result obtained by increasing or diminishing the required number in a given ratio, is increased or diminished by some number which is no known part or multiple of the required number; or when any root or power of the required number, is either directly or indirectly contained in the result given in the question.

SINGLE POSITION.†

RULE. (1.) Assume any number, and perform on it the operations mentioned in the question as being performed on the required number: (2.) Then, as the result thus obtained, is to the assumed number, so is the result given in the question, to the number required.

Exam. 1. Required a number to which if one half, one third, one fourth, and one fifth of itself be added, the sum may be 1644.

Suppose the number to be 60: then, if to 60 one half, one third, one fourth, and one fifth of itself be added, the sum is 137. Hence, according to the rule, as 137 : 1644 : : 60 : 720, the number required. The truth of the result is proved by adding to 720, one half, one third, &c. of itself, and the sum is found to be 1644. The number 60 was here assumed, not as being near the truth, but as being a multiple of 2, 3, 4, and 5; and in this way the operation was kept free from fractions. By the assumption of any other number, however, the answer would have been found correctly, but often not so easily. The reason of the operation is obvious from the principles of proportion.

Ex. 1. Divide £2000 between A, B, and C, giving A as much as B and a fifth part more, and C as much as both together. *Answ.* A's part £545 9 1, B's £454 10 11, C's £1000.

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* This rule is sometimes called *the Rule of False*, or *the Rule of False Position*, or *the Rule of Trial and Error*. It might properly be called *the Rule of Supposition*.

† Every question that can be resolved by this rule, may also be resolved by the rule for Double Position, or without Position, by some of the preceding rules; and hence this rule is of little importance.

2. One third of a ship belongs to A, and one fifth to B, and A's part is worth £1000 more than B's: required the value of the ship. *Answ.* £1500.

3. It is required to divide 252 into three parts, such that one third of the first, one fourth of the second, and one fifth of the third, shall all be equal to one another. *Answ.* 63, 84, and 105.

4. To find a number such that if it be multiplied by 10, and the product be divided by 13, the quotient, increased by the number itself, and by 80, will amount to 1000. *Answ.* 520.

5. Required a number to which if one half of itself, one third of that half, and one fourth of that third, be added, the sum will be 500. *Answ.* $292\frac{2}{11}$.

6. A father bequeaths to his three sons £7000 in such a manner, that if the share of the eldest be multiplied by 5, that of the second by 6, and that of the third by 7, the products are all equal. What are their shares? *Answ.* £2747 13 $3\frac{1}{4}$, £2289 14 $4\frac{3}{4}$, and £1962 12 4.

7. The number of a gentleman's horses is two fifths of the number of his black cattle, and for every four of the latter he has eleven sheep. Required the number of each, the number of the sheep exceeding that of the horses by 141. *Answ.* 24 horses, 60 black cattle, 165 sheep.

DOUBLE POSITION.

RULE I. (1.) Assume two different numbers, and perform on them separately the operations indicated in the question: (2.) Then, as the difference of the results thus obtained, is to the difference of the assumed numbers, so is the difference between the true result and either of the others, to the correction to be applied, by addition or subtraction, as the case may require, to the assumed number which gave this result.

This rule, which was first published in substance by Mr. Bonnycastle in his larger work on Arithmetic, in 1810, is the simplest and easiest that has yet appeared for the resolution of questions in which the given result is *a known number, independent on the required number:* and these questions are generally the most useful. Mr. Bonnycastle appears, however, not to have been aware, that this rule fails in relation to the whole class of questions, in which the result of the operations to be performed, according to the question, on the required number, is *not a known, determinate number, but the required number, or one depending on it*, such as some multiple or part of it. In that case the following rule will be necessary. This rule has also the advantage of being applicable in *every case whatever*.

RULE II. (1.) Having assumed two different numbers, perform on them separately the operations indicated in the question, and find the *errors* of the results. (2.) Then, as the difference of the errors, if both results be too great or both too little, or as the sum of the errors, if one result be too

great and the other too small, is to the difference of the assumed numbers, so is either error to the correction to be applied to the number that produced that error.

Exam. 1. Required a number, from which if 2 be subtracted, one third of the remainder will be 5 less than half the required number.

Here, suppose the required number to be 8, from which take 2, and one third of the remainder is 2. This being taken from one half of 8, the remainder is 2, the *first result*. Suppose again, the number to be 32, and from it take 2: one third of the remainder is 10, which being taken from the half of 32, the remainder is 6, the *second result*. Then, the difference of the results being 4, the difference of the assumed numbers 24, and the difference between 5, the true result, and 6 the result nearer it, being 1; as 4 : 24 : : 1 : 6, the correction to be subtracted from 32, since the result 6 was too great. Hence, the required number is 26.

Exam. 2. If one person's age be now only four times as great as another person's, though 7 years ago it was six times as great: what is the age of each?

Here, suppose the age of the younger to be 12 years; then would the age of the older be 48. Take 7 from each of these, and there will remain 5 and 41, their ages 7 years ago. Now, 6 times 5 is 30, which taken from 41, leaves an error of 11 years. By supposing the age of the younger to be 15, and proceeding in a similar manner, the error is found to be 5 years. Hence, as 6, the difference of the errors, (both results being too small,) is to 3, the difference of the assumed numbers, so is 5, the less error, to $2\frac{1}{2}$, the correction; which being added to 15, the sum, $17\frac{1}{2}$, is the age of the younger, and consequently that of the older must be 70.

Both the rules above given for Double Position depend on the principle, that the differences between the true and the assumed numbers, are proportional to the differences between the result given in the question and the results arising from the assumed numbers. This principle is quite correct in relation to all questions which in Algebra would be resolved by simple equations, but not in relation to any others; and hence, when applied to others, it gives only approximations to the true results. In this case the assumed numbers should be taken as near the true answer as possible. Then, to approximate the required number still more nearly, assume for a second operation the number found by the first, and that one of the two first assumptions which was nearer the true answer, or any other number that may appear to be nearer it still. In this way, by repeating the operation as often as may be necessary, the true result may be approximated to any assigned degree of accuracy. When applied in this way, Double Position is of considerable use in Algebra, affording in many cases a very convenient mode of approximating the roots of equations, and finding the values of unknown quantities in very complicated expressions, without the usual reductions.

Exam. 3. Required a number to which if twice its square be added, the sum will be 100.

It is easy to see that this number must be between 6 and 7. These numbers being assumed, therefore, the sum of 6 and twice its square is 78, and the sum of 7 and twice its square 105. Then, as 105 — 78 : 7 — 6 : : 105 — 100 : ·18; which being taken from 7, the remainder, 6·82, is the required number nearly. To this let twice its square be added, and the result is 99·8448. Then, as 105 — 99·8448 : 7 — 6·82 : : 105 — 100 : ·1746; which being taken from 7, the remainder is 6·8254, the required number still more nearly: and if the operation were repeated with this and the former approximate answer, the required number would be found true for seven or eight figures.

Ex. 1. A merchant increased his capital each year by a fourth of itself, except an expenditure of £300 ℔ annum, and at the end of four years found himself worth £5000. What was his original capital? *Answ.* £2756 9 7½.

2. Suppose every thing to be as in the last exercise, except that at the end of four years the merchant found himself possessed of *twice* his original stock: how much had he to begin with? *Answ.* £3918 11 8.

3. Required as in the two preceding exercises, supposing every thing as before, except that at the end of the time the merchant finds himself possessed of only *half* his original capital. *Answ.* £890 18 11.

4. Required a number from which if 84 be taken, three times the remainder will exceed the required number by a fourth of itself. *Answ.* 144.

5. Required a number such that if it be multiplied by 11, and 320 be taken from the product, the tenth part of the remainder will be 20 less than the number itself. *Answ.* 120.

6. Required a number whose half is as much less than 1000, as its double is greater than 999. *Answ.* 799⅜.

7. A farmer engaged a labourer on condition of paying him 1/4 a day for every day he should work, and of charging him 9d. for his boarding every day he should be idle. Now, at the end of a year (313 days) the man was entitled to £12: how many days then did he work? *Answ.* 227$\frac{13}{16}$ days.

8. How many guineas of £1 2 9, and moidores of £1 9 3 each, will pay a bill of £130, the number of pieces of both kinds being 100? *Answ.* 50 of each.

9. If 1 be added to a number, and 100 divided by the sum, the quotient is 3 less than if 1 had been subtracted from the number, and 100 divided by the remainder: required the number. *Answ.* 8·2259751.

10. Required a number which exceeds 3 times its square root by 11. *Answ.* 26·4201648.

11. Required a number to which if twice its square and 3 times its cube be added, the sum will be 2000. *Answ.* 8·506744.

12. Given the sum of two numbers = 20, and the sum of their squares = 324: required the numbers. *Answ.* 17·8740079, &c.

COMPOUND INTEREST.

The method that naturally presents itself for finding the amount of a sum at compound interest, is to find its amount at simple interest at the end of the first year; then to take this amount as a new principal, and find its amount in like manner, which would be the amount at compound interest at the end of the second year, and the principal for the third year, the amount of which must be found in like manner. Continuing the process, we should thus find the amount at the end of the proposed time. This will be illustrated in the following example.

Exam. 1. Required the amount of £2500 at the end of 4 years, at 6 ℘ cent. ℘ annum, compound interest.

Here, the amount for 1 year is £2650; the amount of which for 1 year also is £2809, the amount at compound interest for two years. The amount of this again for 1 year, or the amount of the given sum at the end of the third year, is £2977 10 9½: the amount of which for the same time is £3156 3 10, the amount of £2500 for four years.—The amount at simple interest would have been £3100, which is less than the amount at compound interest by £56 3 10.

When the time is short, this method may be practised without much trouble; but when the time is long, the labour would become very great. In this case, the methods that follow should be employed.

RULE I. *To find the amount of* one pound *sterling for any number of years, at compound interest:* (1.) Divide the amount of £100 for 1 year by 100, and the quotient will be the amount of one pound for 1 year: (2.) This amount involved to a power denoted by the number of years, will be the amount of 1 pound for that number of years.

The contracted mode of multiplication of decimals is peculiarly useful in this rule, and in computations in compound interest and annuities in general. So also is the contracted method of dividing decimals. (See *pages* 118, 121, *and* 203.)

Exam 2. Required the amount of one pound sterling for 20 years at 4½ ℘ cent. ℘ annum, compound interest.

Here, the amount of £100 for 1 year is £104·5, the hundredth part of which is £1·045, the amount of £1 for a year. The second power of this is £1·092025, the amount of £1 at the end of the second year. The product of this by itself, by the contracted method of multiplication, is £1·192518, the amount at the end of the fourth year. The square of this again is £1·422099, the amount for eight years; the square of which is £2·022366, the amount at the end of the sixteenth year. Finally, the product of

this by £1·192518 (the amount for four years) is £2·411708, or £2 8 2¾ nearly, the amount of £1 for 20 years. At simple interest, the amount would have been only £1 13.

Had the products here been found at full length, the labour would have been immense. In the last multiplication, one of the factors would have contained 49 figures, the other 13, and the product 61. It should be carefully remarked, however, that the decimal part of the amount found as above, will rarely be true in all its places. A trifling error in rejecting or over-estimating a figure at the end of a decimal may accumulate, and render the accuracy of the last figure, or the last two figures doubtful. Thus, in the preceding result, the two last figures should have been 14 instead of 08, which would occasion an error of rather more than a penny in the amount of £1000. When great accuracy is required, therefore, the amounts should be brought out to a greater number of places, and the last figure or two of the final amounts rejected, or not depended on. The larger the sum also whose interest is required, and the longer the time, this is the more necessary, as the effect of the error is the more perceptible.

Exam. 3. What is the amount, true to six places of decimals, of £1 for 6 years, payable half-yearly, at 5 ℘ cent. ℘ annum compound interest?

Here, the payments being *half-yearly*, the amount of £100 for half a year is £102 10, or £102·5; and, consequently, that of £1 for the same time is £1·025: the square of this is 1·050625, or 1·0506250, the amount for two half years, or one year. Multiplying this by 1·050625, by the contracted method, we obtain 1·1038129, the fourth power of £1·025, or the amount of £1 for two years; the third power of which is 1·3448888, the twelfth power of £1·025, or the amount of £1 for six years or if only six figures of decimals be retained, 1·344889.

Exercises. Find the amounts of £1 in the following exercises, at the given rates ℘ cent. ℘ annum:

Exercises.	*Answers.*
1. For 10 years, at 10 ℘ cent., &c.	2·593742
2. ... 17 ... 6¼ ...	2·802799
3. ... 100 ... 3 ...	19·218632
4. ... 100 ... 4 ...	50·504948
5. ... 100 ... 5 ...	131·501258
6. ... 100 ... 6 ...	339·302083

RULE II. *To find the amount, or the interest, of* any sum, *at compound interest, for a given time, and at a given rate:* Find the amount of £1 for the given time by rule I., and multiply it by the given sum; the product will be the amount required.

If the principal be subtracted from the amount, the remainder will be the interest.

Exam. 4. Required the amount of £760 14 4 for 12 years, at 5 ℔ cent. ℔ annum, compound interest.

By rule I., the amount of £1 is found to be £1·795856. This being multiplied by 760, and aliquot parts taken for 14/4, as in the margin, there results for the amount £1366 2 9; and the principal being subtracted from this, there remains for the interest, £605 8 5. The simple interest would have been £456 8 7¼. The same result would have been obtained by using instead of 14/4 the equivalent decimal, and the answer would in that case have been found by multiplication alone.

```
1·795856
      760 14 4
-----------------
107751360
12570992
    897928 for 10/
    359171 ..... 4/
     29931 ..... 4d.
-----------------
£1366,137590, or
£1366  2  9, the amount.
```

Exercises. Find the amounts of the following sums at the given rates ℔ cent. ℔ annum:

	Exercises. £	s.	d.				*Answers.* £	s.	d.
7.	£251	16	6	for 9 years,	at 5 ℔ cent., &c.	.....	390	13	3
8.	£212	0	0	 15	 4	.....	381	16	0
9.	£213	13	4	 14	..... 5½	.....	452	2	9¼
10.	£463	10	10	 12	 6	.....	932	14	8¾
11.	£295	12	6	 17	..... 4½	.....	624	15	4
12.	£495	7	6	 13	..... 3½	.....	774	14	10¾
13.	£649	13	6	 16	 5	.....	1418	3	2
14.	£582	7	6	 5	..... 5½	.....	761	2	9¾

15. If a boy 12 years old, have a legacy of £1396 16 8 left to him, how much will he have to receive at the age of 21, the legacy being improved by compound interest at 5 ℔ cent. ℔ annum.? *Answ.* £2166 18 11¼.

16. Required the amount of £589 10 5 from the third till the twenty-first year of a boy's life, at 5 ℔ cent. ℔ annum, compound interest. *Answ.* £1418 15 0½.

17. £648 from the 6th till the 21st year of a boy's life, at 4¾, &c. *Answ.* £1299 16 6½.

18. If a merchant commence trade with a capital of £1200, and each year, after paying all expenses, increase the capital of the former year by a fifth part of itself; how much will he be worth at the end of 30 years? *Answ.* £284,851 11 6¼.

RULE III. *To find the principal, which at a given rate, and in a given time, will amount to a given sum:* Or, *to find the present worth of a sum at compound interest for a given time, and at a given rate:* Divide the given sum by the amount of £1, found by rule I., and the quotient will be the principal, or present worth required

The present worth of £1 may be found by dividing it by its amount for the given time.

Exam. 5. What sum must be lent at compound interest, at 5 ℘ cent. ℘ annum, at the birth of a child, so that the amount may be £3000 6 8 at the end of 21 years?

Here, the amount of £1 for 21 years being 2·785962, we have for answer £3000·3′ ÷ 2·785962 = £1076·9469 = £1076 18 11¼.

Exercises. Required the present worths of the following sums, or the principals that would produce them, at compound interest, at the given rates ℘ cent. ℘ annum:

Exercises.							*Answers.*		
	£	s.	d.				£	s.	d.
19.	324	18	6	for 9 years,	at 5 ℘ cent.,	&c.	209	8	11¾
20.	264	11	8	12	4¾		151	12	1¼
21.	554	18	4	27	4		192	9	1
22.	1000	0	0	22	8		183	18	9½

23. A brother is to pay his sister a portion of £4500 at the end of 11 years: how much will discharge the debt at the end of 4 years, compound interest being allowed at 4½ ℘ cent ℘ annum on the sum he pays? *Answ.* £3306 14 6¾.

24. With what capital must a merchant commence trade, to be worth £15000 at the end of 12 years, if he may be expected to clear annually an eighth of his capital? *Answ.* £3649 14 7¾.

25. Whether is it better to sell a farm for £1000 payable at present, £1000 payable at the end of 5 years, and £1000 payable at the end of 10 years; or to sell it for £3000 payable at the end of 5 years, compound interest being allowed at 4 ℘ cent. ℘ annum? *Answ.* Better at three payments, by £31 14 2½.

The reason of the first rule will appear from the following considerations. The amount of £1 for a year will evidently be a hundredth part of the amount of £100: and as £1 is to its amount for a year, so is any other principal to its amount for the same time. Hence, to take a particular instance, the amount of £1 for a year at 5 per cent., will be 1·05; and by the nature of compound interest, this will be the principal for the second year. Then, as the principal £1 : £1·05, its amount : : the principal £1·05 : £$1·05^2$, which will be the amount at the end of the second year, and the principal for the third year. Again, as £1 : £1·05, its amount : : the principal £$1·05^2$: £$1·05^3$, the amount at the end of the third year, and the principal for the fourth year. In this manner it will appear, that the amount of one pound for any number of years will be equal to £1·05 raised to the power denoted by the number of years. The amount of £1 being thus determined, it is evident, that the amount of any other principal will be had by multiplying the amount of £1 by that principal, since the amount will evidently be proportional to the principal; which proves the second rule.

The third rule is evidently the converse of the second, and hence its correctness is evident.*

It may be proper here to observe, that, if interest be payable *yearly*, the amount of £1 at the end of 6 months will be the square root of its mount for a year; its amount for 4 months, or one-third of a year, the cube root of the same; for three months the fourth root; for a day the 365th root, &c. Farther, also, the amount of £1 for 2 days will be the quare, for 3 days the cube, &c. of its amount for one day; its amount or 8 months will be the square of its amount for 4 months; its amount for 9 months the cube of its amount for 3 months: and, finally, its amount for a year and a quarter will be the product of its amounts for a year, and for 3 months; and its amount for 6 years and a half will be the product of its amounts for 6 years, and for half a year; and so in similar cases. All this will appear obvious from a due consideration of the nature of compound interest.

If the interest were payable *half-yearly*, however, at a given rate, per cent. *per annum*, as in the third example, the amount at the end of a year would be more than if it were payable yearly. Thus at 4 per cent. per annum, payable half-yearly, the amount at the end of half a year would be 1·02; at the end of a year, or two half years, $1{\cdot}02^2$, or 1·0404; at the end of a year and a half, or 3 half years, $1{\cdot}02^3$; at the end of 2 years, or 4 half years, $1{\cdot}02^4$, &c.—Had it been payable quarterly, the

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* If r be put = the amount of £1 for a year, p = the principal, t = the time, and a = the amount; the second rule expressed algebraically will become $a = pr^t$.....(1); or, by taking the logarithms of both sides, $\log a = \log p + t \log r$.....(2.) From equation 1 we have $p = \frac{a}{r^t}$, which is the algebraic expression for rule III.: also from equation 2, we have $\log p = \log a - t \log r$, which is a logarithmic formula for the same purpose.

From equation 1, we have also $r = \left(\frac{a}{p}\right)^{\frac{1}{t}}$; and from equation 2, $\log r = \frac{\log a - \log p}{t}$

Each of these serves for the resolution of the problem in which it is required *to find the rate at which a given principal will amount to a given sum in a given time.* The first of them expressed in words gives this rule: *Divide the amount by the principal, and that root of the quotient which is denoted by the number of years, will be the amount of £1 for 1 year, whence the rate will be known.* Thus, if it be required to find at what rate £1000 would amount to £1360 17 3 in seven years, let the latter, with its shillings and pence reduced to a decimal, be divided by the former: the quotient is 1·3608625, the *seventh* root of which is 1·045 nearly. Hence, the rate must be 4½ per cent. per annum. The difficulty of the extraction of high roots, renders this mode of calculation inferior almost beyond comparison to that by the logarithmic formula above exhibited, unless, which is rarely the case, a degree of accuracy may be required which cannot be attained by logarithmic tables. In general, indeed, the great facility afforded by logarithms in calulation, will be as much felt in Compound Interest as in almost any other part of Mathematics.

We have also, from equatioh 2, $t = \frac{\log a - \log p}{\log r}$, — a formula which will serve *to find the time in which a given principal at a given rate will amount to a given sum.* This may also be approximated without logarithms by Position: for if the given amount be divided by the principal, the quotient will be the amount of £1 for the required time: then this and the amount of £1 for a year, being known, the time will be found, either exactly or nearly so, without much difficulty.

amounts at the end of 1, 2, 3, &c. quarters would have been $1{\cdot}01$, $1{\cdot}01^2$, $1{\cdot}01^3$, &c. In such cases, *to find the amount of £1 at the end of the proposed time, raise its amount at the end of the first payment, to the power denoted by the number of payments.*

As calculations in Compound Interest are much facilitated by the use of interest tables, a table constructed by rule I., is given at the end of the book showing the amount of £1 at compound interest for any number of years not exceeding 50, at the most useful rates. The pupil, after having wrought the preceding exercises by the rules already given, should be taught, instead of finding the amount of £1 by rule I, to take it, when he can, from the table. This table will also be useful in finding by inspection, in many instances, the time, from the principal, rate, and amount; and the rate, from the principal, time, and amount.

ANNUITIES.

An ANNUITY is a fixed sum of money payable at the ends of equal periods of time, such as years, half-years, or quarters.

Annuities are of two kinds, *Certain* and *Contingent*.

ANNUITIES CERTAIN are those which commence at a fixed time, and continue for a determinate number of years.

ANNUITIES CONTINGENT are those whose commencement, or continuance, or both, depend on some contingent event, usually the life or death of one or more individuals.

The PRESENT* VALUE of an an annuity at compound interest, is such a sum as would, if lent at compound interest for the given time, amount to the same sum to which the annuity itself would amount, if forborn during the same time.

When an annuity does not come into possession till a given time has elapsed, or some particular event has taken place, it is said to be an ANNUITY IN REVERSION.

ANNUITIES CERTAIN

RULE I. *To find the amount of an annuity, payable yearly, the payments of which are forborn for a given time, compound interest being charged on them as they become due:* (1.) Subtract a unit from the amount of £1 for a year, and from its amount for the given time at compound interest: (2.) Di-

* An annuity is commonly said to be worth as many *years' purchase* as there are pounds in the present value of an annuity of £1. Thus in the case of an annuity for 20 years at 5 per cent. per annum, because the present value of an annuity of £1 is £12·462, &c. the annuity is said to be worth about 12½ years' purchase.

vide the latter remainder by the former, and the quotient will be the amount of an annuity of £1 forborn for the proposed time: (3.) Multiply this amount by the given annuity, and the product will be the amount required.

When the payments are not yearly, instead of the amount of £1 for a year, use its amount for the interval between the payments; and instead of the number of years, use the number of payments that would have been made during the time they were remitted, and then proceed as before.

Exam. 1. If a person save £120 ℔ annum, and improve it at 5 ℔ cent. ℔ annum, compound interest, how much will he be worth at the end of 20 years?

The amounts of £1 for 1 year and for 20 years, at 5 ℔ cent. per annum, are (as found by rule I. Compound Interest) 1·05 and 2·6532977; from each of which if a unit be subtracted, there remain ·05, and 1·6532977. Let the latter of these be divided by the former, and the quotient, 33·065954, is the amount of an annuity of £1 for 20 years; then let this be multiplied by 120, and the product, £3967·91448, or £3967 18 3½, is the amount required. In this case, the gain by interest is £1567 18 3½, since the person's savings without interest, would have been £120 × 20, or £2400.

Exam 2. Let every thing be as in the last example, except that the annuity is payable *half-yearly*, instead of *yearly*.

Here, since the payments are half-yearly, there would have been 40 payments; and the amount of £1 at the end of half a year, in these circumstances, is £1·025, the 40th power of which is 2·6850723, the amount of £1 at compound interest at the end of 20 years. Then, 1·6850723 ÷ ·025 = 67·402892, is the amount, at the end of 20 years, of an annuity of £1 payable at the end of each period of 6 months. Multiply this by £60, the sum payable each half year, and the product, £4044·17352, or £4044 3 5½, is the amount required, which is £76 5 2 more than the answer of the last question. It is evident, that the more frequent the payments are, the greater is the amount: for the several gains by interest are thus put sooner to gain more interest. *(See exercise* 11.*)*

The theory of the preceding rule is much more easily and satisfactorily explained by an algebraic investigation, than it can by common Arithmetic. For the use of those, however, who are unacquainted with Algebra, the following illustration of a particular case is annexed. Let it be required to find the amount of an annuity of £1 for 8 years at 5 per cent. per annum, compound interest. At the end of the time, the eighth payment would be simply £1: the value of the seventh would be £1·05, as it would remain at interest for 1 year: that of the sixth £$1{\cdot}05^2$, as it would remain at interest 2 years: that of the fifth £$1{\cdot}05^3$; of the fourth £$1{\cdot}05^4$; of the third £$1{\cdot}05^5$; of the second £$1{\cdot}05^6$; and of the first £$1{\cdot}05^7$. Hence, the entire amount to be re-

ceived at the end of the time would be the sum of the series of continual proportionals 1, 1·05, 1·05², 1·05³, 1·05⁴, 1·05⁵, 1·05⁶, and 1·05⁷. But, by rule II., page 216, the sum of this series is $(1{\cdot}05^6 - 1) \div {\cdot}05$, which agrees with the rule here given for finding the amount of an annuity of £1. The rest is obvious.

It may serve to illustrate the nature of annuities, to show another method of resolving the first exercise, which method might also be employed in solving all questions of a similar nature.—As £5 : £100 : : £120 : : £2400, the principal which would gain £120 per annum. Then, at compound interest, the amount of £2400 for 20 years is £6367 18 3½; from which £2400 being subtracted, we have remaining £3967 18 3½, for the interest, or improvement of this imaginary principal, which is also the amount of the annuity.

When the pupil shall have learned to perform the exercises on this rule and the next, he may be taught to use Tables II. and III. at the end of the book as often as they are applicable. By this means the labour will often be greatly abridged, in the same manner as operations in compound interest are often much shortened by the use of Table I.

Exercises. Find the amounts, at compound interest, of the following annuities, payable yearly, and forborn during the given times, and at the given rates per cent. per annum:

	Exercises.						*Answers.*		
	£	s.	d.				£	s.	d.
1. Annuity	100	0	0	forborn	10 years, at	4	1200	12	2½
2. ———	13	15	9	———	14 ———	6	289	14	10¾
3. ———	56	17	6	———	9 ———	6	653	11	4¼
4. ———	11	7	6	———	12 ———	5	181	1	1¾
5. ———	34	2	6	———	8 ———	5½	331	14	11¾
6. ———	14	15	9	———	15 ———	7	371	11	10¾
7 ———	51	2	8½	———	14 ———	4½	968	2	0¼

8. If a person rent a farm at £57 14 3 ℔ year, payable yearly, and forbear paying rent for 16 years: how much will he owe to the proprietor at the end of that time, allowing him compound interest at 5 ℔ cent. ℔ annum? *Answ.* £1365 6 8.

9. Suppose a person who has a salary of £75 a year, payable half-yearly, to allow it to remain unpaid 17 years: how much will he be entitled to receive at the end of that time, compound interest being allowed at 6 ℔ cent. ℔ annum? *Answ.* £2164 17 7½.

10. Suppose a salary of £11 7 6 a year, payable at periods of two years each, to be forborn for 12 years at 5 ℔ cent. ℔ annum, compound interest: what will be its amount? *(See exercise 4.)* *Answ.* £175 10 7¼.

11. Suppose a salary of £120 ℔ annum, payable quarterly, to be forborn at compound interest for 20 years: to what sum will it amount at 5 ℔ cent. ℔ annum? *Answ.* 4083 11 3½.

RULE II. *To find the present value of an annuity at compound interest:* (1.) Find by the last rule the amount of an

annuity of £1 forborn for the given time, and at the given rate: (2.) Divide this by the amount of £1 at compound interest for the given time, and the quotient will be the present value of an annuity of £1 for the given time: (3.) Multiply this by the given annuity to find the present value required.*

In case of an annuity to continue for ever, or, as it is called, *a perpetuity*, subtract a unit from the amount of £1 for a year, or for the interval between the payments, and divide a unit by the remainder: the quotient is the present value of a perpetuity of £1; which multiply by the given perpetuity.

Or, as the given rate : £100 : : the perpetuity : its present value.

Exam. 3. Required the present value of a house held on a lease of which 22 years are unexpired, and bringing a profit rent of £45 10 per annum, payable yearly, compound interest being allowed at 6 per cent. per annum.

Here, the amount of £1 for 22 years is 3·603537. Then dividing 2·603537 by ·06, we get 43·3923; the quotient of which by 3·603537, is 12·041583, the present value of an annuity of £1 for 22 years at 6 per cent. per annum. Let this be multiplied by £45 10, and the product is £547·892026, or £547 17 10, the required value.

Exam. 4. Let every thing be the same as in the last example, except that the annuity is payable *half-yearly* instead of yearly.

In this case the amount of £1, at the end of 6 months is 1·029563, the square root of 1·06; the 44th power of which (44 being the number of payments,) or its equal, the 22d power of 1·06, is 3·603537. From this take 1, and divide the remainder, 2·603537, by ·029563, and the result by 3·603537: the quotient, 24·43916, is the present value of £1 of each payment, which being multiplied by £22 15, the half-yearly payment, the product is £555·99089, or £555 19 9¾, the amount required.

Exam. 5. Required the value of a perpetuity of £80 a year, at 6 per cent. per annum.

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* Perhaps some might prefer the following very simple rule for this problem: *As the given rate is to £100, so is the annuity to the imaginary principal which would annually produce the annuity. Then from this principal subtract its present worth found by rule* III. *for Compound Interest: the remainder will be the present worth of the annuity.*

It may be remarked, that when the payments are not yearly, different writers have viewed the subject in different lights, and given different rules for computing the present value. The method employed in example 4, is perhaps preferable; but the reader who is well acquainted with the principles of compound interest will feel it easy to form other rules founded on different suppositions. The work of example 2, proceeds on a different principle.

As £6 : £100 : : £80 : £1333 6 8 : or £80 ÷ ·06 = £1333⅓, the value required.

Exam. 6. Suppose the same perpetuity as in the last question, payable half-yearly: what is its present value?

At 6 per cent., the amount of £1 at the end of half a year, is £1·029563; and in this question each payment is £40: therefore the value of the perpetuity is £40 ÷ ·029563 = £1353·0426, or £1353 0 10¼, exceeding that found in the last question, in consequence of the frequency of the payments, by £19 14 2¼.

The reason of the first part of this rule is evident from the definition of the present value of an annuity (page 229,) and from rule I. in this article, and rule III. in Compound Interest. It might also be easily shown from rule III. of Compound Interest, that at any particular rate, as 5 per cent. the present value of an annuity of £1 is the sum of the decreasing series of continual proportionals, whose terms are the present worths of £1 at compound interest for 1 year, 2 years, 3 years, and so on; the first term being 1 ÷ 105, the ratio 1·05, and the number of terms equal to the number of years. Now the summation of this series, according to rule II. in the note, page 218, agrees exactly with the first part of the rule given above: and the summation of the *infinite* series of present worths, according to the second part of the rule in the text of the same page, agrees with the rule here given for a perpetuity.

Exercises. Find the present values of the following annuities, for the given times, and at the given rates per cent. per annum, compound interest:

	Exercises.							*Answers.*		
	£	s.	d.					£	s.	d.
12. Annuity	84	7	9	to continue	9 years,	at	5	599	16	2¾
13. ———	46	12	9	———	12	———	6	391	0	0¼
14. ———	75	0	0	———	6	———	2½.....	413	2	2¼
15. ———	58	10	0	———	5	———	3½.....	264	2	7¼
16. ———	113	15	0	———	10	———	4	922	12	3½
17. ———	95	5	6	———	13	———	10	676	15	5¼

18. How much must a person pay to have a salary of £224 per annum for 19 years, being allowed compound interest at 5 per cent. per annum? *Answ.* £2707 2 2¾.

19. Suppose a widow to be entitled to an annuity of £40 a year, payable half-yearly, from a fund, for 8 years: what is it worth at 6 per cent. per annum, compound interest? *Answ.* £252 1 3¼.

20. If a farmer have a lease of 65 acres for 36 years, at £1 12 per acre, what fine must he pay to reduce the rent to 15/ per acre, compound interest being allowed at 6 per cent. per annum? *Answ.* £807 16 2¼.

21. Required the present values of a perpetuity of £1, and of

another of £68 5, payable annually, at $4\frac{3}{4}$ per cent. per annum *Answ.* £21 1 $0\frac{1}{13}$, and £1436 16 $10\frac{2}{19}$.

22. A perpetuity of £126 6 8 at $4\frac{1}{4}$ per cent., &c. *Answ.* £2972 10 $11\frac{13}{17}$.

23. A perpetuity of £96 7 6, and another of £1, at $3\frac{1}{2}$ per cent., &c. *Answ.* £2753 11 $5\frac{1}{7}$, and £28 11 $5\frac{1}{7}$.

24. Required the present values, at 5 per cent. per annum, of a perpetuity of £1, payable yearly; of the same payable half-yearly; and of the same payable quarterly. *Answ.* £20; £20 4 $11\frac{1}{4}$; and £20 7 5.

RULE III. *To find the present value of an annuity in reversion:* (1.) Find by rule II. the present value of the annuity from the present time till the end of the period of its continuance: (2.) Find also its value for the time before it comes into possession: (3.) The difference of these two results will be the present value required.

The reason of this rule is so obvious as to require no explanation. The following rule is also founded on obvious principles, and may perhaps be preferred by some.

RULE IV. (1.) Find by rule II. the present value of the annuity during the time it is to be possessed: (2.) Then the present value of this result, found by rule III. of Compound Interest, will be the present value of the reversion.

The next rule which is *in principle* the same as the last, will perhaps be found preferable for the purposes of calculation to either of the preceding.

RULE V. Subtract a unit from the amount of £1 at compound interest during the period in which the annuity is to be possessed: divide the remainder by the amount of £1 at compound interest during the period from the present time till the termination of the annuity; and the quotient by the amount of £1 for a year, diminished by a unit; the result will be the present value of a reversion of £1.

Exam. 7. A father leaves to his eldest child for 8 years a profit rent of £280 per annum, payable yearly, and the reversion of it for the 12 years succeeding to his second child. What is the present value of the legacy of the second at 4 per cent. per annum, compound interest?

By rule III. Here, by rule II., the value of an annuity of £1 for 20 years at 4 per cent, is 13·590325, and for 8 years 6·732745; the difference of which is 6·85758, the present value of a reversion of £1 in the proposed circumstances. This being multiplied by £280, the product, £1920·1224, or £1920 2 $5\frac{1}{4}$, is the value required.

By rule IV. The present value of an annuity of £1 for 12 years is found by rule II. to be 9·385073; and by rule III. of Compound Interest, the present value of this for 8 years is found to be 6·857579, as before.

By rule V. The amount of £1 at compound interest for 12 years, diminished by a unit, is ·601032; and the amount of £1 for 20 years is 2·191123. Divide the former by the latter, and the result by ·04, and there will finally result 6·857579, the same present value as before.

Exercises. Required the present values of the following annuities in reversion, at the given rates per cent. per annum:

	Exercises.							*Answers.*		
	£	s.	d.					£	s.	d.
25. Ann.	135	10	9,	after 6 yrs.	& for	8 yrs.	at 5½..	622	13	6
26.	79	12	6,	 4	.....	6	..... 5 ..	332	9	11¼
27.	58	9	10,	 3	.....	7	..... 4½..	302	0	8¾
28.	54	12	3,	 4	.....	8	..... 7 ..	248	15	8½
29. A Perpetuity of	£84	7	6,	after 8 years,			at 7 ..	701	10	7
30.	136	17	9,	 8	.....		5 ..	1853	0	4¼

31. What fine must be paid, to change into a perpetuity, a lease for 16 years, which brings a profit rent of £71 13 3 per annum, payable yearly, compound interest being allowed at 4¼ per cent. per annum? *Answ.* £866 6 8½.

32. What fine must be paid to add 25 years to a lease which brings a profit rent of £112 10, and of which 14 years are unexpired, compound interest being allowed at 5 ℔ cent. ℔ annum? *Answ.* £800 16 4¾.

33. What is the present value of the reversion of a perpetuity of £60 ℔ annum, payable yearly, but not to come into possession till the expiration of 100 years, compound interest being allowed at 6 ℔ cent. ℔ annum?* *Answ.* £2 18 11¼.

ANNUITIES CONTINGENT, OR LIFE ANNUITIES.

LIFE ANNUITIES are those whose commencement or termination, or both, depend on the extinction of one or more lives.

When life annuities are in possession, they are often called simply ANNUITIES ON LIVES; but when they are in reversion, they are generally called ANNUITIES ON SURVIVORSHIPS.

* This question may tend to correct a mistake that pretty generally prevails respecting the comparative values of long leases and perpetuities, the latter being supposed to exceed the former in value in a far greater degree than they really do. In the case on which this exercise is founded, the difference of present values is no more than £2 18 11¼.

The VALUE OF A LIFE is the present value of an annuity of £1 to continue during that life.

The EXPECTATION OF A LIFE of a given age, is the mean period during which persons of that age live.

The COMPLEMENT OF A LIFE is double the expectation of the same life.

The calculation of life annuities depends on the joint application of the rules of compound interest, and of the doctrine of chances, to tables deduced from observations on the duration of human life. In what follows on this subject, a selection of the rules most generally useful will be given. For the theory of these rules, which is of a nature too complicated to be given in a work like the present, the reader who wishes to become thoroughly acquainted with this interesting and difficult subject, may have recourse to the writings of Simpson, De Moivre, and more particularly of Dr. Price and Mr. Morgan, where the subject will be found treated at great length, both in theory and practice.

The duration of life being different in different countries, calculations have been founded on the registers of births and deaths kept in London, Breslaw, Northampton, and various other places. Tables IV. and V at the end of the book, which are employed in what follows, are founded on the Northampton register, which is thought to serve, for the generality of places, better than any other.

RULE I. *To find the present value of an annuity to continue during the life of a person whose age is given:* Take from table IV. the value of £1 for the given age and rate, and multiply it by the given annuity.

Exam. 1. What should be given at 6 ℘ cent. ℘ annum, for a farm worth £36 ℘ annum, held on a lease of one life aged 58 years?

Here, by the table, the value of an annuity of £1 on the life of a person aged 58 years, is, at 6 ℘ cent., 8·173, the product of which by £36 is £294·228, or £294 4 6¾, the value required.

Ex. 1. If a person aged 38 years, have a salary of £138 10 ℘ year for life, what is its present value at 5 ℘ cent. ℘ annum? *Answ.* £1678 1 3¾.

2. What should a person aged 32 years, pay, at 4 ℘ cent. ℘ annum, to have for life a yearly salary of £180? *Answ.* £2609 2.

3. If a farm of 28 acres be held at £1 14 6 ℘ acre, on a life aged 41 years, what fine must be paid, at 5 per cent. per annum to reduce the rent to 10 shillings per acre? *Answ.* £401 2 9¼.

RULE II. *To find the present value of an annuity which is to continue during* the joint lives *of two persons, and to cease when either of them dies:* In table V. find the age of the younger, or of either if they be equal, in the first column; and in the same division of the table, in the second column, find the age of the other; opposite to the latter is the value of an annuity of £1, which multiply by the given annuity.

Exam. 2. What is the value at 4 per cent., of an annuity of £90 per annum, to continue during the joint lives of two persons, whose ages are 15 and 50 years respectively?

Here, by table V., the value of an annuity of £1 is £9·872, the product of which by £90 is £888·480, or £888 9 7¼, the value required.

Exam. 3. Required the present value, at 6 per cent., of an annuity of £120 per annum, which is to cease, when either of two persons, aged 14 and 57 years respectively, shall die.

Neither of these ages being in the table, recourse must be had to the method of proportional parts. (See page 191.) Thus, the table gives, for 10 and 55, 7·951, and for 15 and 55, 7·812. The difference of these is ·139, four fifths of which being subtracted from 7·951, the remainder, 7·840, is the value of £1 for the ages 14 and 55. In like manner, the table gives for 10 and 60, 7·250, and for 15 and 60, 7·135; four fifths of the difference of which being taken from 7·250, the remainder, 7·158, is the value of £1 for the ages 14 and 60. Hence we have, for 14 and 55, 7·840, and for 14 and 60, 7·158; two fifths of the difference of which being subtracted from 7·840, the remainder, 7·567, is very nearly the present value of £1 for the ages 14 and 57: and this being multiplied by £120, the product, £908·04, or £908 0 9½, is the value required.

Ex. 4. Required the present value, at 6 per cent., of an annuity of £39 10, on the joint continuance of two lives of 25 and 73 years. *Answ.* £182 8 10¼.

5. Required, at 5 per cent., the present value of an annuity of £43 12 6 on the joint lives of two persons, whose ages are 44 and 51 years respectively. *Answ.* 342 9 5¼.

RULE III. *To find the present value of an annuity to continue during the* longer *of two lives:* From the sum of the values of the single lives (found in table IV.) subtract the value of the joint lives (found by rule II.:) the remainder is the present value of an annuity of £1 on the longer of the two lives.

Exam. 4. For how much should a house be sold which brings a profit rent of £41 5 per annum, and is held by a lease on the longer of two lives aged 20 and 35 years, compound interest being allowed at 6 per cent. per annum?

By table IV. the values of the lives are 12·398 and 11·236, the sum of which is 23·634. Also, by table V., the value of them jointly is 9·451, which taken from the preceding sum, leaves 14·183, the value of an annuity of £1 on the life of the longest liver; which being multiplied by £41 5, the product, £585·049, or £585 0 11¾, is the present value required.

Ex. 6. What should be paid, at 5 per cent., for the purchase of

a profit rent of £68 5 per annum, to continue during the longer of two lives, aged 38 and 42 years? *Answ.* £1001 18 4¼.

7. What should a man, aged 44, pay, at 5 per cent., to secure, during his own life, and that of his wife,* aged 39, an annuity of £200 a year? *Answ.* £2892 9 3¼.

RULE IV. *To find the value of an annuity during the joint continuance of three lives; or which is to terminate on the extinction of any one of them:* Find by rule II., the value of the two eldest jointly; and, by table IV., find what single life would have this same value. Then find, by rule II., the value of the joint continuance of the single life thus found, and of the youngest, and this will be the value of the three proposed lives jointly.

Exam. 5. Required the present value at 4 per cent., of an annuity of £140 on the joint continuance of three lives aged 15, 30, and 35 years.

Here, at 4 per cent., the value of the joint continuance of two lives aged 30 and 35, is 10·948; which in table IV. is found to be the value of a single age of 51½ years nearly. Then, by rule II., the value of the joint continuance of two lives, of 15 and 51½, is 9·6335, the value of an annuity of £1 on the joint continuance of the three given lives. The product of this by £140 is £1348·69, or £1348 13 9½, the value of the given annuity.

RULE V. *To find the value of an annuity on the* longest *of three lives; or which is to continue, till they are all extinct:* From the sum of all the values of the single lives, found in table IV. and of the value of the three jointly, found by rule IV.; subtract the sum of the joint values of the lives combined two and two, by rule II.; and multiply the remainder by the given annuity.

Exam. 6. Required the present value, at 4 ℘ cent., of a house and farm which yield a profit rent of £32 10 ℘ annum, held by a lease of three lives, aged 35, 30, and 15 years.

Here, by the last example, the value of the three lives jointly is 9·6335, and their values singly are 14·039, 14·781, and 16·791. Also, by rule II., the value of the first and second jointly is 10·948; of the first and third 11·787; and of the second and third 12·246. From the sum of the first four of these numbers take the sum of the others, and the remainder, 20·2635, is the value of an annuity

* The duration of the lives of females is found to be somewhat greater than that of males. It is inconsistent with the nature of this work, however, to take into calculation this circumstance, which in general produces but a slight difference in the result.

of £1 on the life of the longest liver of the three. This multiplied by £32 10, gives for the value required, £658·56375, or £658 11 3¼.

Ex. 8. How much should a man pay at 5 ℘ cent. ℘ annum, to purchase an annuity of £320 a year to continue till himself aged 55, his wife aged 49, and his son aged 20, shall all be extinct? *Answ.* £5103 11 0½.

RULE VI. *To find the value of the reversion of an annuity after the death of the present possessor:* From the present value of an annuity of £1 for the entire continuance of the annuity, subtract the value of an annuity of £1 during the life of the possessor: the remainder will be the value of a reversion of £1 in the given circumstances, which multiply by the given annuity.

If there be two or three lives, the same rule will serve, their values being used instead of the value of the single life.

Exam. 7. If a person aged 47 years, possess a perpetuity of £500 per annum; what is its present value at 5 ℘ cent. to his son, who is to possess it at his death?

Here, the value of £1 in perpetuity is £20, and, by table IV, the value of an annuity of £1 on a life of 47 years is 10·784. Then 20 — 10·784 = 9·216; which multiplied by £500 produces £4608, the value required.

Exam. 8. A man leaves an annuity of £165 ℘ annum, of which 30 years are yet to come, to his nephew, aged 38 years, during life, or till its termination; and the reversion of it to a charitable institution, in case the nephew die before the termination of the annuity. How much is the present value of the reversion at 4 ℘ cent.?

In this case, the present value of an annuity of £1 for 30 years, is by rule II. of Annuities Certain, 17·292; and, by table IV. the present value of £1 on a life of 38 years, is 13·548. Then, 17·292 — 13·548 = 3·744; and 3·744 × 165 = £617·76, or £617 15 2½, the value required.

Ex. 9. What is the present value at 5 ℘ cent., of the reversion of a perpetuity of £400 ℘ annum, not to come into possession till the death of the present incumbent aged 70, and of his intended successor aged 30? *Answ.* £2538 16 0.

RULE VII. *To find the present value of a proposed sum payable at the decease of a person whose age is given:* From the value of £1 in perpetuity, take the value of an annuity of £1 on the life; divide the remainder by the value of the perpetuity increased by a unit; and multiply the result by the given sum.

If the present value thus found be divided by the value of the life, the quotient will be the annuity payable yearly.

By using the value of two or three lives found by the preceding rules, we may apply the same rule.

Exam. 9. How much must a person aged 32 years, pay, at 5 ℘ cent., to secure to his family at his death the sum of £1000?

At 5 ℘ cent., the perpetuity of £1 is £20, and, by table IV. the value of a life of 32 years, is 12·854. The difference of these is 7·146; and this being divided by 21, the quotient is ·340286, the present value of £1 payable at the decease of the person. Then, ·340286 × £1000 = £340 5 8½, the value required: and £340·286 ÷ 12·854 = £26·473 = £26 9 5½, the annuity. Hence it appears, that at 5 ℘ cent. ℘ annum, if a person either at one payment deposit £340 5 8½, or pay each year during life the sum of £26 9 5½, his heirs at his death will be entitled to receive £1000.

Ex. 10. At 6 ℘ cent. ℘ annum, how much ℘ year, must a person aged 66 years, pay during life, to entitle his heirs at his death to receive £1500? *Answ.* £128 13 10½.

11. If a husband and his wife be each aged 45 years; how much ℘ annum must be paid during the longer of the two lives, to entitle the family after the decease of both, to receive £4000, compound interest being allowed at 6 ℘ cent. ℘ annum? *Answ.* £75 8 5½.

Exam 10. How much must a man, aged 39, pay per year, at 6 ℘ cent., during his marriage, or during life, to entitle his wife aged 32 years, in case she survive him, to an annuity of £50 a year during the remainder of her life?

By rule II., the value of the two lives jointly is 8·774, and the value of a life of 32 years is, by table IV., 11·512. The difference of these, 2·738, is the present value of the reversion of an annuity of £1 to be paid to the wife after her husband's death, in case she survive him. The product of this by £50 is £136·9, or £136 18, the sum to be paid at a single payment. Let this be divided by 8·774, the value of their lives jointly, the quotient £15·603, or £15 12 0¾, is the annual payment. Hence, if there be paid annually, during the joint continuance of both lives, the sum of £15 12 0¾, or at present, in a single payment, £136 18, the wife, if she survive the husband, will be entitled to an annuity of £50 per annum during life.

This solution proceeded on the supposition in regard to the annual payments, that the first of these was not made till the end of the first year. If it be made at present, however, we must divide by the value of the joint continuance increased by a unit; in the present case by 9·774. In this case the annual payment would be found to be £14 0 1½.

Exam. 11. If a number of persons form themselves into an association for providing annuities of £40 ℔ annum for their surviving widows, and each person, at the age of 30, contribute £75 to the fund, and equal annual contributions during the rest of his life: how much must each of these annual contributions be, money being improvable at 6 ℔ cent. ℔ annum, and the wife of each being supposed to be at an average three years younger than himself?

The value of a life of 27 years is 11·917, and the value of two lives of 27 and 30 jointly is 9·481. The difference of these, 2·436, multiplied by £40, gives for the entire sum to be paid at present, at a single payment, £97·44. But as only £75 are to be paid at present, there will still remain £22·44 to be paid by annual contributions, by dividing which by 9·481, we obtain £2 7 $3\frac{3}{4}$ for each of these contributions.

Exam. 12. Suppose that, as in the last question, a man aged 30 years, pays £50 into a fund; what annual contribution must he pay during life to entitle his family to an annuity of £40 per annum for 8 years after his death, interest being at 6 ℔ cent?

The present value of an annuity of £40 for 8 years at 6 ℔ cent. is £248·3918, (by rule II. of Annuities Certain.) Then, by rule VII. of this article, the present value of this sum is found to be £70·084 from which £50 being subtracted, the remainder, £20·084, is the sum still remaining to be paid at present; and this being divided by 11·682, the value of a life of 30, the quotient, £1 14 $4\frac{1}{2}$, is the annual contribution required.

CONTINUED FRACTIONS.*

It often happens that fractions, even when reduced to their simplest forms, are expressed in numbers inconveniently large; and hence it is often desirable to approximate their values in smaller numbers. Thus, if the fraction $\frac{1117}{2467}$, which is in its lowest terms, be proposed, we may wish, even for the purpose of forming a more correct idea of its magnitude, to find other fractions, in smaller terms, which will be nearly of the same value. The method that most naturally presents itself for this purpose is to divide both terms by the smaller of them, as the smaller will by this means become 1, and we shall thus be enabled to compare the other with that number with which the

* For full information respecting the nature and applications of continued fractions, see Bonnycastle's *larger Treatise on Algebra*, Lacroix's *Complément des Elémens d' Algèbre*, Legendre's *Essai sur la Théorie des Nombres*, Barlow's *Theory of Numbers*, Lagrange's *Additions to Euler's Algebra*, and several other works. What is here given is one of the most interesting applications, and one which cannot fail to be useful to the more advanced arithmetician.

mind can most easily compare it in respect of magnitude. By this means the fraction becomes $\frac{1}{2\frac{217}{1115}}$, the denominator of which, being between 2 and 3, we conclude that the value of the given fraction is between $\frac{1}{2}$ and $\frac{1}{3}$; and therefore $\frac{1}{2}$ is a first approximation to its value, being too great. Divide again both terms of the the fraction in the denominator by 217, and it will become $\frac{1}{5\frac{30}{217}}$, which is between $\frac{1}{5}$ and $\frac{1}{6}$. By taking $\frac{1}{5}$ instead of $\frac{217}{1115}$, we shall have, for a second approximate value of the proposed fraction, $\frac{1}{2\frac{5}{11}}$, or $\frac{5}{11}$, which is too small, as instead of $\frac{217}{1115}$, we used in the denominator $\frac{1}{5}$, which is greater than $\frac{217}{1115}$. To continue the approximation still farther, we divide both terms of the fraction $\frac{30}{217}$ by 30: the result is $\frac{1}{7\frac{7}{30}}$, which is less than $\frac{1}{7}$ and greater than $\frac{1}{8}$. Hence, if instead of using $\frac{1}{5}$ in the preceding fraction, we use $\frac{1}{5\frac{1}{7}}$ or its equal $\frac{7}{36}$, we shall have for the next approximation to the value of the given fraction $\frac{1}{2\frac{7}{36}}$ or $\frac{36}{79}$ which is too great, because $\frac{1}{5\frac{1}{7}}$, or its equal $\frac{7}{36}$, is too small, in consequence of $\frac{1}{7}$ being too great; and therefore $\frac{1}{2\frac{7}{36}}$ must be too great, since the denominator is too small. By continuing the process in a similar manner, we find $\frac{148}{325}$, $\frac{499}{1095}$, and $\frac{1114}{2445}$, for the succeeding fractions, the last of which is the given fraction,* and the first an approximate fraction smaller, and the second another greater, than the given fraction. These successive fractions have the remarkable property that *each of them approaches more nearly than the one which precedes it, to the value of the given fraction.* Thus, $\frac{1114}{2445}$, is nearly equal to $\frac{1}{2}$; more nearly equal to $\frac{5}{11}$; more nearly equal to $\frac{36}{79}$; still more nearly equal to $\frac{148}{325}$; and, finally, more nearly still to $\frac{499}{1095}$. That there is this continual approach to the true value of the fraction, will appear evident, if it be considered that in each of the results in the preceding operations, a correction is made on the result which goes before it.

The several converging fractions above obtained, if the last or supplementary simple fraction be rejected from each, except when its numerator, like that of the rest, is a unit, may be written at full length, as follows:

$$\frac{1}{2};\quad \frac{1}{2+\frac{1}{5}};\quad \frac{1}{2+\frac{1}{5+\frac{1}{7}}};\quad \frac{1}{2+\frac{1}{5+\frac{1}{7+\frac{1}{4}}}};$$

* If the given fraction be finite, the last of the converging fractions will always be equal to it, as in this example.

$$\frac{1}{2}+\frac{1}{5}+\frac{1}{7}+\frac{1}{4}+\frac{1}{3}; \qquad \frac{1}{2}+\frac{1}{5}+\frac{1}{7}+\frac{1}{4}+\frac{1}{3}+\frac{1}{2}.$$

In this form they are called *continued fractions.* It appears, therefore, that a *CONTINUED FRACTION is that which has for its denominator a whole number with a fraction annexed, which fraction has also for its denominator a whole number with a fraction annexed, the denominator of which latter fraction is also a whole number with a fraction: and so on, however far the fraction may be continued; and each numerator is a unit.**

It will be seen by a review of the preceding processes, that the denominators of the continued fraction are the quotients which would be found in using the rule given in page 89, for finding the greatest common measure. Hence, we have the following rule:

RULE I. *To convert a given simple fraction into a continued fraction:* Divide the greater term by the less, the less by the remainder, &c. as in finding the greatest common measure: the quotients will be the denominators of the several fractions in the continued fraction, and the numerator of each will be a unit.

The successive fractions which approach continually to the value of a given fraction expressed in large numbers, may be found by reducing it to a continued fraction, and perating in the way already employed in approximating the value of $\frac{1111}{4447}$: the following method, however, will be found much preferable.

RULE II. "*A fraction expressed by a great number of figures being given, to find all the fractions in less terms, which approach so near the truth, that it is impossible to approach nearer without employing greater terms:*" Reduce the proposed fraction to a continued one, or at least find the quotients, by the preceding problem. Then write the several quotients in a line; and if the given fraction be greater than a unit, take the first quotient as the numerator, and a unit as the denominator, of the first fraction, which set below the second quotient; but if the given fraction be less than a unit, make the first quotient the denominator, and a unit the numerator of the first fraction. Then, for the second fraction, multiply both the terms of the first by the quotient which stands above it, and add a unit to the product of that term which was the first quotient: the result is the second fraction, which is to be set below the third quotient. To find the succeeding fractions, multiply the terms of each fraction, when found by the quotient, which stands above it, and to the products add separately the terms of the preceding fraction.

Exam. 1. The height of Slieve Donard, in county Down, is 2654

* It is not essential to the nature of continued fractions that each numerator be a unit; but fractions of this kind only are used as instruments of calculation.

feet, and that of Ben Lomond 3262 feet: it is required to find a series of converging fractions, expressing as nearly as possible the ratio of the heights of these mountains.

Here the fraction expressing the height of Slieve Donard in relation to that of Ben Lomond, is $\frac{3224}{3262}$, or, in its lowest terms, $\frac{1327}{1631}$, and the quotients found in the manner shown in the margin, are 1, 4, 2, 1, 2, 1, 4, 1, 4: consequently the continued fraction is

```
1327)1631(1
     ----
      304)1327(4
          1216
          ----
           111)304(2
                &c.
```

$$\frac{1}{1}+\frac{1}{4}+\frac{1}{2}+\frac{1}{1}+\frac{1}{2}+\frac{1}{1}+\frac{1}{4}+\frac{1}{1}+\frac{1}{4}.$$

The successive converging fractions will be found thus:

1 4 2 1 2 1 4 1 4

$\frac{1}{1}$, $\frac{4}{5}$, $\frac{9}{11}$, $\frac{13}{16}$, $\frac{35}{43}$, $\frac{48}{59}$, $\frac{227}{279}$, $\frac{275}{338}$, $\frac{1327}{1631}$,

Here, the quotients, or denominators, being arranged in succession, we take for the denominator of the first fraction, 1, the first quotient, and we make its numerator 1 also. Then, by multiplying both terms by 4, the quotient which stands above them, and adding 1 to the product resulting from the denominator, because it was the first quotient, we obtain $\frac{4}{5}$, for the second fraction. We then multiply 4 by 2, the figure above it, and add to the product the preceding numerator; and we multiply 5 by 2, and add to the product the preceding denominator; and thus we obtain $\frac{9}{11}$ for the third fraction. We next multiply the terms of this fraction by 1, and add to the results 4 and 5 severally: we thus find the next fraction to be $\frac{13}{16}$. In this way, rejecting the first fraction, $\frac{1}{1}$, which is evidently far from the truth, we find that the height of Slieve Donard is nearly $\frac{4}{5}$ that of Ben Lomond; more nearly $\frac{9}{11}$ it: still more nearly, $\frac{13}{16}$; more nearly still, $\frac{35}{43}$, &c.; $\frac{4}{5}$ being too small, $\frac{9}{11}$ too great, $\frac{13}{16}$ too small, $\frac{35}{43}$ too great, &c.*

Exam. 2. The circumference of the circle whose diameter is 1, is found to be greater than 3·1415926, but less than 3·1415927:

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* Thus, $\frac{4}{5}$ of 3262=2699$\frac{2}{5}$; $\frac{9}{11}$ of 3262=2668$\frac{10}{11}$; $\frac{13}{16}$ of 3262=2650$\frac{3}{8}$; $\frac{35}{43}$ of 3262=2655$\frac{9}{43}$; $\frac{48}{59}$ of 3262=2653$\frac{49}{59}$, which are evidently approaching 2654, the true value, being alternately greater and less.—It may be observed, that in all cases, the error of each fraction is a less part of the integer than a unit divided by the product of its own and the succeeding denominator; but a greater part than a unit divided by the product of its own denominator, and the sum of that and the succeeding denominator. Thus, in the preceding example, the error of $\frac{4}{5}$ is less than $\frac{1}{5\times11}\times3262$, but greater than $\frac{1}{5\times16}\times3262$.

required the series of fractions converging to the ratio of the circumference and the diameter.

In questions like this, in which one term of the fraction or ratio is not precisely given, but is contained between given limits, as when one of the terms is an infinite decimal, it is proper to work for the quotients by both limits, and to use those only which result from both. Thus, in the proposed exercise, by dividing 3·1415926 by 1·0000000, this divisor by the remainder, &c. and by proceeding in like manner with 3·1415927, we find in both cases, the quotients 3, 7, 15, 1, after which the quotients would be different, and are therefore not to be used. Hence the converging fractions are found as in the margin.

3	7	15	1
$\frac{3}{1}$,	$\frac{22}{7}$,	$\frac{333}{106}$,	$\frac{355}{113}$,

It appears, therefore, that the diameter of a circle is to its circumference nearly as 1 to 3; more nearly, as 7 to 22; more nearly again, as 106 to 333; and still more nearly, as 113 to 355. The degree of approximation of each of these to the given ratio, will be discovered by dividing the first term of each by the second. In this way the last gives 3·14159292, which exceeds the truth by less than a ten-millionth part of the circumference. Had the circumference been taken to a greater number of places, (see note, page 112,) the succeeding fractions would have been found to be $\frac{103993}{33102}$, $\frac{104348}{33215}$, &c.

Exam. 3. Let it be required to approximate the ratio of 1·41421* to 1.

Here the quotients are found to be 1, 2, 2, 2, 2, &c. and consequently the continued fraction the same as in the margin. The converging fractions also are thus found;

$$1+\frac{1}{2+}\frac{1}{2+}\frac{1}{2+}\frac{1}{2+}\frac{1}{\&c.}$$

1	2	2	2	2	2
$\frac{1}{1}$,	$\frac{3}{2}$,	$\frac{7}{5}$,	$\frac{17}{12}$,	$\frac{41}{29}$,	$\frac{99}{70}$, &c.

Ex. 1. The height of mount Etna is 10963 feet, and that of mount Vesuvius 3900 feet; required the approximate ratios of their heights. *Answ.* $\frac{1}{2}$, $\frac{1}{3}$, $\frac{4}{14}$, $\frac{18}{45}$, $\frac{27}{104}$, $\frac{99}{258}$, $\frac{127}{337}$.

2 The height of mount Hecla is 4900 feet, and that of mount Perdu, the highest of the Pyrenees, 11,283 feet: required the approximate ratios of their heights. *Answ.* $\frac{1}{2}$, $\frac{3}{7}$, $\frac{10}{23}$, $\frac{33}{76}$, $\frac{76}{175}$, &c.

3 Find the approximate ratios of 1 and 3·6055513, (that is, of 1 to the square root of 13.) *Answ.* $\frac{1}{3}$, $\frac{1}{4}$, $\frac{3}{7}$, $\frac{9}{11}$, $\frac{4}{18}$, $\frac{33}{119}$, $\frac{38}{137}$, $\frac{71}{256}$, &c.

4. Required the series of ratios approaching the ratio of English and Irish acres. *Answ.* $\frac{1}{1}$, $\frac{1}{2}$, $\frac{3}{5}$, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{8}{13}$, $\frac{21}{34}$, $\frac{50}{81}$, $\frac{121}{196}$.

5. The weights of equal bulks of pure water and fluid mercury

* This is the square root of 2, which therefore is expressed by a unit, with a continued fraction, each of whose denominators is 2 Hence the fraction may be continued with-

are as 1 to 13·568; required the series of fractions converging to this ratio. *Answ.* $\frac{1}{13}, \frac{1}{14}, \frac{2}{27}, \frac{7}{95}, \frac{37}{502}, \frac{44}{597}, \frac{125}{1696}$.

6 It has been computed, that between the years 1696 and 1800, the value of money decreased so much, that in the former year £1 would have procured as much of the necessaries of life as £2 7 11 in the latter. Required the series of fractions approaching to the ratio of the values of money at these periods. *Answ.* $\frac{1}{2}, \frac{2}{5}, \frac{3}{7}, \frac{5}{12}, \frac{49}{116}$.

ON ARITHMETICAL SCALES.

The system of notation for which we are indebted to the Arabians, and which has been employed and exemplified in the preceding pages, proceeds according to the combinations of the number *ten*, and is such as to correspond to the names given to numerals in almost all languages. The language of every civilized nation, ancient or modern, furnishes names for *ten* times *ten*, and for *ten* times that product; but none of them furnishes distinct names, of general use, for the powers of seven, eight, twelve, &c.; these, as well as other numbers which are not powers of ten, being denominated from a combination of the names of the powers of ten, with the names of units,* when necessary. Thus, for the second power of eleven we have no distinct name, but we call it *one hundred and twenty one*, that is, *the second power of* TEN *with twice* TEN, *and one unit:* and the third power of fifteen we call *three thousand, three hundred, and seventy five;* that is, *three times the third power of* TEN, *three times the second power of* TEN, *seven* TENS, *and five units.* In like manner, in the natural succession of numbers, the first after one hundred, is called one hundred and *one*, the next one hundred and *two*, &c., no new names being given, but merely combinations of those previously formed.

This remarkable agreement in the numerical language of almost all nations, seems evidently to arise from the use that is made of the fingers in arithmetical computations, in the ruder periods of society, and very generally by those who have not been instructed in better modes of calculation. Such persons, in counting a number of objects, would naturally distribute them into parcels, each consisting of *ten*, from the number of the fingers employed in reck-

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out limit, the law of continuation being manifest; which is not the case when the root is expressed decimally. The same holds respecting the square root of every number which is not a square. Thus, the square roots of 11 and 35 are expressed by the continued fractions in the margin, the law of continuation in each of which is manifest.

$$\sqrt{11}=3+\cfrac{1}{3+\cfrac{1}{6+\cfrac{1}{3+\cfrac{1}{6+\&c.}}}}$$

$$\sqrt{35}=5+\cfrac{1}{1+\cfrac{1}{10+\cfrac{1}{1+\cfrac{1}{1+\&c.}}}}$$

* The mathematical reader will know, that the number *one* may be regarded as that power of ten (or indeed of any number) whose index is zero.

oming them: and, if the number of these parcels should be great, it would be natural, in ascertaining *their* number, to form them into *larger* parcels, each containing *ten* of the smaller; and it is easy to see how this principle would be extended to the numeration of any number of objects however great. It is also evident, that when names would be invented, by some process of formation now unknown, for all the numbers so far as ten, and also for the powers of ten, the names of all other numbers would be obtained by a proper combination of these. Thus, the number eleven might be called *one and ten;* twelve, *two and ten;* thirteen, *three and ten;* twenty, *two tens;* thirty, *three tens*, &c.: and we find this method of denominating numbers strictly followed, except in some of the smaller numbers, such as eleven, twelve, twenty, sixty, &c. which, from their frequent use, have been more liable to have their names corrupted and altered, but which, when their derivations can be discovered, are always found to be formed on correct anological principles, according to the foregoing explanation.

Of the advantages and excellence of this system of notation we can scarcely be duly sensible. Instructed in its use from the earliest lessons we receive in Arithmetic; never comparing it, or comparing it but slightly, with other modes of expressing numbers by characters; and finding no deficiency, no need of improvement, nothing to call our thoughts to the subject, we use it without feeling its superiority, and with a very inadequate idea of its power. We do not reflect, that merely by means of the different positions and combinations of no more than ten simple characters, we can adequately and correctly express any number, however great. With the Roman, or even with the Greek notation, on the contrary, we find it impossible to express numbers that exceed a certain magnitude: and, even were additional characters formed to supply this defect, we would find, that calculations, which are performed with great facility and despatch by the decimal notation, would, by either the Greek or Roman system, be excessively tedious and intricate, while the performance of many others would be almost impracticable.

But, though the decimal system of notation has so far the advantage over these and all other systems not depending on the same principle, we must not conclude, that no other system of equal excellence could be invented. There may be an indefinite number of systems of notation founded on the same principle, and possessing various degrees of excellence. In the decimal notation, we distribute numbers into classes or parcels of *ten* each; these classes again into higher classes, each containing *ten* of the lower; these into still higher classes, each containing *ten* classes of the second order, and so on, till the numbers are exhausted. But if we proceed still in the same manner, only making the classes consist of *two*, instead of *ten*, each, we have the *binary* scale of notation, in which only the *two* characters, 1 and 0, are requisite for expressing all numbers: and if the classes be made

to consist of *three* each, we have the *ternary* scale, in which only *three* characters, 1, 2, 0, are requisite. In the same manner, it is obvious we may have a *quaternary, quinary, duodecimal, trigesimal, sexagesimal, centesimal,* or any other scale, by merely taking 4, 5, 12, 30, 60, 100, or any other assigned number, as the number contained in each class. This number may be called the RADIX, ROOT, or BASE, of the system; and it is obvious, that in each system there will be as many distinct characters required, as there are units in the radix. Thus, in the decimal scale *ten* characters are necessary, but in the duodecimal *twelve* would be required, which number would be made up by adding to the characters at present in use, two others to denote *ten* and *eleven*.

In what follows, D will be used to denote *ten*, and H to denote *eleven*;* and, in the duodecimal scale, twelve will of course be written 10.

Hence, *to express a given number in any assigned scale: Divide the given number by the radix of the scale; divide the result also by the radix, and the result arising from this again by the radix. Continue the division in this manner as long as possible, and to the final quotient annex the several remainders in a retrograde order, placing ciphers where there is no remainder.* Thus, the expression of 592835 in the decimal scale, will be 2470DH in the duodecimal scale, as will appear from the annexed operation.

```
12)592835
   ------
12)49402.....11
   -----
12)4116......10
   ----
12)343......0
   ---
12)28......7
   --
    2.....4
```

Here it is evident, that by dividing the given number by 12, it is distributed into 49402 classes each containing 12, with the remainder 11. By the second division by 12, these classes are distributed into 4116 classes, each containing 12 times 12, or the second power of 12, with a remainder of 10 of the former classes, each containing 12. By the third operation, the classes last found are distributed into 343 classes, each containing 12 of the latter, which were each the second power of 12, and therefore these are each the third power of 12; and the remainder is 0. In like manner, the next quotient expresses 28 times the fourth power of 12, and the remainder, 7 times the third power of 12; and the final quotient expresses twice the fifth power of 12, with a remainder of 4 times the fourth power of 12. Hence the

* These characters, which may serve the intended purpose as well as any others in the few instances in which they will be employed in what follows, may be easily recollected by conceiving the first to be formed by running together 1 and 0, the characters which express 10 in the common notation and by joining with a line 1 and 1, which express eleven.

given number is analysed into $2 \times 12^5 + 4 \times 12^4 + 7 \times 12^3 + 0 \times 12^2 + 10 \times 12 + 11$, or 2470DH, according to the notation above adopted.

It will be easily found by proceeding in the same manner, that for seven thousand, eight hundred, and fifty four, the expression in the *binary* scale will be 1111010101110; in the *ternary*, 101202220; in the *quaternary*, 1322232; in the *quinary*, 222404; in the *senary*, 100210; in the *septenary*, 31620; in the *octary*, 17256; in the *nonary*, 11686; in the *denary* or *decimal*, 7854; in the *undenary* or *undecimal*, 59D0; in the *duodenary* or *duodecimal*, 4666; in the *vigesimal*, or that whose radix is twenty, (19)(12)(14); in the *trigesimal* (radix thirty), 8(21)(24); in the *quinquagesimal* (radix fifty), 374; in the *sexagesimal* (radix sixty), 2D(54); and in the *centesimal* (radix one hundred), (78)(54): where each pair of the figures enclosed in brackets would be represented by a single character, were there a sufficient number of distinct characters for each scale.

The converse of this problem, or *the reduction of a number to the decimal scale from any other, will be performed by finding the values of the several digits, and collecting those values into one sum: or, more easily, by multiplying the left hand digit by the radix, and adding to the product the next digit; then by multiplying this sum by the radix, and adding to the product the next digit, and so on till all the digits shall have been employed.** Thus, 4503142 in the senary scale is equivalent to $4 \times 6^6 + 5 \times 6^5 + 0 \times 6^4 + 3 \times 6^3 + 1 \times 6^2 + 4 \times 6 + 2$, or 226214; which result will be obtained more easily by the operation in the margin.

```
4503142
      6
   ----
     29
      6
   ----
    174
      6
   ----
   1047
      6
   ----
   6283
      6
   ----
  37702
      6
   ----
 226214
```

Thus it appears that the decimal scale is only one out of many on the same principle; and, upon due consideration, it will be found *not* to be the best scale that might be adopted. It does not suit the plan or the limits of the present work to add much to what has been said on this curious and interesting subject. For farther information, recourse may be had to various works, especially to Professor Leslie's Philosophy of Arithmetic, to Mr. Anderson's Treatise on Arithmetic in the Edinburgh Encyclopædia, and Barlow's Theory of Num-

* It is scarcely necessary to remark, that both this rule and the preceding are the same in principle, as the rules for the reduction of quantities of different denominations.

bers. it may be sufficient to observe, that the *binary* is chiefly important from its unfolding some curious properties of numbers; that from its employing no character of higher value than a unit, operations would be performed by it, though tediously, yet with great facility, and with little exertion of memory: but an insuperable obstacle to the general use of this scale, is that in expressing numbers, the characters 1 and 0, must generally be repeated a great number of times. The same advantages, and the same disadvantages belong, but in a less degree, to the *ternary* scale, and in a still less degree to the *quaternary*. The *quinary*, *septenary*, and *undenary*, all labour under the disadvantage of having their bases prime numbers, and of thus giving origin to a great number of interminate fractions.* The *octary* and *nonary* scales are not so objectionable on this account, but they have no advantage of a different kind to recommend them. The decimal scale does not give origin to so many interminate fractions as either of the latter, and has besides the advantage of expressing numbers more concisely. The *senary* and *duodenary* scales, having each so many integral aliquot parts in proportion to their magnitudes, and those of so convenient a kind, give origin to so much fewer interminate fractions, than any of the above mentioned. The radix of the senary scale is so small however, that numbers are not expressed by it with sufficient conciseness,—an objection which does not apply to the duodenary. The latter therefore, for this and other reasons, is much preferable to any other scale that could be adopted. To introduce this scale now, however, when men are accustomed to the decimal scale; when the languages of all civilized nations are suited to it; and when so many valuable works, particularly tables, in which it is adopted, would be rendered comparatively useless, would be unadviseable, and perhaps impracticable: but we must regret that the decimal scale was adopted at a time, when any other scale might have been introduced with equal facility.

The following exercises are annexed for the use of those who may wish to familiarize themselves with this subject:

Ex. 1. Reduce 1000000 in the decimal scale, to the ternary scale, and also to the nonary. *Answ.* 1,212,210,202,001, and 1,783,661.

2. Reduce 1000000 in the quaternary scale, to the denary and binary scales. *Answ.* 4096; and 1,000,000,000,000.

3. Reduce 123454321 in the senary scale, to the duodenary scale. *Answ.* 987,3D1.

4. How will 476897 in the decimal scale, be expressed in the duodecimal scale? *Answ.* 1DH,H95.

5. How will 6666 in the decimal scale, be expressed in the binary and quinary scales? *Answ.* 1,101,000,001,010; and 203,131.

* That is, interminate fractions having for their denominators the radix of the or its powers, such as interminate decimals in the common scale.

6. Reduce 13579 in the duodecimal scale, to the undecimal scale. *Answ.* 190D3.

7. Express nine hundred and three pounds, eighteen shillings, and eleven pence, in the quinary scale. *Answ.* £12,103 33 21.

QUESTIONS WITH THEIR SOLUTIONS.*

1. What number taken from the square of 48, will leave 16 times 54?

Solution.—$48^2=2304$, and $54\times16=864$; then $2304-864=1440$, the answer.

2. Divide £1000 between A, B, and C, and give A £120 more than C, and C £95 more than B.

Sol. A's share is evidently to be $215(=120+95)$ more than B's: therefore $1000-215-95=690$; and $690\div3=£230=$ B's share. Hence, $230+95=£325$, C's share; and $325+120=£445$, A's share.

3. A father left to his eldest son $\frac{44}{75}$ of his property, to his second $\frac{3}{8}$ of the remainder, and to his third son what was left. What was the share of each, the shares of the first and second differing by £500?

Sol.—$\frac{3}{8}$ of $\frac{44}{75}=\frac{132}{1024}$, the part of the whole property belonging to the second son. Then, $\frac{44}{75}+\frac{132}{1024}=\frac{4721}{6144}$, and $1-\frac{4721}{6144}=\frac{1423}{6144}$, the part of the youngest. Also $\frac{44}{75}-\frac{132}{1024}=\frac{3521}{6144}$, the difference of the parts of the first and second. Then, as $\frac{3521}{6144}:\frac{44}{75}$, or as $2251:3476::£500:£772\frac{328}{2251}$, the share of the eldest: as $2251:1225::£500:£272\frac{328}{2251}$, the share of the second: and as $2251:1540::£500:£342\frac{158}{2251}$, the share of the third.

4. A gets £4 of a legacy for £3 that B gets, and C £5 for £6 that B gets, and A's share is £5000. What is the whole legacy?

Sol.—As $£3:£6::£4:£8$, A's part, when B gets £6, and C £5. Then as $8:8+6+5::£5000.£11875$, the whole legacy.

5. A person possessed of $\frac{3}{8}$ of a ship, sold $\frac{2}{3}$ of his share for £1260. What is the value of the whole ship at the same rate?

Sol.—$\frac{2}{3}$ of $\frac{3}{8}=\frac{1}{4}$; and as $\frac{1}{4}:1::£1260:£5040$, the answer.

6. A person being asked the hour of the day, said, that the time past noon was $\frac{2}{5}$ of the time till midnight. What was the hour?

Sol.—As $1+\frac{2}{5}:\frac{2}{5}$, or as $9:4::12$ hours : 5 hours, 20 minutes; and therefore the time was 20 minutes past 5 o'clock in the afternoon.

7. A, B, and C purchase a ship; A pays $\frac{3}{8}$, B $\frac{2}{5}$, and C £2000, the cost. What are the sums paid by A and B?

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* These questions are selected from various authors. Their solutions are annexed for the purpose of showing the more advanced student, how they and similar arithmetical questions may be resolved. They will be found useful in preparing the pupil for working the series of questions that follow them, and for resolving any of the more difficult questions that may be proposed for resolution by common Arithmetic.

Sol.—$\frac{3}{5}+\frac{2}{3}=\frac{19}{15}$, and $1-\frac{19}{15}=\frac{4}{15}$, C's part. Then, as $\frac{19}{15}:\frac{2}{3}$, or as 31 : 14 : : £2000 : £903$\frac{7}{31}$; the part paid by A; and as $\frac{4}{15}:\frac{2}{3}$, or 31 : 18 : : £2000 : £1161$\frac{9}{31}$, the part paid by B.

8. Required the stocks of A and B, A's gain being £160, and B's £130, and A's stock £175 more than B's.

Sol.—£160—£130=£30. Then, as £30 : £175 : : £160 : £933$\frac{1}{3}$, A's stock; and as £30 : £175 : : £130 : £758$\frac{1}{3}$, B's stock.

9. A's stock £240, B's £210; whole gain £120, and C's part of it £30. Required A's and B's gains, and C's stock.

Sol.—£120—£30=£90, the sum of A's and B's gains; also £240+£210=£450, the sum of their stocks. Then, as £450 : £90, or 5 : 1 : : £240 : £48, A's gain; as 5 : 1 : : £210 : £42, B's gain; and as 1 : 5 : : £30 : £150, C's stock.

10. A gains £12 in 6 months, B £15 in 5 months, and C £21 in 9 months. What is the whole stock, C's part of it being £40?

Sol.—In compound fellowship the gains are proportional to the *products* of the stocks and times; and, conversely, the stocks are proportional to the *quotients* obtained by dividing the gains by the times. Hence, as $\frac{21}{9}:\frac{12}{6}+\frac{15}{5}+\frac{21}{9}$: : £40 : £125$\frac{4}{7}$, the whole stock.—A's stock would be found thus : as $\frac{21}{9}:\frac{12}{6}$, or as $2\frac{1}{3}:2$, or as 7 : 6 : : £40 : £34$\frac{2}{7}$.

11. A and B gain £13 10, B and C £12 12, and A and C £11 16 6. What is the gain of each?

Sol.—From £18 19 3, half the sum of the given monies, which is evidently equal to the gains of all the three, take £12 12, and there will remain £6 7 3, A's part. In like manner, £18 19 3 —£11 16 6=£7 2 9, B's share; and £18 19 3—£13 10=£5 9 3, C's share.

12. If A can do a piece of work in 10 days, and B in 13; in what time will both do it, at the same rate?

Sol.—In one day A does $\frac{1}{10}$ and B $\frac{1}{13}$ of the work; therefore both together do in one day $\frac{1}{10}+\frac{1}{13}=\frac{23}{130}$ of it. Hence, as $\frac{23}{130}$ of the work : 1, the whole work; or as 23 : 130 : : 1 day : 5$\frac{18}{23}$ days, the time required. It appears from this work, that the answer will be found by dividing the product of 10 and 13 by their sum.

13. A, B, and C, can trench a field in 12 days; B, C, and D, in 14 days; C, D, and A, in 15 days; and D, A, and B, in 18 days. In what time will it be done by all of them together, and by each of them singly?

Sol.—In one day, A, B, and C, will do $\frac{1}{12}$ of the whole work B, C, and D, $\frac{1}{14}$; C, D, and A, $\frac{1}{15}$; and D, A, and B, $\frac{1}{18}$. Let these fractions be added together, and the sum, $\frac{349}{1260}$, is the part done by all working 3 days, since in each of the three parts, $\frac{1}{12}$, $\frac{1}{15}$, and $\frac{1}{18}$, of the whole work, there is one day's work of A; in each of the three parts, $\frac{1}{12}$, $\frac{1}{14}$, and $\frac{1}{18}$, one day's work of B, &c. Dividing the sum by 3, therefore, we have $\frac{349}{3780}$, the part done by all four in one day. Hence, as $\frac{349}{3780}$ of the work : 1, the whole

work; or as 349 : 3780 : : 1 day : $10\frac{329}{349}$ days, the time in which it would be performed by all of them working together. Now, from $\frac{349}{3780}$, the part done by A, B, C, and D, take $\frac{1}{14}$, the part done by B, C, and D, and the remainder $\frac{79}{3780}$, is the part done in one day by A. Then, as $\frac{79}{3780}$: 1, or as 79 : 3780 : : 1 day : $47\frac{27}{79}$ days, the time in which the work would be finished by A alone. By proceeding in the same manner, we should find, that B would perform it $38\frac{34}{97}$ days; C in $27\frac{37}{137}$ days; and D in $111\frac{3}{17}$ days.

14. A and B, at the opposite extremities of a wood 135 fathoms in compass, begin to go round it the same way at the same time; A at the rate of 11 fathoms in 2 minutes, and B of 17 fathoms in 3 minutes. How many rounds will each make, before the one will overtake the other?

Sol.—As 2 minutes : 3 minutes : : 11 fathoms : $16\frac{1}{2}$ fathoms, the space gone by A in three minutes. Hence it appears, that B, in going 17 fathoms, gains $\frac{1}{2}$ fathom on A; and the object of the question being to find how many rounds he will make in gaining half a round, we have this analogy; as $\frac{1}{2}$ fathom : 17 fathoms : : $\frac{1}{2}$ round : 17 rounds, the space to be gone over by B: consequently A will make $16\frac{1}{2}$ rounds.

15. Suppose A, B, and C, to start from the same point, and to travel in the same direction, about an island 73 miles in compass, A at the rate of 6, B of 10, and C of 16 miles per day: in what time will they be next together?

Sol.—Since B gains 4 miles each day on A, as 4 miles : 73 miles : : 1 day : $18\frac{1}{4}$ days, the time in which B would gain a round on A, or in which these two would first be together again. Also, as 6 miles, the space gained each day by C on B : 73 miles : : 1 day : $12\frac{1}{6}$ days, the time in which B and C will first be together. Now the least common multiple of $18\frac{1}{4}$ and $12\frac{1}{6}$, which are both divisible by $6\frac{1}{12}$, is readily found to be $36\frac{1}{2}$, the number of days required.

16. A person remarked that when he counted over his basket of nuts two by two, three by three, four by four, five by five, or six by six, there was one remaining; but when he counted them by sevens there was no remainder. How many had he?

Sol.—The least common multiple of 2, 3, 4, 5, and 6, being 60, it is evident, that if 61 were divisible by 7, it would answer the conditions of the question. This not being the case, however, let $60 \times 2 + 1$, $60 \times 3 + 1$, $60 \times 4 + 1$, &c. be tried successively, and it will be found that $301 = 60 \times 5 + 1$, is divisible by 7; and consequently this number answers the conditions of the question. If to this we add 420, the least common multiple of 2, 3, 4, 5, 6, and 7, the sum, 721, will be another answer; and by adding perpetually 420, we may find as many answers as we please.

17. At what time, between twelve and one o'clock, do the hour and minute hands of a common clock or watch, point in directions exactly opposite?

Sol.—This is the same as to find in what time after twelve, at which time the hands are together, the minute hand will have gained half a round on the hour hand. Now it is evident, that in 12 hours the minute hand gains eleven rounds; and consequently one round is gained in the eleventh part of 12 hours, and half a round in half that time, or the eleventh part of 6 hours; that is, in $32\frac{8}{11}$ minutes. The time required, therefore, is $32\frac{8}{11}$ minutes after twelve o'clock.

18. If one ship, containing 150 hogsheads of wine, pay for toll at the Sound, the value of 2 hogsheads wanting £6; and another, containing 240 hogsheads, pay at the same rate, the value of 2 hogsheads, and £18 besides; what is the value of the wine ℔ hogshead?

Sol.—The tolls must evidently be in the ratio of 150 to 240, or of 5 to 8. Hence the value of 2 hogsheads less by £6, must be to the value of 2 hogsheads together with £18,—or, the half of each being taken, the value of 1 hogshead wanting £3, must be to the value of 1 hogshead together with £9, as 5 to 8. $\frac{5}{8}$ of the value of 1 hogshead with $\frac{5}{8}$ of £9, therefore, must be equal to the value of 1 hogshead wanting £3; that is, $\frac{5}{8}$ of the value of 1 hogshead, with £$5\frac{5}{8}$, must be equal to the value of 1 hogshead wanting £3. Hence, £$5\frac{5}{8}$ must be equal to $\frac{3}{8}$ of the value of 1 hogshead wanting £3: and consequently $\frac{5}{8}$ of the value of a hogshead must be equal to £8^{5}. Hence, as $\frac{3}{8}$ hhd. : 1 hhd. : : £$8\frac{5}{8}$: £23, the value of 1 hogshead.

19. If 3 men, or 4 women, can do a piece of work in 56 days, in what time will one man and one woman together, perform it?

Sol.—In 56 days, one man will do $\frac{1}{8}$, and one woman $\frac{1}{4}$ of the work; and consequently, in the same time, one man and one woman will do $\frac{1}{8}+\frac{1}{4}$, or $\frac{7}{12}$, of the work. Hence, as $\frac{7}{12}$ of the work : 1, the whole work : : 56 days : 96 days, the time required.

20. If 7 gallons of brandy cost as much as 9 gallons of rum, and 9 gallons of rum as much as 12 gallons of geneva, and the price of 3 gallons of these, taking 1 of each kind, was £2 2 6; what was the value of each ℔ gallon?

Sol.—It appears from the question, that the prices of 1 gallon of brandy, 1 of rum, and 1 of geneva are as $\frac{1}{7}$, $\frac{1}{9}$, and $\frac{1}{12}$; whence, by reducing these fractions to equivalent ones having a common denominator, and using the numerators, we find, for the ratios of the prices, 36, 28, and, 21. We have, therefore, by the method of dividing into proportional parts, the following analogies: as 36 + 28 + 21, that is, as 85 : 36 : : £2 2 6 : 18/, the value of the brandy ℔ gallon; as 85 : 28 : : £2 2 6 : 14/, that of the rum; and as 85 : 21 : : £2 2 6 : 10/6, that of the geneva.

MISCELLANEOUS QUESTIONS.*

1. If a person gain $8\frac{1}{3}$ per cent. by selling apples at the rate of 8 for $6\frac{1}{2}$d., how much does he gain per cent. by selling them at the rate of 3 for $2\frac{1}{2}$d? *Answer*, $11\frac{1}{3}$.

2. If eggs be bought at the rate of 5 for a penny, how must they be sold to gain 40 per cent.? *Answ.* At the rate of 25 for 7d.

3. If 150 apples cost 9/$4\frac{1}{2}$, how many of them must be sold at the rate of 8 for $6\frac{1}{2}$d., and how many at the rate of 3 for $2\frac{1}{2}$d., that the gain on the whole may be 10 per cent.? *Answ.* 90 at 3 for $2\frac{1}{2}$d., and 60 at 8 for $6\frac{1}{2}$d.

4. A merchant engages a clerk at the rate of £20 for the first year, £25 for the second, £30 for the third, &c., thus augmenting his salary by £5 each year. How long must the clerk retain his situation, so as to receive on the whole as much as he would have received, had his salary been fixed at £52 10 ℔ annum? *Answ.* 14 years.

5. Three gentlemen contribute £164 5 towards the building of a church at the distance of 2 miles from the first, $2\frac{3}{4}$ miles from the second, and $3\frac{1}{2}$ miles from the third; and they agree that their shares shall be reciprocally proportional to their distances from the church. How much must they severally contribute? *Answ* £72 9, £50 8, and £41 8.

6. If a person purchase pins, when there are 18 in the row, which sells for a farthing, and sell them when there are only 11 in the row, how much is his gain ℔ cent.? *Answ.* $63\frac{7}{11}$.

7. A hosier sells 90 pair of stockings and gloves for £12 10, the stockings at 3/, and the gloves at 2/6 ℔ pair. Required the number of each. *Answ.* 50 pair of stockings, and 40 pair of gloves.

8. A son having asked his father's age, the father replied: "Your age is twelve years; to which if five-eighths of both our ages be added, the sum will be equal to mine." What was the father's age? *Answ.* 52 years.

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* The questions contained in this article are intended to exercise the advanced student in the use of the several rules and modes of operation, exhibited both in the text and in the notes of the preceding part of this work. They are not adapted for the majority of arithmetical pupils; as for their purpose they are too difficult, and possess too little practical utility. It is hoped, however, that, besides affording much practice in calculation, and in the application of the rules already delivered, they will form useful exercises for the reasoning powers of those who have taste or ability for such speculations. The great and principal object with every teacher of Arithmetic, should be, to make his pupils acquire an extensive and substantial *practical* knowledge of this science, without occupying their time and attention with many puzzling or difficult questions. At the same time, however, when he meets with pupils of capacity, and of considerable proficiency, it may be very proper to direct their attention to such questions as are contained in this article. By this means he will have a farther proof of their capacity, and he may lay the foundation of future proficiency in other departments of mathematical science.

Some of the following questions may be solved perhaps most easily by Position. *They may all, however, be wrought without Position:* and the student should endeavour to do them without this rule, as they will thus, form a much better exercise for his thinking powers, and fit him in a greater degree, for solving questions of difficulty, either in Arithmetic, or in other departments of Mathematics.

9. The population of Great Britain was 10,820,100 in 1801, and 12,596,803 in 1811. Hence, it is required to find its amounts in 1810 and 1821, the yearly increase being supposed to be proportional to the population. *Answ.* 12,406,733 in 1810, and 14,665,248 in 1821.

10. Three merchants having formed a joint stock of £1064, A's stock continues in trade [illegible] months, B's 8 months, C's 12 months; and A's share of the gain is £114, B's £133 4, and C's £165. What was the stock of each? *Answ.* A's £456, B's £333, and C's £275.

11. The stocks of three partners, X, Y, and Z, continue in trade 8, 10, and 7 months respectively; and their respective gains are £115 10, £204 15, and £183 15. Hence, it is required to find their several stocks, the difference between those of Y and Z being £220. *Answ.* X's stock £550, Y's £780, and Z's £1000.

12. The joint sum of two series' of continual proportionals, consisting of five terms each, and having a common mean, is $80\frac{7}{10}$, and their ratios are $1\frac{1}{2}$ and $2\frac{1}{2}$. Required the series'. *Answ.* $2\frac{2}{5}$, $3\frac{1}{2}$, 5, $7\frac{1}{2}$, $11\frac{1}{4}$; and $\frac{4}{5}$, 2, 5, $12\frac{1}{2}$, and $31\frac{1}{4}$.

13. If A, B, and C could pave a street in 18 days; B, C, and D in 20 days; C, D, and A in 24 days; and D, A, and B in 27 days; in what times would it be done by all of them together, and by each of them singly? *Answ.* By all in $16\frac{56}{199}$ days; by A in $87\frac{31}{37}$ days; by B in $50\frac{5}{8}$ days; by C in $41\frac{1}{19}$ days; and by D in $170\frac{10}{19}$ days.

14. If A can reap a field in 13 days, and B in 16 days, in what time would both together reap it? *Answ.* In $7\frac{5}{29}$ days.

15. If A and B, with C working half time, could build a wall in 21 days; B and C, with D working half time, in 24 days; C and D, with A working half time, in 28 days; and D and A, with B working half time, in 32 days; in what times would it be built by all of them together, and by each of them singly? *Answ.* A would finish it in $52\frac{1}{3}$ days; B in $57\frac{27}{49}$ days; C in $44\frac{4}{19}$ days; D in 280 days; and all in 16 days.

16. X, Y, and Z can build $\frac{4}{15}$ of a wall in 10 days; Y, Z, and V, $\frac{4}{17}$ of it, in 6 days; Z, V, and X, $\frac{4}{19}$ of it in 7 days; and V, X, and Y, the remainder of it in 9 days. Hence, it is required to find in what times it would by done by all of them together, and by each of them separately. *Answ.* By all in $23\frac{8885}{19203}$ days; and by X, Y, Z, and V respectively, in $292\frac{7849}{15653}$, $79\frac{3575}{5753}$, $93\frac{2721}{4133}$, and $62\frac{4387}{7167}$ days.

17. The stocks of three partners, A, B, and C, are £350, £220, and £250, and their gains £112, £88, and £120, respectively; and B's stock continued in trade 2 months longer than A's. Required the time the money of each continued in trade. *Answ.* 8, 10, and 12 months respectively.

18. In what arithmetical scale would five hundred and fifty four be expressed by 95? *Answ.* In the scale whose radix is 61.

19. The population of Great Britain was 6,523,000 in 1700; 7,860,000 in 1750; 10,820,100 in 1801; and 12,596,803 in 1811. Hence it is required to find the annual rates of increase on each million of the population between the first and second, the second and third, and the third and fourth, of the above-mentioned years, the rate of increase at any time during each period, being supposed to be proportional to the population at that time. *Answ.* During the first period, 3736; during the second, 6287; and during the third, 15320.

20. If a gallon of water were resolved into the oxygen and hydrogen of which it is composed, it is required to determine the bulk into which it would thus be expanded, water being 741 times heavier than an equal bulk of oxygen, and 9699 times heavier than an equal bulk of hydrogen. *(See exer. 5, p. 192.)* *Answ.* The two gases would fill $1794\frac{5}{17}$ gallons.

21. The Neperian logarithm* of any number, is to the common logarithm of the same number as 1 is to ·43429448, nearly. Required the series of ratios converging to this ratio. *Answ.* 2 to 1; 7 to 3; 23 to 10; 76 to 33; 99 to 43; 175 to 76; (or 700 to 304;) 624 to 271; 3919 to 1702; 12381 to 5377; 16300 to 7079, &c.

22. Reduce the fraction whose numerator is the square root of 5, and denominator the square root of 11, to a continued fraction, and find the first seven of the series of fractions converging to its value. *Answ.* $\frac{1}{1}, \frac{2}{3}, \frac{28}{43}, \frac{69}{89}, \frac{140}{221}, \frac{358}{531}$, and $\frac{5181}{7850}$.

23. A person, in discounting a bill, at 6 per cent. per annum, according to the common or false method, finds that he has $6\frac{1}{2}$ per cent. per annum for his money. How long must the bill have been discounted before it was due? *Answ.* 1 year and 103 days, nearly.

24. A person pays £54 for the insurance of goods at $3\frac{3}{4}$ per cent.; and he finds, that in case of the goods being lost, he will by this means be entitled to the value of the goods, the premium of insurance, and £5 besides. What is the value of the goods? *Answ.* £1381.

25. Reduce five ninths to a fraction in the septenary scale of notation, whose denominator in that scale may be expressed by a unit with as many ciphers annexed as there are figures in the numerator. *Answ.* The numerator will be ·361361361, &c. or ·3′61′.

26. If a merchant each year increase his capital by a fifth part of itself, except an expenditure of £400 per annum, and at the end of 15 years be worth £12000; find, without Position, his original capital. *Answ.* £2649 1 1.

27. A servant draws off one gallon each day for 20 days, from a cask containing 10 gallons of rum, each time supplying the deficiency by the addition of a gallon of water; and then, to escape

* Neperian logarithms are also frequently, but improperly, called *hyperbolic* logarithms.—The number ·4342944819, &c. is called the *modulus* of the system of common logarithms.

detection, he again draws off 20 gallons, supplying the deficiency each time by a gallon of rum. It is required to determine how much water still remains in the cask. *Answ.* 1·0679577 gallon, or rather more than a gallon and half a pint.

28. A sells a quantity of tea, which cost him £246 12, to B; and B sells it to C, who disposes of it for £391 11 10. Required the prices at which A and B sold it, each of the three merchants having gained at the same rate per cent. *Answ.* A sold it for £287 14, and B for £335 13.

29. A and B set out from the same place, and in the same direction. A travels uniformly 18 miles per day, and after 9 days turns and goes back as far as B has travelled during those 9 days; he then turns again, and pursuing his journey, overtakes B $22\frac{1}{2}$ days from the time they first set out. It is required to find the rate at which B uniformly travelled. *Answ.* 10 miles per day.

30. A merchant every year gains 50 per cent. on his capital, of which he spends £300 per annum in house and other expenses, and at the end of 4 years he finds himself possessed of a capital 4 times as great as what he had at commencing business. Find his original capital without using the rule of Position. *Answ.* £$2294\frac{2}{17}$.

31. At what time does the sun set, when the length of the day (from sunrise till sunset) is four times the length of the morning or evening twilight, and the evening twilight two sevenths of the time from its termination till day-break? *Answ.* At $3\frac{3}{19}$ minutes past 5 o'clock.

32. How will 13579 in the trigesimal scale, be expressed in the duodecimal scale? *Answ.* 372433.

33. Find the first nine fractions approaching to the ratio of 1 to the cube root of 2. *Answ.* $\frac{1}{1}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{23}{29}$, $\frac{27}{34}$, $\frac{50}{63}$, $\frac{227}{286}$, $\frac{277}{349}$, and $\frac{504}{635}$.

34. Required, the approximate ratios of the English foot to the French *metre*, and also to the *toise*. (*See note, page* 43.) *Answ.* The foot to the metre, as 1 to 3, 3 to 10, 4 to 13, 7 to 23, 25 to 82, 32 to 105, 57 to 187, 89 to 292, 146 to 479, &c.; and the foot to the toise, as 1 to 6, 2 to 13, 3 to 19, 5 to 32, 33 to 211, &c.

35. How will that fraction be expressed in the decimal scale, whose denominator in the octary scale is a unit with as many ciphers annexed as there are figures in its numerator, and its numerator 644 repeated without end? *Answ.* $\frac{60}{73}$.

36. What is the difference between $\frac{4}{12}$ in the quinary scale, and $\frac{7}{21}$ in the nonary scale? *Answ.* Twenty-seven one-hundred-and thirty-thirds.

37. What is the product of $\frac{54}{7}$ in the duodecimal scale, and $\frac{7}{54}$ in the octary scale? *Answ.* One and five-elevenths.

38. It is required to find a sum of money, of which, in the space 4 years, the *true* discount, at simple interest, is £5 more at the rate of 6 than of 4 per cent. per annum. *Answ.* £89 18.

39. One third of a quantity of flour being sold to gain a certain

rate per cent., one-fourth to gain twice as much per cent., and the remainder to gain three times as much per cent.: it is required to determine the gain per cent. on each part, the gain upon the whole being 20 per cent. *Answ.* The gains per cent. are $9\frac{3}{5}$, $19\frac{1}{5}$, and $28\frac{4}{5}$.

40. A man travels from his own house to Belfast in 4 days, and home again in 5 days, travelling each day, during the whole journey, one mile less than he did the preceding. How far does he live from Belfast? *Answ.* 90 miles.

41. What is the radix of the arithmetical scale, in which 9(20)(12)609 in the trigesimal notation will be expressed by 5000004? *Answ.* 19.

42. The men employed by a gentleman work 12 hours, the women 9 hours, and the boys 8 hours, each day: for labouring the same number of hours, each man receives a half more than each woman, and each woman a third more than each boy: the entire sum paid to all the women each day is double of the sum paid to all the boys; and for every five shillings earned by all the women each day, twelve shillings are earned by all the men. Hence it is required to find the number of each class employed, the entire number being 59. *Answ.* 24 men, 20 women, and 15 boys.

43. A man leaves to his eldest child one-fourth of his property; to his second, one-fourth of the remainder, and £350 besides; to his third one-fourth of the remainder and £975; to his youngest one-fourth of the remainder and £1400; and what still remains he bequeaths to his wife, whose share is found to be one-fifth of the whole. Hence it is required to find the value of the whole property. *Answ.* £20,000.

44. The less of two bales of cloth is bought at the rate of twice as many pence per yard as it contains yards, and costs £31 0 2 more than the greater, which contains 4 yards for every 3 in the less, and is bought at the rate of as many pence per yard as it contains yards. How many yards are contained in each? *Answ.* 244 yards in the greater, and 183 in the less.

45. It is required to find a sum of money such that its true discount for one year at 5 per cent. will be £1 more than the sum of the true discounts of one-half of it at 4 per cent. and the rest at 6 per cent. *Answ.* £11575 4.

46. A property of £10,000 is left to four children whose ages are 6, 8, 10, and 12 years respectively; and it is divided among them in such a manner, that their several shares being improved at $4\frac{1}{2}$ per cent. per annum, compound interest, they shall all have equal properties at the age of 21. It is required to determine the sum left to each. *Answ.* £2180 3 $4\frac{1}{4}$, £2380 15 $11\frac{1}{4}$, £2599 17 $9\frac{3}{4}$, and £2839 2 $10\frac{3}{4}$.

47. A property is left to four children, one aged 6 years, two aged 9 years each, and one aged 11 years, in such a manner that all their properties are to be equal on their coming to age, compound interest being allowed at 4 per cent. per annum. Now, one

of the twins dying before the period at which the eldest would be of age, his share is divided among the rest in such a manner that their properties may be still equal, when they come to age; and, in consequence, each is found at that period to have £1000 more than he would otherwise have had. What was the value of the entire property bequeathed? *Answ.* £7367 2 $11\frac{1}{4}$.

48. A man owes a debt, to be paid in four equal instalments at the end of 4, 9, 12, and 20 months respectively; and he finds that, discount being allowed, according to the true method, at 5 ℔ cent. ℔ annum, £750 paid at present will discharge the whole debt. How much did he owe? *Answ.* £$784\frac{978194}{1321009}$.

49. If a merchant commence trade with a capital of £5000, and gain so much, that, after paying all expenses, his capital, each year, is increased by a tenth part of itself wanting £100, how much will he be worth at the end of 20 years? *Answ.* £27910.

50. A man borrows £500, and agrees to pay simple interest at 5 ℔ cent. ℔ annum. At the end of 11 months he pays one part of the principal with its interest; 8 months after he pays another part with its interest from the time it was borrowed; and 11 months after that he pays the remainder of the principal with its interest in like manner from the time it was borrowed. What was the amount of each payment, each of the two last being double of the first? *Answ.* The first payment £$108\frac{669717}{670976}$, and each of the others £$217\frac{334229}{335488}$.

51. Mercury revolves round the sun in 87 days, 23 hours, 15 minutes, 44 seconds, and the earth in 365 days, 6 hours, 9 minutes, 12 seconds. Required the seven first approximate ratios of these periods. *Answ.* $\frac{1}{4}$, $\frac{6}{25}$, $\frac{7}{29}$, $\frac{13}{54}$, $\frac{33}{137}$, $\frac{46}{191}$, and $\frac{217}{901}$.

52. Venus revolves round the sun in 224 days, 16 hours, 49 minutes, 11 seconds, and the earth in the time stated in the preceding question. Required the first eight fractions approaching the ratios of these periods. *Answ.* $\frac{1}{1}$, $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{8}{13}$, $\frac{235}{382}$, $\frac{478}{777}$, and $\frac{14575}{23692}$.

53. 35 lbs. of tea being mixed with 20 lbs. of a better quality, the mixture is found to be worth 7/4 per lb. Required the value of each kind, the difference of their values being 1/10 ℔ lb. *Answ.* 8/6 and 6/8 ℔ lb.

54. Express three hundred and fifty seven in the scale of notation whose radix is $3\frac{1}{2}$. *Answ.* $21101\frac{1}{2}$.

55. If a person lend £7000 at 6 ℔ cent. ℔ annum, compound interest, and allow the interest to accumulate in the hands of the creditor, except £240 ℔ annum, which he lifts for family expenditure; how much will the creditor owe him at the end of 16 years? *Answ.* £11621 1 $1\frac{1}{4}$.

56. If a boy read each day 2 lines more of Virgil than he did the day before, and find, that having read a certain quantity in 18 days, he will read, at this rate of increase, the same quantity in the next 14 days; how much will he read in the whole time? *Answ.* 4032 lines.

57. Two men, A and B, are on a straight road, on the opposite sides of a gate, and distant from it 308 yards and 277 yards respectively, and travel each towards the original station of the other. How long must they walk till their distances from the gate will be equal, B travelling 2 yards, and A $2\frac{1}{3}$ yards, per second? *Answ.* 1 minute, 33 seconds, or 2 minutes, 15 seconds.

58. Every thing being supposed to be as in the preceding question, at what time will each be at the same distance from the original station of the other, as the other is from his? *Answ.* In $4\frac{1}{2}$ minutes after starting.

59. Suppose a person to mix 11 lbs. of tea with 5 lbs. of an inferior quality, and to gain 16 per cent. by selling the mixture at 7/3 per lb.: it is required to determine the prime costs, when a pound of the one cost a shilling more than a pound of the other. *Answ.* $6/6\frac{3}{4}$ and $5/6\frac{3}{4}$ per lb.

60. If a person borrow £1100 at 6 per cent. per annum, compound interest, and agree to pay both principal and interest in eleven equal annual payments, how much must each payment be, the first being made at the end of the first year? *Answ.* £139 9 $5\frac{1}{4}$.

64. If a farm of 84 acres be held at £1 7 6 per acre, on a lease of which 48 years are unexpired, what fine must be paid at present to reduce the rent to 10/ per acre during the last 30 years of the lease, compound interest being allowed at 6 per cent. per annum? *Answ.* £354 8 $11\frac{1}{2}$.

62. A man aged 45 has a pension of £300 a year during his own life; but he wishes to exchange it for another to continue not only during his own life, but also during that of his wife, aged 40. What will this pension be, money being supposed to be improvable at 6 per cent. per annum, compound interest? *Answ.* £236 14 $7\frac{1}{4}$.

63. Required the converging fractions approaching to the ratio of 5 hours, 48 minutes, 48 seconds, and 24 hours. *Answ.* $\frac{1}{4}$, $\frac{7}{29}$, $\frac{8}{35}$, $\frac{31}{135}$, and $\frac{32}{139}$.

64. If the acting partner in a mercantile concern contribute £1000 to the original joint stock of the company, and annually increase this sum by £150 saved from his salary; to how much will his share of the joint stock amount, at the end of 11 years, on the supposition, that, after all expenses are paid, there is a clear gain of 10 per cent. per annum on the entire capital? *Answ.* £5632 15 10.

65. Suppose 17 gallons of spirits, at 10/6 per gallon, to be mixed with 7 gallons at a different price. What was the price of the latter per gallon, when 20 per cent. is gained by selling the mixture at 13/ per gallon? *Answ.* $11/7\frac{4}{7}$.

66. If a grocer mix together 13 parts of better and 7 parts of worse sugar, the price of the latter per cwt. being only $\frac{5}{8}$ of that of the former; what were their prime costs, if by selling 17cwt. 3qrs. 20lbs. at £4 6 6 per cwt., he gain £12 11? *Answ.* The better £3 16 $11\frac{191}{113}$, and the worse £3 4 $1\frac{191}{113}$ per cwt. respectively.

67. A merchant bequeaths £1000 among six clerks in proportion to their salaries, and the periods they have held their situations. Now, one of them has held his situation five years, and his salary is £120; two of them four years, with salaries of £75 each; and the rest two years, with salaries of £60 each. Required their several shares. *Answ.* The share of the first £384 12 $3\frac{9}{13}$; of the two next £192 6 $1\frac{11}{13}$ each; and of the rest £76 18 $5\frac{7}{13}$ each.

68. If a shopkeeper each year double his capital, except an expenditure of £240 per annum, and, at the end of 4 years, be worth only three-fourths of his original capital; find without Position, what he had at commencing trade.
Answ. £236 1 $3\frac{45}{61}$.

69. What sum will amount to £1 more at simple than at compound interest, in four months, at 5 per cent. per annum? *Answ.* £3699 9 2.

70. Required the sum of the infinite series, $\frac{2}{5}-\frac{4}{15}+\frac{8}{45}-\frac{16}{135}+\frac{32}{405}$—&c. *Answ.* $\frac{6}{25}$.

71. Given the extremes of three numbers in harmonical proportion=10 and 18: required the mean. *Answ.* $12\frac{6}{7}$.

72. Given the first and second of three numbers in harmonical proportion=70 and 126 respectively: required the third term. *Answ.* 630.

73. Given the first, second, and third of four numbers in harmonical proportion=6, 7, and 8 respectively: required the fourth. *Answ.* $9\frac{3}{5}$.

74. Given the first, third, and fourth of four numbers in harmonical proportion=5, 9, and 10 respectively: required the second term *Answ.* $5\frac{1}{2}$.

75. If a bank borrow £10,000 at 4, and employ it in discounting bills at 6 $\frac{20}{21}$ cent. $\frac{20}{21}$ annum, and afterwards borrow money at 2, and discount bills with it at 4 $\frac{20}{21}$ cent. $\frac{20}{21}$ annum; how much must be borrowed, and so employed, in the latter case, that the gain may be the same as in the former, the bills being discounted in both cases six months before they would be due? *Answ.* £10,499$\frac{1447}{4947}$.

APPENDIX,

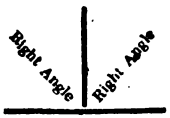
CONTAINING

AN INTRODUCTION TO MENSURATION.

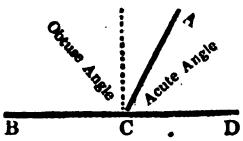
DEFINITIONS.

I. AN ANGLE is the inclination of two straight lines which meet one another in a point, which is called the VERTEX of the angle: or it is the degree of their opening or divergence.

II. When one straight line standing on another, makes with it two angles which are equal, each of these angles is called a RIGHT ANGLE; and the straight line which stands on the other is said to be PERPENDICULAR, or AT RIGHT ANGLES, to it, or to be a PERPENDICULAR to it.

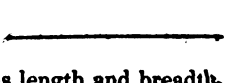


III. An OBTUSE ANGLE is greater than a right angle: an ACUTE ANGLE is less than a right angle.



An angle is usually named by three letters, the middle one being placed at the vertex, and the other two somewhere on the lines which contain the angle. Thus, the obtuse angle in the last diagram, may be called the angle ACB, or BCA, and the acute one the angle ACD, or DCA. When there is only one angle at the same point, it may be named by a single letter placed at that point.

IV. PARALLEL STRAIGHT LINES are those which have every where equal perpendicular distances from each other.



V. A SURFACE, or SUPERFICIES, has length and breadth, without thickness.

VI. A BODY, or as it is often called, a SOLID, has length, breadth, and thickness, or depth.

VII. A FIGURE is a portion of space inclosed by one or more boundaries.

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* The small introductory tract on Mensuration which is here given, is intended for the use of such pupils as may not have time or opportunity for studying a more extended course. For this reason, the easiest and most useful parts are selected, and the definitions and illustrations are delivered in plain and familiar terms, rather than with a view to Mathematical precision. The rules are also given without demonstrations, as the pupils for whom this abstract is intended, are not supposed to have read a preparatory course of Mathematics. Those who may wish to prosecute the subject more extensively may have recourse to the treatises of Bonnycastle, Hutton, and others on the subject. Hutton's larger treatise, in particular, is a very extensive and valuable work.

VIII. A FIGURE in said to be EQUILATERAL, if it have equal sides; and EQUIANGULAR, if it have equal angles.

IX. If a figure be contained by three lines, it is called a TRIANGLE; if bv four, a QUADRILATERAL; if by more than four, a POLYGON.

X. An equilateral and equiangular polygon is often called a REGULAR POLYGON.

XI. Polygons of five, six, seven, eight, nine, ten, eleven, and twelve sides, are often called respectively, PENTAGONS, HEXAGONS, HEPTAGONS, OCTAGONS, NONAGONS, DECAGONS, HENDECAGONS, and DODECAGONS.

XII. A quadrilateral which has its opposite sides parallel, is called a PARALLELOGRAM: and a parallelogram which has its angles right angles, is called a RECTANGLE.

XIII. A quadrilateral which has its sides equal, and its angles right angles, is termed a SQUARE; and a quadrilateral which has its sides equal, but its angles not right angles, is called a RHOMBUS.

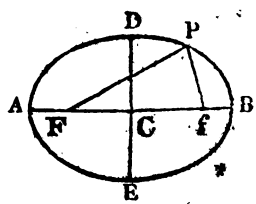
XIV. A quadrilateral which has two sides parallel, and the other two not, is called a TRAPEZOID.

XV. Any quadrilateral, except a parallelogram or trapezoid, is called a TRAPEZIUM.

XVI. A DIAGONAL of a figure is a line passing through two of its remote angles.

XVII. A CIRCLE is a figure contained on a flat surface, by one line which is called the CIRCUMFERENCE; and is such that all straight lines drawn to the circumference from a certain point within the figure, called the CENTRE, are equal to each other. Any of those equal lines is called a RADIUS: and a line drawn through the centre, and terminated both ways by the circumference, is called a DIAMETER. Hence, a diameter is evidently double of a radius.

XVIII. If the ends of a thread, FPf, be fastened to two pins, F,f, fixed at a less distance asunder than the length of the thread, and if the point of a pen or pencil, P, be carried round in such a manner as to keep the thread constantly stretched, the figure inclosed by the curve line thus described, is called an ELLIPSE; the points F and f, where the pins are fixed, are



called the FOCI (and each of them a FOCUS); the line AB, drawn through the foci, and terminated both ways by the curve, is called the GREATER AXIS; and the line DE, drawn perpendicular to this axis through its middle point, and terminated by the curve, is called the LESS AXIS.

XIX. MENSURATION is the method of determining by computation the comparative magnitudes of figures; and is divided into two branches, *the mensuration of surfaces*, and *the mensuration of bodies or solids*.

XX. The AREA of a figure is the space which it contains. In Mensuration, the magnitude of this space is ascertained by the number of times that a given space, called the *measuring unit*, is contained in it.

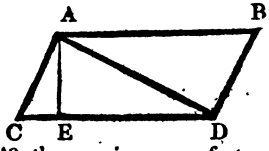
XXI. The MEASURING UNIT which is adopted for surfaces, is a square whose side is some of the common measures of length, such as *a square inch, a square foot, a square yard*, &c. (*See the Table of Square Measure, page* 39.)

MENSURATION OF SURFACES.

RULE I. *To find the area of a parallelogram:* Multiply its length by its perpendicular breadth.

Hence, to find the area of a square, we are only to multiply a side by itself.

Exam. 1. Required the area of the parallelogram ABDC, the length AB, or CD, being 36 feet, and the perpendicular breadth, AE, 15 feet.



Here the product of 36 and 15 is 540, the area in square feet.

Exam. 2. If the length of the floor of a rectangular room be 22 feet 5 inches, and its breadth 16 feet 11 inches, how many square feet does it contain?

This and similar examples, in which the dimensions are given in feet and inches, may be wrought in different ways.

1. By reducing 5 inches and 11 inches each to the decimal of a foot, we obtain for the dimensions, 22·416′, and 16·916′; the product of which is 379·21527′ square feet, or 379¼ square feet nearly, the area required.

2. As a second method, by reducing the dimensions to inches, we obtain 269 and 203; the product of which is 54607 square inches, the area required: and by dividing this by 144, we get 379 square feet, 31 square inches, a result which agrees with that found by the former method.

3. Another mode, which is perhaps preferable to either of the preceding, is the method of *duodecimals*, or, as it is often improperly called, the method of *cross multiplication*. This will be understood from the operation in the margin, in which we multiply successively by 16 and 11. The product of 16 and 5 is divided by 12, and the remainder is set down, and the quotient carried. In multiplying by 11, both the products are divided by 12, and the remainders set one place towards the right hand. The sum of the two partial products is then taken, and the result is found to be 379 square feet, 2 twelfths of a square foot, or 24 square inches, and 7 square inches; or 379 square feet, 31 square inches, as before. The answer might also be obtained by multiplying by 16, and working for 11 inches by means of aliquot parts.

Feet	In.	
22	5	
16	11	
358	8	
20	6	7
379	2	7

The results obtained by this latter method are often called, very improperly, *feet*, *inches*, and *parts*, instead of square feet, twelfths of such feet, and square inches. According to this mode, the last answer would be read 379 feet, 2 inches, and 7 parts. It may be written, 379 f. 2′ 7″.

The multiplication of feet and inches might also be performed very simply and easily by reducing the feet to the duodecimal scale, and then carrying for 12 in the multiplication. Thus, by reduction to the duodecimal scale, (see page 248) the dimensions in the foregoing example become 1D·5 and 14·H, and the work will stand as in the margin, the answer in the duodecimal scale being 277·27, or by reduction to the decimal scale, 379 feet, 2 twelfths of a foot, and 7 inches. The learner, after a little practice, will perhaps prefer this method to any other.

1D·5
14·H
1867
758
1D5
277·27, or
379f. 2′ 7″, *Answer*.

Exam. 3. Required the area of a square field, each of whose sides is 6 chains 30 links.

Here, (note 1, page 40) each side is 630 links; and multiplying this by itself we obtain for the area 396900 square links; or 3·969 acres, by division by 100,000; whence by multiplying by 4 and 40, we get for the required content 3a. 3r. 35p. with a small remainder.

Links.
630
630
189
378
3·96900
4
3·876
40
35·040

Ex. 1. Required the content of a field in form of a parallelogram, the length and breadth of which are 12 chains 76 links, and 9 chains 43 links. *Answ.* 12 acres, 0 roods, 5 per.

2. Given the length of a street = 937 feet 6 inches, and its breadth = 66 feet 8 inches; required the cost of paving it at 8½d. ℔ square yard. *Answ.* £245 18 11¾.

3. At 9¾d. ℔ yard, required the cost of painting a room, the sum of the lengths of whose sides is 70 feet 10 inches, and its height 10 feet 1 inch. *Answ.* £3 4 5¾.

4. Required the content of a rectangular garden whose length is 98 yards, and its breadth 81 yards. *Answ.* 1 acre, 2 perches, Irish measure; or 1 acre, 2 roods, 22 perches, 12½ yards, English measure.

5. What is the content of a deal-board, 9 feet 8 inches long, and 8½ inches broad? *Answ.* 6 f. 10′ 9″.

6. If each side of a square table be 3 feet, 10 inches, what is its content? *Answ.* 14 f. 8′ 4″.

RULE II. *To find the area of a triangle:* Multiply the base by the perpendicular, and take half the product: or multiply one of these lines by half the other.

Exam. 4. Required the area of the triangle ACD (see the diagram page 265) whose base CD is 15 feet 4 inches, and its perpendicular AE, 8 feet 7 inches.

Here, by the method of duodecimals already explained, and by halving, we find for the area, 65 feet, 9 twelfths of a foot, and 8 inches, or 65¾ feet, nearly.

	Feet	In.	
	15	4	
	8	7	
	122	8	
	8	11	4
2)	131	7	4
	65	9	8

Ex. 7. Base = 13 chains 24 links, perpendicular = 8 chains 50 links. *Answ.* Area = 5 a. 2 r. 30 p.

8. Base = 21 feet 7 inches, perpendicular = 17 feet 10 inches. *Answ.* Area = 192 f. 5′ 5″.

RULE III. *To find the area when the three sides are given.* (1.) Add the three sides together, and take half the sum: (2.) From the half sum take the three sides severally: (3.) Find the continual product of the half sum and the three remainders: (4.) Extract the square root of this product.

The area of an equilateral triangle may be found by multiplying the square of one of the sides by ·4330127.

Exam. 5. Required the area of a triangle, whose sides are 2, 3, and 4 feet respectively.

Here, half the sum of the sides is 4·5; and subtracting from this

the three sides successively, we find the three remainders 2·5, 1·5, and ·5. Then $4{\cdot}5 \times 2{\cdot}5 \times 1{\cdot}5 \times {\cdot}5 = 8{\cdot}4375$; and $\sqrt{8{\cdot}4375} = 2{\cdot}9047375$, the area required.

Ex. 9. Sides, 9 chains 62 links; 6 chains 38 links; and 7 chains 20 links. *Answ.* Area, 2 acres, 1 rood, $7\frac{1}{2}$ perches, nearly.

10. Sides, 13, 14, and 15 feet. *Answ.* Area, 84 feet, or $9\frac{1}{3}$ yards.

11. Sides, 3 feet 8 inches; 4 feet 7 inches; and 6 feet 5 inches. *Answ.* Area, 8·233 feet.

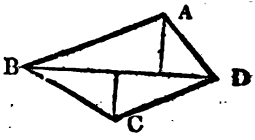
RULE IV. *To find the area of a trapezoid:* Multiply the sum of the parallel sides by the perpendicular breadth of the figure, and take half the product.

Ex. 12. Given the parallel sides of a trapezoid = 33 and 28 inches, and its breadth = 11 inches: required the area. *Answ.* $335\frac{1}{2}$ inches.

13. Parallel sides = 75 and 33 feet, breadth = 20. *Answ.* Area, 1080 feet, or 120 yards.

RULE V. *To find the area of a trapezium:* Multiply either of the diagonals by the sum of the perpendiculars drawn to it from the opposite angles, and take half the product. Or, Find, by rule II. or III., the areas of the triangles that compose the trapezium, and add them together.

Exam. 6 Given BD, the diagonal of the trapezium ABCD, = 15 perches 6 yards; and the perpendiculars drawn from A and C = 6 perches 5 yards, and 5 perches 4 yards; to find the area.

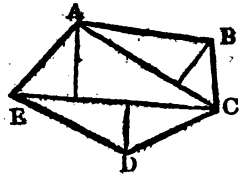


Here, the sum of the perpendiculars is 12 perches 2 yards, or 86 yards; and the diagonal being 15 perches 6 yards, or 111 yards, we have $86 \times 111 = 9546$, the half of which is 4773, the area in square yards, which, by division by 49 and 40, is reduced to 2 roods, 17 perches, and 20 yards.

Ex. 14. Perpendiculars 8 chains 82 links, and 7 chains 75 links; diagonal 17ch. 56l. *Answ.* Area = 14 acres, 2 roods, 5 perches.

15. Given BD = 21 perches, BA = 18 perches, AD = 9 perches, BC = 12 perches, and CD = 15 perches. *Answ.* Area = 168·68008.

RULE VI. *To find the area of a polygon:* (1.) Divide it by diagonals into triangles and trapeziums: (2.) Find the areas of these by some of the foregoing rules: (3.) Add all these areas together.



Ex. 16. Given EC = 58 perches, and the perpendiculars to it from A and D = 27 and 18 perches respectively; given also AC = 46 perches, and the perpendicular to it from B = 20 perches; to find the area. *Ans.* 11a. 0r. 5p.

17. Given AB = 6 feet, BC = 3 feet, CD = 4 feet, DE = 5 feet, AE = 4 feet, AC = 7 feet, and EC = 8 feet. Required the area. *Answ.* 31·123574 feet.

RULE VII. The *area of a regular polygon* is most easily found by multiplying the square of one of the sides by the number standing opposite to the name of the polygon in the annexed table.

Pentagon	1·7204774	Nonagon	6·1818240
Hexagon	2·5980762	Decagon	7·6942088
Heptagon	3·6339126	Hendecagon	9·3656411
Octagon	4·8284272	Dodecagon	11·1961524

Ex. 18. Side of a Hexagon = 5 inches. *Answ.* Area = 64·951905.

19. Side of an Octagonal inclosure = 60 yards. *Answ.* Area = 2a. 0r. 34p. 36yds. Irish measure, or 3a. 2r. 14p. 19yds English measure.

RULE VIII. *The diameter of a circle being given to find the circumference:* Multiply the diameter by 3·1416, or more accurately, 3·141593: Or,

As 113 is to 355, so is the diameter to the circumference: or, when much accuracy is not necessary, as 7 is to 22, so is the diameter to the circumference.

The diameter would be found from the circumference, by reversing any of the foregoing processes.

Ex. 20. Given the diameter = 13 inches. *Answ.* Circumference = 40·8407.

21. If the diameter of a circular cask be 2 feet 9 inches, what will be the length of a hoop for it? *Answ.* 8 feet, 7·67257 inches.

22. If the girt of a round tree be 12 feet 5 inches, what is its diameter? *Answ.* 3 feet, 11·42817 inches.

RULE IX. *To find the area of a circle:* Multiply the diameter by the circumference, and take one fourth of the product: Or,

Multiply the square of the diameter by ·7854, or, for greater accuracy, by ·785398: Or,

Multiply the square of the radius by 3·141593: Or,

Multiply the square of the circumference by ·07958.

The *area of an ellipse* is found by multiplying the product of the axes by ·7854, or more accurately, ·785398.

Ex. 23. Required the area of a circular pond whose diameter is 31 yards. *Answ.* 754·7694 square yards.

24. Required the area of the space on which a horse may graze, when confined by a cord 7½ perches in length, having one of its ends fixed at a certain point. *Answ.* 1 a. 0 r. 16·7 p.

25. Required the content of a circular grove, 56·5 perches in circumference. *Answ.* 1 a. 2 r. 14 p.

26. What is the area of a circular table whose diameter is 5 feet 8 inches? *Answ.* 25 feet 31¾ inches, nearly.

27. What is the area of an elliptic ceiling, the axes of which are 33 feet 5 inches, and 20 feet 3 inches? *Answ.* 59 yards, 0 feet, 67 inches.

MENSURATION OF BODIES, OR SOLIDS.

DEFINITIONS.

I. A figure whose ends, or bases, are parallel, and its sides parallelograms, is called a PRISM. Such a figure is termed a RIGHT PRISM, if each of its bases be perpendicular to its other sides; and it is said to be TRIANGULAR, HEXAGONAL, &c. if its bases be triangles, hexagons, &c.

II. A prism whose bases, as well as its other sides, are parallelograms, is called a PARALLELEPIPED. A common chest, bar of iron, brick, &c. afford instances of *right parallelepipeds*.

III. A right parallelepiped whose sides are equal squares, is called a CUBE. Such are dice.

IV. A PYRAMID is a body bounded by plane surfaces meeting in a point called the VERTEX of the pyramid, and by a rectilineal base terminated by those planes.

V. A CYLINDER is a round body which is of equal thickness throughout, and which has circular bases, parallel to each other, such as a rolling stone for walks, a round pillar, &c.

VI. A body which has a circular base, and which tapers uniformly to a point named the VERTEX, is called a CONE. A sugar-loaf, and the top of a round pillar are nearly of this form.

VII. A FRUSTUM of any body is what remains, when the top is taken away.

VIII. A GLOBE, or SPHERE, is a body of such a figure

that all points of the surface are equally distant from a point within it called the CENTRE.

IX. If one of the parts into which an ellipse is divided by either of the axes, revolve about that axis, the figure which it describes is called a SPHEROID:—PROLATE, if the revolution be performed round the greater axis; OBLATE, if round the less. An egg is nearly of the former figure; and a watch, or a flat turnip, nearly of the latter.

X. The CONTENT, or VOLUME, or as it is often improperly called, the SOLIDITY of a body, is the space contained within it. The magnitude of this space is expressed by the number of times that it contains a given space called the *measuring unit*.

XI. The MEASURING UNIT which is adopted for bodies, is a cube whose base is the measuring unit for surfaces, such as a cubic inch, a cubic foot, &c. (*See the Table of Cubic Measure, page* 40.)

RULE I. *To find the content of a prism or cylinder:* (1.) Find the area of the base: (2.) Multiply this by the perpendicular height, or distance between the ends.

Hence, *to find the content of a right parallelepiped*, take the continual product of the length, breadth, and thickness, or depth: and, *to find the content of a cube*, find the third power of one of the sides of its base.

Exam. 1. Required the content of a box of cloth, the length, breadth, and depth of which are 4 feet 10 inches; 2 feet 11 inches; and 2 feet 2 inches, respectively.

Here, by the method of duodecimals, we multiply 4 feet 10 inches by 2 feet 11 inches: the product, 14 feet, 1 twelfth, and 2 inches, is the area of the base. We then multiply in a similar manner by 2 feet 2 inches, the depth, setting each product in multiplying by the inches, one place towards the right hand. The final product or content of the box, is 30 cubic feet, 6 twelfths of a foot, 6 one-hundred-and-forty-fourths of a foot, and 4 inches, or 30½ feet, nearly.

Feet	In.		
4	10		
2	11		
9	8		
4	5	2	
14	1	2	
2	2		
28	2	4	
2	4	2	4
30	6′	6″	4‴.

In measures of capacity, since the cubic foot contains 1728 cubic inches, the twelfths of a foot are each 144 cubic inches, and the hundred-and-forty-fourths are each 12 inches. Hence the foregoing answer is easily reduced to

30 feet and 940 inches. The same result would be obtained by reducing the dimensions to inches, finding their continual product, and dividing it by 1728; or by reducing the inches to decimals, and proceeding in a similar manner. The results in such cases as the present, are often improperly called feet, inches, parts, and seconds.

Ex. 1. Required the number of gallons of water, of 217·6 cubic inches each, contained in a rectangular cistern, the length, breadth, and depth of which are 16 feet, 10 feet 6 inches, and 8 feet 4 inches. *Answ.* 11117·647.

2. If each side of the base of a triangular prism be 2 inches, and its length 14 inches, what is its content? *Answ.* 24·2487 inches.

3. Required the number of cubic feet contained in a room whose length, breadth, and height are 24 feet, 18 feet 6 inches, and 10 feet 7 inches. *Answ.* 4699.

4. Given the diameter of the base of a cylindric column = 3 feet 1 inch, and its height = 18 feet 9 inches; required its content. *Answ.* 140·0016 feet.

5. Required the content of a bale, the length, breadth, and thickness of which are, 4 feet 8 inches, 3 feet 3 inches, and 2 feet 6 inches. *Answ.* 37 feet, 11 twelfths.

6. Given the length, breadth, and thickness of a uniform plank equal to 22 feet 7 inches, 1 foot 5 inches, and 6½ inches, respectively. Required the content. *Answ.* 17·32957 feet.

RULE II. *To find the content of a pyramid or cone:* Multiply the area of the base by the perpendicular height, and take one third of the product.

Ex. 7. Given each side of the base of a square pyramid = 10 inches, and the perpendicular height, or altitude = 9 feet 9 inches: required the content. *Answ.* 2f. 3′ 1″.

8. Given the diameter of the base of a conical glass-house = 37 feet 8 inches, and the altitude = 79 feet 9 inches. Required the entire space enclosed. *Answ.* 29622 feet, nearly.

9. The height of the largest of the Egyptian pyramids is 477 feet, and each side of its base, which is a square, is 720 feet. Required the content. *Answ.* 82425600 cubic feet, or 3052800 cubic yards.

10. Given the altitude of a conical sugar-loaf = 17 inches, and the diameter of its base = 9 inches. Required the content. *Answ.* 360·4986 inches.

11. How often may a conical glass, 3 inches deep, and 1¾ inches in diameter at the mouth, be filled out of an Irish liquid gallon? *Answ.* 90½ times nearly.

RULE III. *To find the content of a frustum of a pyramid, made by a plane parallel to its base:* (1.) To the product of two

corresponding sides of the greater and less ends, add one third of the square of their difference; the sum will be the square of a side of the mean base: (2.) From this find the area of the mean base by some of the rules already given for the measurement of plane surfaces: (3.) Multiply the result by the height.

To find the content of a like frustum of a cone: (1.) To the product of the diameters of the two ends add one third of the square of their difference; the sum will be the square of a mean diameter: (2.) Multiply this square by ·7854, and the product by the height.

Exam. 2. Given the sides of the greater and less bases of a regular octagonal pyramid = 19 and 10 inches respectively, and the length 5 feet 6 inches: required the content.

Here, $19-10=9$, and $19\times10+9^2\div3=217$, the square of a side of the mean base, which being multiplied by 4·8284272, the tabular number (see page 271) we find for the area of the mean base 1047·7687024: and the product of this by 5·5, the length, is 5762·7278632: and dividing this by 144, we obtain 40·01894349 feet, the content required.

Ex. 12. If the length of a frustum of a square pyramid be 18 feet 8 inches, the side of its greater base 27 inches, and that of its less 16 inches, what is the content? *Answ.* 61·228395 cubic feet.

13. Required the content of an ale glass in form of the frustum of a cone, the diameter at the mouth being 2½ inches, that of the bottom 1 inch, and the depth 5 inches. *Answ.* 12·76275 cubic inches.

Rule IV. *To find the content of a globe:* Multiply the cube of the diameter by ·5236.

To find the content of a spheroid: Multiply the fixed axis by the square of the revolving axis, and the product by ·5236.

Ex. 14. Required the contents of three globes, whose diameters are 12, 15, and 21 inches respectively. *Answ.* 904·7808, 1767·15, and 4849·0596 cubic inches respectively.

15. Required the content of a balloon in form of a prolate spheroid, having its longest diameter 48 feet, and its shortest 38 feet. *Answ.* 36291·7632 cubic feet.

Rule V. *To find the area of the surface of a body bounded by plane surfaces:* Find the areas of those surfaces separately, and add them together.

To find the area of the curve surface of a right cone: Multiply the circumference of the base by the slant height, and take half the product.

To find the area of the surface of a globe: Multiply the square of the diameter by 3·1416.

Ex. 16. Required the area of the surface of a square pyramid, each side of the base of which is 2 feet 8 inches, and its slant height, measured from the vertex to the middle of any side of the base, 3 feet 9 inches. *Answ.* 27 feet, 1 twelfth, and 4 inches.

17. Required the area of the entire surface of a right cone, the slant height of which is 4 feet 7 inches, and the diameter of its base 2 feet 11 inches. *Answ.* 27·679896 square feet.

18. If the earth were a sphere 7912 miles in diameter, what would be its superficial content? *Answ.* 196,663,355·7504 square miles.

19. Required the superficial contents of the three globes mentioned in exercise 14. *Answ.* 452·3904, 706·86, and 1385·4456 square inches respectively.

RULE VI. *To find the content of round or squared timber:* (1.) Take the girt of the tree at the middle, and divide it by 4 (which may be done by halving the line used in taking the girt, and then halving the half thus obtained): (2.) Multiply the square of the quarter girt thus found, by the length.

If the breadth and depth of squared timber differ considerably, measure them, and multiply their product by the length.

If the tree do not taper uniformly, measure the girts at the middle and ends, or at other equal distances; add the results together, and divide the sum by the number of girts taken; use the quotient as a mean girt, and proceed as before.

If great accuracy were required, timber which tapers uniformly, should be measured as the frustum of a pyramid or cone; but when it tapers slowly, as is generally the case, any dimension taken at the middle may be used as a mean without much error. The measurement of round timber, however, by taking, according to the foregoing rule, which is universally adopted in practice, the quarter girt as the side of a square to a mean section of the tree, is very erroneous, giving the content far too small. If it should be wished *to correct the result thus found, it may be done by adding to it one fourth of itself, and to the sum one foot for every 54 contained in it.* Another method of approximating the true content is, to *multiply the square of one fifth of the girt by twice the length;* or, which is perhaps easier, to *multiply the square of the girt by* ·08, *and the product by the length.* By both these methods the result is rather too great, and requires to be diminished by one foot in 190.

Ex. 20. Given the mean girt of a square piece of timber = 6 feet 8 inches, and its length = 44 feet 4 inches: required the content. *Answ.* 123·1′48″ cubic feet.

21. Required the content of a

is 32 feet 6 inches, and its mean girt 5 feet 10 inches. *Answ.* 69·119 cubic feet; or, more correctly, 87·9 feet.

22. Given the length of a piece of squared timber = 31 feet 4 inches, and its breadth and depth at the middle = 2 feet 7 inches, and one 1 foot 9 inches respectively. Required its content. *Answ.* 141·65 cubic feet.

23. Given the girts of a round tree at the ends, and at two intermediate points equally distant from them and from each other, equal to 10 feet 6 inches, 5 feet 6 inches, 8 feet 8 inches, and 7 feet respectively, and the length 21 feet 5 inches. Required the content. *Answ.* 88·89 cubic feet; or, more correctly, 106·8 feet.

RULE VII. *To find the content of a common cask in gallons:* (1.) To the square of the head diameter add double the square of the bung diameter, and from the sum take four-tenths of the square of the difference of those diameters: (2.) Multiply the remainder by the length: (3.) Then multiply the result by ·0012, to find Irish liquid gallons; by ·0011$\frac{1}{3}$, to find English wine gallons; or by ·0009$\frac{1}{4}$, to find English ale gallons.

The result thus found in Irish gallons may be corrected by adding to it a four-hundredth of itself.

Ex. 24. If the head diameter of a cask be 25 inches, the bung diameter 34 inches, and the length 43 inches, how many gallons does it contain? *Answ.* 150$\frac{1}{4}$ Irish, 141$\frac{1}{3}$ wine gallons, or 115$\frac{1}{2}$ ale gallons.

Ex. 25. Given the length of a rectangular field = 15 perches: required its breadth so that it may contain an acre. *Answ.* 10$\frac{2}{3}$ perches.

26. If a horse be bound in the middle of a field by a cord, one end of which is fixed at a certain point; what must be the length of the cord, that the horse may be allowed to graze on exactly an acre? *Answ.* 7·1365 perches.

27. Given the height of a stone column in form of a frustum of a cone = 28 feet 6 inches, and the diameters of its ends 3 feet, and 2 feet 3 inches, respectively. Required its weight, a cubic foot of the stone weighing 2568 ounces, avoirdupois. *Answ.* 11 tons, 2 cwt. 2 qrs. 3 lbs.

28. Required the diameter of a circle whose area is a square foot. *Answ.* 13·54054 inches.

29. Required the diameter of a globe whose content is a cubic foot. *Answ.* 14·8884 inches.

30. Given the diameter of the base of a cone = 50 inches, and its content = 50 cubic feet: required its height. *Answ.* 11 feet nearly.

TABLE I. SHOWING THE AMOUNT OF £1, AT COMPOUND INTEREST

Yrs.	3 per cent.	4 per cent.	5 per cent.	6 per cent.
1	1·030,000	1·040,000	1·050,000	1·060,000
2	1·060,900	1·081,600	1·102,500	1·123,600
3	1·092,727	1·124,864	1·157,625	1·191,016
4	1·125,509	1·169,859	1·215,506	1·262,477
5	1·159,274	1·216,653	1·276,282	1·338,226
6	1.194,052	1·265,319	1·340,096	1·418,519
7	1·229,874	1·315,932	1·407,100	1·503,630
8	1·266,770	1·368,569	1·477,455	1·593,848
9	1·304,773	1·423,312	1·551,328	1·689,479
10	1·343,916	1·480,244	1·628,895	1·790,848
11	1·384,234	1·539,454	1·710,339	1·898,299
12	1.425,761	1·601,032	1·795,856	2·012,196
13	1.468,534	1·665,074	1·885,6 9	2·132,928
14	1·512,590	1·731,676	1·979,932	2·260,904
15	1·557,967	1·800,944	2·078,928	2·396,558
16	1·604,706	1·872,981	2·182,875	2·540,352
17	1·652,848	1 947,900	2·292,018	2·692,773
18	1·702,433	2·025,817	2·406,619	2·854,339
19	1·753,506	2·106,849	2·526,950	3·025,600
20	1.806,111	2·191,123	2·653,298	3·207,135
21	1·860,295	2·278,768	2·785,962	3·399,564
22	1·916,103	2·369,919	2·925,261	3·603,537
23	1·973,587	2·464,716	3·071,524	3·819,750
24	2·032,794	2·563,304	3·225,101	4·048,935
25	2·093,778	2·665,836	3·386,356	4·291,871
26	2·156,592	2·772,470	3·555,673	4·549,383
27	2·221,289	2·883,369	3·733,456	4·822,346
28	2·287,928	2·998,703	3·920,129	5·111,687
29	2·356,566	3·118,651	4·116,136	5·418,388
30	2·427,262	3·243,398	4·321,942	5·743,491
31	2·500,080	3·373,133	4·538,039	6·088,101
32	2·575,083	3·508,059	4·764,941	6·453,386
33	2·652,335	3·648,381	5·003,189	6·840,590
34	2·731,905	3·794,316	5·253,348	7·251,025
35	2·813,862	3·946,089	5·516,015	7·686,087
36	2.898,278	4·103,933	5·791,816	8·147,252
37	2·985,227	4·268,090	6·081,407	8·636,087
38	3·074,783	4·438,813	6·385,477	9·154,252
39	3·167,027	4·616,366	6·704,751	9·703,507
40	3·262,038	4·801,021	7·039,989	10·285,718
41	3·359,899	4·993,061	7·391,988	10·902,861
42	3·460,696	5·192,784	7·761,588	11·557,033
43	3·564,517	5·400,495	8·149,667	12·250,455
44	3·671,452	5·616,515	8·557,150	12·935,482
45	3·781,596	5·841,176	8·985,008	13·764,611
46	3·895,044	6·074,823	9·434,258	14·590,487
47	4·011,895	6·317,816	9·905,971	15·465,917
48	4·132,252	6·570,528	10·401,270	16·393,872
49	4·256,219	6·833,349	10·921,333	17·377,504
50	4·383,906	7·106,683	11·467,400	18·420,154

TABLE II. SHOWING THE AMOUNT OF AN ANNUITY OF £1.

Yrs.	3 per cent.	4 per cent.	5 per cent.	6 per cent.
1	1·000,000	1·000,000	1·000,000	1·000,000
2	2·030,000	2·040,000	2·050,000	2·060,000
3	3·090,900	3·121,600	3·152,500	3·183,600
4	4·183,627	4·246,464	4·310,125	4·374,616
5	5·309,135	5·416,322	5·525,631	5·637,092
6	6·468,409	6·632,975	6·801,912	6·975,318
7	7·662,462	7·898,294	8·142,008	8·393,837
8	8·892,336	9·214,226	9·549,108	9·897,467
9	10·159,106	10·582,795	11·026,564	11·491,315
10	11·463,879	12·006,107	12·577,892	13·180,794
11	12·807,795	13·486,351	14·206,787	14·971,642
12	14·192,029	15·025,805	15·917,126	16·869,941
13	15·617,790	16·626,837	17·712,982	18·882,137
14	17·086,324	18·291,911	19·598,631	21·015,065
15	18·598,913	20·023,587	21·578,563	23·275,969
16	20·156,881	21·824,531	23·657,491	25·672,528
17	21·761,587	23·697,512	25·840,366	28·212,879
18	23·414,435	25·645,412	28·132,384	30·905,652
19	25·116,868	27·671,229	30·539,003	33·759,991
20	26·870,374	29·778,078	33·065,954	36·785,591
21	28·676,485	31·969,201	35·719,251	39·992,726
22	30·536,780	34·247,969	38·505,214	43·392,290
23	32·452,883	36·617,888	41·430,475	46·995,827
24	34·426,470	39·082,604	44·501,998	50·815,577
25	36·459,264	41·645,908	47·727,098	54·864,512
26	38·553,042	44·311,744	51·113,453	59·156,382
27	40·709,633	47·084,214	54·669,126	63·705,765
28	42·930,922	49·967,582	58·402,582	68·528,111
29	45·218,850	52·966,286	62·322,711	73·639,798
30	47·575,415	56·084,937	66·438,847	79·058,186
31	50·002,678	59·328,335	70·760,789	84·801,677
32	52·502,758	62·701,468	75·298,829	90·889,778
33	55·077,841	66·209,527	80·063,770	97·343,164
34	57·730,176	69·857,908	85·066,959	104·183,754
35	60·462,081	73·652,224	90·320,307	111·434,779
36	63·275,944	77·598,313	95·836,322	119·120,866
37	66·174,222	81·702,246	101·628,138	127·268,118
38	69·159,449	85·970,336	107·709,545	135·904,205
39	72·234,232	90·409,149	114·095,023	145·058,458
40	75·401,259	95·025,515	120·799,774	154·761,965
41	78·663,297	99·826,536	127·839,762	165·047,683
42	82·023,196	104·819,597	135·231,751	175·950,544
43	85·483,892	110·012,381	142·993,338	187·507,577
44	89·048,409	115·412,876	151·143,005	199·758,031
45	92·719,861	121·029,392	159·700,155	212·743,513
46	96·501,457	126·870,567	168·685,163	226·508,124
47	100·396,500	132·945,390	178·119,421	241·098,612
48	104·408,395	139·263,206	188·025,392	256·564,528
49	108·540,647	145·833,734	198·426,662	272·958,400
50	112·796,867	152·667,083	209·347,995	290·335,9[illegible]

TABLE III. SHOWING THE PRESENT VALUE OF AN ANNUITY OF £1.

Yrs.	3 per cent.	4 per cent.	5 per cent	6 per cent.
1	·970874	·961538	·952381	·943396
2	1·913470	1·886194	1·859410	1·833392
3	2·828612	2·775190	2·723248	2·673011
4	3·717099	3·629994	3·545950	3·465105
5	4·579708	4·451821	4·329476	4·212363
6	5·417192	5·242136	5·075691	4·917324
7	6·230284	6·002054	5·786372	5·582381
8	7·019693	6·732744	6·463211	6·209793
9	7·786110	7·435331	7·107820	6·801691
10	8·530204	8·110895	7·721733	7·360086
11	9·252625	8·760576	8·306412	7·886874
12	9·954005	9·385073	8·863249	8·383843
13	0·634956	9·985647	9·393570	8·852682
14	11·296074	10·563122	9·898638	9·294983
15	11·937936	11·118487	10·379655	9·712248
16	12·561103	11·652395	10·837767	10·105894
17	13·166119	12·165668	11·274064	10·477258
18	13·753514	12·659396	11·689585	10 827602
19	14·323800	13·133938	12·085319	11·158115
20	14·877476	13·590325	12·462208	11·469920
21	15·415025	14·029159	12·821150	11·764075
22	15·936918	14·451114	13·163000	12·041580
23	16·443610	14·856840	13·488571	12·303377
24	16·935544	15·246961	13·798639	12·550356
25	17·413150	15·622078	14·093942	12·783355
26	17·876845	15·982767	14·375183	13·003165
27	18·327034	16·329584	14·643031	13·210533
28	18·764111	16·663061	14·898125	13·406163
29	19·188457	16·983712	15·141071	13·590720
30	19·600441	17·292031	15·372448	13·764830
31	20·000431	17·588491	15·592807	13·929085
32	20·388768	17·873549	15·802673	14·084042
33	20·765794	18·147643	16·002546	14·230228
34	21·131839	18·411195	16·192901	14·368140
35	21·487222	18·664610	16·374191	14·498245
36	21·832254	18·908279	16·546848	14·620986
37	22·167237	19·142576	16·711284	14·736779
38	22·492463	19·367861	16·867889	14·846018
39	22·808217	19·584482	17·017037	14·949074
40	23·114774	19·792771	17·159083	15·046296
41	23·412402	19·993049	17·294365	15·138015
42	23·701361	20·185624	17·423205	15·224542
43	23·981904	20·370792	17·545909	15·306172
44	24·254276	20·548838	17·662770	15·383181
45	24·518715	20·720036	17·774067	15·455831
46	24·775452	20·884650	17·880064	15·524369
47	25·024711	21·042933	17·981013	15·589027
48	25·266710	21·195128	18·077155	15·650025
49	25·501660	21·341469	18·168719	15·707571
50	25·729767	21·482182	18·255923	15·761859

TABLE IV. SHOWING THE VALUE OF AN ANNUITY ON A SINGLE LIFE.

Ages.	4 per cent.	5 per cent.	6 per cent.	Ages.	4 per cent.	5 per cent.	6 per cent
1	13·465	11·563	10·107	49	11·475	10·443	9·563
2	15·633	13·420	11·724	50	11·264	10·269	9·417
3	16·462	14·135	12·348	51	11·057	10·097	9·273
4	17·010	14·613	12·769	52	10·849	9·925	9·129
5	17·248	14·827	12·962	53	10·637	9·748	8·980
6	17·482	15·041	13·156	54	10·421	9·567	8·827
7	17·611	15·166	13·275	55	10·201	9·382	8·670
8	17·662	15·226	13·337	56	9·977	9·193	8·509
9	17·625	15·210	13·335	57	9·749	8·999	8·343
10	17·523	15·139	13·285	58	9·516	8·801	8·173
11	17·393	15·043	13·212	59	9·280	8·599	7·999
12	17·251	14·937	13·130	60	9·039	8·392	7·820
13	17·103	14·826	13·044	61	8·795	8·181	7·637
14	16·950	14·710	12·953	62	8·547	7·966	7·449
15	16·791	14·588	12·857	63	8·291	7·742	7·253
16	16·625	14·460	12·755	64	8·030	7·514	7·052
17	16·462	14·334	12·655	65	7·761	7·276	6·841
18	16·309	14·217	12·562	66	7·488	7·034	6·625
19	16·167	14·108	12·477	67	7·211	6·787	6·405
20	16·033	14·007	12·398	68	6·930	6·536	6·179
21	15·912	13·917	12·329	69	6·647	6·281	5·949
22	15·797	13·833	12·265	70	6·361	6·023	5·716
23	15·680	13·746	12·200	71	6·075	5·764	5·479
24	15·560	13·658	12·132	72	5·790	5·504	5·241
25	15·438	13·567	12·063	73	5·507	5·245	5·004
26	15·312	13·473	11·992	74	5·230	4·990	4·769
27	15·184	13·377	11·917	75	4·962	4·744	4·542
28	15·053	13·278	11·841	76	4·710	4·511	4·326
29	14·918	13·177	11·763	77	4·457	4·277	4·109
30	14·781	13·072	11·682	78	4·197	4·035	3·884
31	14·639	12·965	11·598	79	3·921	3·776	3·641
32	14·495	12·854	11·512	80	3·643	3·515	3·394
33	14·347	12·740	11·423	81	3·377	3·263	3·156
34	14·195	12·623	11·331	82	3·122	3·020	2·926
35	14·039	12·502	11·236	83	2·887	2·797	2·713
36	13·880	12·377	11·137	84	2·708	2·627	2·551
37	13·716	12·249	11·035	85	2·543	2·471	2·402
38	13·548	12·116	10·929	86	2·393	2·328	2·266
39	13·375	11·979	10·819	87	2·251	2·193	2·138
40	13·197	11·837	10·705	88	2·131	2·080	2·031
41	13·018	11·695	10·589	89	1·967	1·924	1·882
42	12·838	11·551	10·473	90	1·758	1·723	1·689
43	12·657	11·407	10·356	91	1·474	1·447	1·422
44	12·472	11·258	10·235	92	1·171	1·153	1·136
45	12·283	11·105	10·110	93	0·827	0·816	0·806
46	12·089	10·947	9·980	94	0·530	0·524	0·518
47	11·890	10·784	9·846	95	0·240	0·238	0·236
48	11·685	10·616	9·707	96	0·000	0·000	0·000

TABLE V. SHOWING THE PRESENT VALUE OF AN ANNUITY OF £1. ON THE JOINT CONTINUANCE OF TWO LIVES.

Ages.		4 per cent.	5 per cent.	6 per cent.
	5	13·591	11·984	10·691
	10	13·943	12·315	11·010
	15	13·479	11·954	10·716
	20	12·993	11·561	10·391
	25	12·633	11·281	10·170
	30	12·220	10·959	9·913
	35	11·732	10·572	9·602
5	40	11·150	10·102	9·219
	45	10·500	9·571	8·778
	50	9·742	8·941	8·248
	55	8·931	8·256	7·665
	60	8·011	7·466	6·982
	65	6·963	6·546	6·171
	70	5·768	5·472	5·209
	75	4·357	4·362	4·181
—	—	—	—	—
	10	14·277	12·665	11·345
	15	13·641	12·302	11·048
	20	13·355	11·906	10·719
	25	12·998	11·627	10·497
	30	12·586	11·304	10·239
	35	12·098	10·916	9·925
	40	11·513	10·442	9·537
10	45	10·851	9·900	9·088
	50	10·085	9·260	8·548
	55	9·256	8·560	7·951
	60	8·314	7·750	7·250
	65	7·236	6·803	6·414
	70	6·008	5·700	5·418
	75	4·725	4·522	4·350
	80	3·517	3·395	3·281
—	—	—	—	—
	15	13·411	11·960	10·767
	20	12·961	11·585	10·453
	25	12·630	11·324	10·244
	30	12·246	11·021	10·001
	35	11·787	10·655	9·703
	40	11·234	10·205	9·333
15	45	10·607	9·690	8·905
	50	9·872	9·076	8·386
	55	9·077	8·403	7·812
	60	8·170	7·622	7·135
	65	7·127	6·705	6·325
	70	5·933	5·631	5·355
	75	4·695	4·495	4·310
	80	3·492	3·372	3·259
—	—	—	—	—
	20	12·535	11·232	10·156
	25	12·229	10·989	9·960
	30	11·873	10·707	9·732
	35	11·445	10·393	9·451
	40	10·924	9·937	9·100
	45	10·330	9·448	8·692
20	50	9·630	8·861	8·195
	55	8·869	8·216	7·643
	60	7·995	7·463	6·990
	65	6·986	6·576	6·205
	70	5·826	5·532	5·262
	75	4·619	4·424	4·242
	80	3·443	3·325	3·214
—	—	—	—	—
	25	11·944	10·764	9·776
	30	11·618	10·499	9·561
	35	11·217	10·175	9·295
	40	10·725	9·771	8·960
	45	10·160	9·304	8·569
	50	9·483	8·739	8·069
25	55	8·754	8·116	7·555
	60	7·906	7·383	6·919
	65	6·920	6·515	6·151
	70	5·780	5·489	5·223
	75	4·589	4·396	4·216
	80	3·425	3·308	3·198

Ages.		4 per cent.	5 per cent.	6 per cent.
	30	11·313	10·255	9·360
	35	10·948	9·954	9·112
	40	10·490	9·576	8·795
	45	9·939	9·135	8·424
	50	9·321	8·596	7·965
30	55	8·619	7·999	7·453
	60	7·802	7·292	6·857
	65	6·844	6·447	6·069
	70	5·729	5·442	5·180
	75	4·557	4·365	4·185
	80	3·406	3·290	3·181
—	—	—	—	—
	35	10·612	9·680	8·883
	40	10·196	9·331	8·589
	45	9·706	8·921	8·248
	50	9·110	8·415	7·809
35	55	8·448	7·849	7·322
	60	7·669	7·174	6·732
	65	6·747	6·360	6·010
	70	5·663	5·382	5·125
	75	4·516	4·327	4·152
	80	3·383	3·268	3·160
—	—	—	—	—
	40	9·820	9·016	8·322
	45	9·381	8·643	8·063
	50	8·834	8·177	7·607
40	55	8·221	7·651	7·146
	60	7·490	7·015	6·590
	65	6·614	6·240	5·901
	70	5·571	5·298	5·047
	75	4·457	4·272	4·101
	80	3·349	3·236	3·130
—	—	—	—	—
	45	8·990	8·312	7·716
	50	8·503	7·891	7·353
45	55	7·948	7·411	6·935
	60	7·274	6·822	6·418
	65	6·453	6·094	5·769
	70	5·460	5·195	4·953
	75	4·386	4·206	4·040
	80	3·308	3·191	3·093
—	—	—	—	—
	50	8·081	7·522	7·030
	55	7·593	7·098	6·658
	60	6·989	6·568	6·189
50	65	6·236	5·897	5·594
	70	5·306	5·054	4·825
	75	4·235	4·112	3·951
—	80	3·247	3·140	3·039
—	—	—	—	—
	55	7·179	6·735	6·336
	60	6·659	6·272	5·924
55	65	5·986	5·671	5·384
	70	5·132	4·893	4·674
	75	4·171	4·006	3·852
—	80	3·180	3·076	2·978
—	—	—	—	—
	60	6·426	5·888	5·379
	65	5·658	5·372	5·112
60	70	4·900	4·680	4·478
	75	4·021	3·866	3·721
	80	3·092	2·992	2·899
—	—	—	—	—
	65	5·201	4·960	4·736
65	70	4·573	4·378	4·198
	75	3·806	3·665	3·533
	80	2·965	2·873	2·786
—	—	—	—	—
	70	4·087	3·930	3·781
70	75	3·471	3·347	3·226
	80	2·757	2·675	2·598
—	—	—	—	—
	75	3·015	2·917	2·827
75	80	2·448	2·381	2·323
—	—	—	—	—
80	80	2·069	2·018	1·969

ANSWERS TO EXERCISES.

NUMERATION.

Ex. 1. Twenty four. 2. One hundred and forty four. 3. Three hundred and sixty five. 4. One thousand. 5. One thousand, seven hundred, and twenty eight. 6. Two thousand, two hundred, and forty. 7. Nine thousand, seven hundred, and ninety. 8. Thirty seven thousand, and forty eight. 9. Thirty thousand and nine. 10. Four millions, fifty five thousand, and seventy. 11. Three hundred thousand, four hundred and five. 12. Seventy nine millions, five hundred and three thousand, and forty six. 13.* Eight billions, five millions, six hundred thousand, four hundred, and eighty. 14. Five hundred and fifty seven millions, two hundred and ninety thousand. 15. Six hundred and eighty millions, and forty two. 16. Ninety three millions, ninety thousand, and ninety three. 17. One hundred and thirteen thousand, three hundred, and fifty five. 18. Seven hundred and eighty five thousand, three hundred, and ninety eight. 19. Seven millions, thirty thousand, four hundred, and sixty two. 20. Twenty four millions, nine hundred and two thousand, four hundred, and ninety. 21.* Nine billions, three millions, eight thousand, and five. 22. Forty millions, six hundred and fifty seven thousand, two hundred. 23.* One hundred billions, and one thousand. 24.* Sixty trillions, six hundred and six billions, sixty millions, seven hundred thousand, seven hundred, and seven. 25.* One hundred and two trillions, thirty billions, four hundred and five millions, sixty thousand, seven hundred and eight. 26*. Nine hundred and one trillions, one billion, one hundred and one millions, two hundred and one thousand, three hundred and one. 27.* Two hundred billions, thirty millions, forty thousand, five hundred and thirty eight. 28.* Seventy three quadrillions, eight hundred and twenty trillions, seven hundred and sixty billions, five millions, one hundred and ninety two thousand, six hundred and forty five. 29.*

According to the common Notation :

* Ex. 13. Eight thousand and five millions, six hundred thousand, four hundred and eighty. 21. Nine thousand and three millions, eight thousand and five. 23. One hundred thousand millions, one thousand. 24. Sixty billions, six hundred and six thousand, and sixty millions, seven hundred thousand, seven hundred and seven. 25. One hundred and two billions, thirty thousand, four hundred and five millions, sixty thousand seven hundred and eight. 26. Nine hundred and one billions, one thousand one hundred and one millions, two hundred and one thousand, three hundred and one. 27. Two hundred thousand and thirty millions, forty thousand, five hundred and thirty eight. 28. Seventy three thousand eight hundred and twenty billions, seven hundred and sixty thousand and five millions, one hundred and ninety two thousand six hundred and forty five. 29. Ten thousand and one billions, ten millions, one hundred. 30. Forty thousand and fifty billions, sixty thousand seven hundred millions, eight hundred and nine thousand. 31. Seven quadrillions, nine hundred and forty six thousand two hundred and eighty nine trillions, six thousand four hundred billions, three hundred thousand and forty seven millions, two hundred and sixty four thousand, seven hundred and ninety one.

Ten quadrillions, one trillion, ten millions, one hundred. 30.* Forty quadrillions, fifty trillions, sixty billions, seven hundred millions, eight hundred and nine thousand. 31.* Seven septillions, nine hundred and forty six sextillions, two hundred and eighty nine quintillions, six quadrillions, four hundred trillions, three hundred billions, forty seven millions, two hundred and sixty four thousand, seven hundred and ninety one.

NOTATION.

Ex. 1. 52	Ex. 9. 5070	Ex. 17. 1010200000*
2. 313	10. 4504	18. 11002000000000*
3. 504	11. 20084	19. 1050000
4. 1024	12. 650090	20. 1000200000*
5. 2048	13. 7007010	21. 1000000700000*
6. 1815	14. 64000300	22. 1000000019000000*
7. 7854	15. 11002000	23. 70000010088*
8. 3008	16. 110020000	24. 900068000020*

25. 37000000, 69000000, 95000000, 145000000 494000000, 907000000, 1810000000.

SIMPLE ADDITION.

Ex. 1. 21555	Ex. 13. 10871894825
2. 206343	14. 145322086
3. 1508939	15. 207305
4. 1383458	16. In 1727.
5. 21225092	17. In 1399.
6. 904388	18. 1527560, 332281
7. 121888988	352853, 937000
8. 27457989	19. 136,570,103
9. 86171735	20. 3371
10. 20967445	21. 36633 miles.
11. 463140294	22. 1,451,678
12. 445684135	23. 11,261,437

Ex. 24.

	Acres	Population
Ulster	3,201,200	2,001,966
Leinster	2,792,550	1,785,702
Munster	3,377,150	2,005,363
Connaught	2,630,300	1,053,918
Total	12,001,200	6,846,949

Ex. 25. 2,093,456

According to the common Notation:

* Ex. 17. 1000010200000
18. 100010000000000000
20. 1000000200000
21. 1000000000000700000
22. 1000000000000000019000000
23. 70000000010088
24. 900000068000020

SIMPLE SUBTRACTION.

Ex.1. 13031
2. 15708
3. 17368
4. 32131
5. 590731
6. 8285367
7. 3209877
8. 763544529
9. 10101001
10. 333232333
11. 7468053687
12. 327504427
13. 99579930
14. 89998999
15. 3995996

Ex.16. 2997000
17. 991
18. 299997
19. 1011924
20. 861928
21. 75
22. 5747
23. 11895
24. £39958
61848
208926
38651¼
25. 1127757
8808
4447

14816
12250
22926
Ex.26. £200060
1210235
27. £308755
1092766
28. 2282155lbs.

29. 1202/281
833 236
728 234
609 233
571 224
533 222
525 214
494 210
413 205
412 181
384 170
371 138
332 43
327 23
307 19
294 9

SIMPLE MULTIPLICATION.

Ex.1. 432
2. 3180
3. 3645
4. 5616
5. 11193
6. 1000
7. 27414
8. 80388

Ex.9. 20748
10. 18160
11. 1615040
12. 161504000
13. 573440000
14. 7068600
15. 49152000

SIMPLE DIVISION.

Ex.1. 156950
2. 457441¼
3. 853196⅘
4. 630341¼
5. 1023648¼

Ex.6. 3432416⅘
7. 8288202⅔
8. 4840788 7/11
9. 4106265 9/15

COMPOUND ADDITION.

	£	s.	d.
Ex.1.	3852	15	10
2.	36	12	3¼
3.	21849	18	2¼
4.	48	0	6¼
5.	2431	9	2¾
6.	3765	15	1½
7.	1861	19	10½
8.	19011	17	7¼
9.	1503	4	6¼
10.	10	4	6
11.	2612	8	2½
12.	331	10	10½
13.	515	1	2
14.	5211	11	9
15.	3169	8	6¼

	£	s.	d.
Ex.16.	4281	0	9¾
17.	6219	7	1¾
18.	20351	16	8½

19. 224c. 1q. 11lbs.
20. 97lbs. 1oz. 15drs.
21. 383 tons. 8 cwt.
22. 362lbs.
23. 693 cwt. 0q. 23lbs.
24. 616 cwt. 2q. 9lbs.
25. 65lbs.
26. 429t. 17c. 2q. 20lbs.
27. 468c. 3q. 11lbs.
28. 517a. 0r. 9p.
29. 1271a. 3r. 4p.

COMPOUND SUBTRACTION.

	£	s.	d.
Ex. 1.	10	8	6¼
2.	451	1	9⅓
3.	103	14	7
4.	263	12	7¾
5.	7	2	4½
6.	98	17	3
7.	324	8	3½
8.	669	6	5¼

	c.	q.	lbs.
9.	4	1	8
10.	5	1	11
11.	25	2	15
12.	9t. 14	3	14

	d.	h.	min.	sec.
13.	3	6	13	46

	d.	h.	min.	sec.
Ex. 14.	24	9	50	·30
15.	6°	57′	12″	
	2	39	43	
	1	50	11	
	2	36	57	
	3	58	26	
16.	35	3	30	
17.	22	3	15	
18.	1	15	32	

	Days	h.	m.
19.	136	17	33
	140	13	20
	321	17	·22
	3645	14	56
	.6426	11	24
	19978	16	9

COMPOUND MULTIPLICATION.

	£	s.	d.
Ex. 1.	7	9	7
2.	5	13	2¼
3.	3	15	9
4.	7	11	8
5.	22	15	1½
6	7	19	3

	£	s.	d.
Ex. 7.	23	10	0
8.	60	0	0
9.	7	18	6½
10.	103	4	4
11.	47	15	6
12.	21	19	0

COMPOUND DIVISION.

	£	s.	d.	Rem.
Ex. 1.	5	5	10¼	
2.	0	18	10¼	...¼d.
3.	1	1	5¼	...¾d.
4.	1	17	10¾	...⅓d.
5.	1	8	10½	...1d.
6.	0	5	7½	...1d.
7.	0	9	5¼	...1d.
8.	0	13	1¼	...¼d.

	£	s.	d.	Rem.
Ex. 9.	0	15	4	...2d.
10.	0	10	3	
11.	1	13	5¼	
12.	0	3	4	
13.	0	2	10¼	...¼d
14.	0	3	9	
15.	0	3	3	

FINIS.

p. 27 Places of dividend + [illegible]

[illegible]

YA 02437

p 72 [illegible]

p 107 9.7

p 176 [illegible] = [illegible]

152

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